

大阪公立大学

Development of the Reconstruction Procedure of the Fluorescence detector Array of Single-pixel Telescopes for measuring Ultra-High Energy Cosmic Rays

極高エネルギー宇宙線の測定へ向けた新型大気蛍光望遠鏡アレイの再構成手法の開発

Osaka Metropolitan University, Graduate School of Science

Fraser Bradfield

2026/02/20

Adapted from PhD Defense



Contents

Background

- Cosmic rays and extensive air showers
- Current largest cosmic ray experiments
- The Fluorescence detector Array of Single-pixel Telescopes **(FAST)**

Research motivation

Results

- Top-down reconstruction improvements
- First Guess I – Neural Networks
- Coincidence event data analysis
- First cosmic ray energy spectrum and mass composition with FAST

Cosmic Rays and Extensive Air Showers

Cosmic Rays: Overview

- Cosmic rays are **extremely** energetic particles from outer space ($10^9 \sim 10^{20}$ eV)
- Accelerated in extreme astrophysical environments
- Cosmic rays with energy $E > 10^{18.5}$ eV are known as **“Ultra-High-Energy Cosmic Rays” (UHECRs)**

The Big 3 Questions of UHECRs

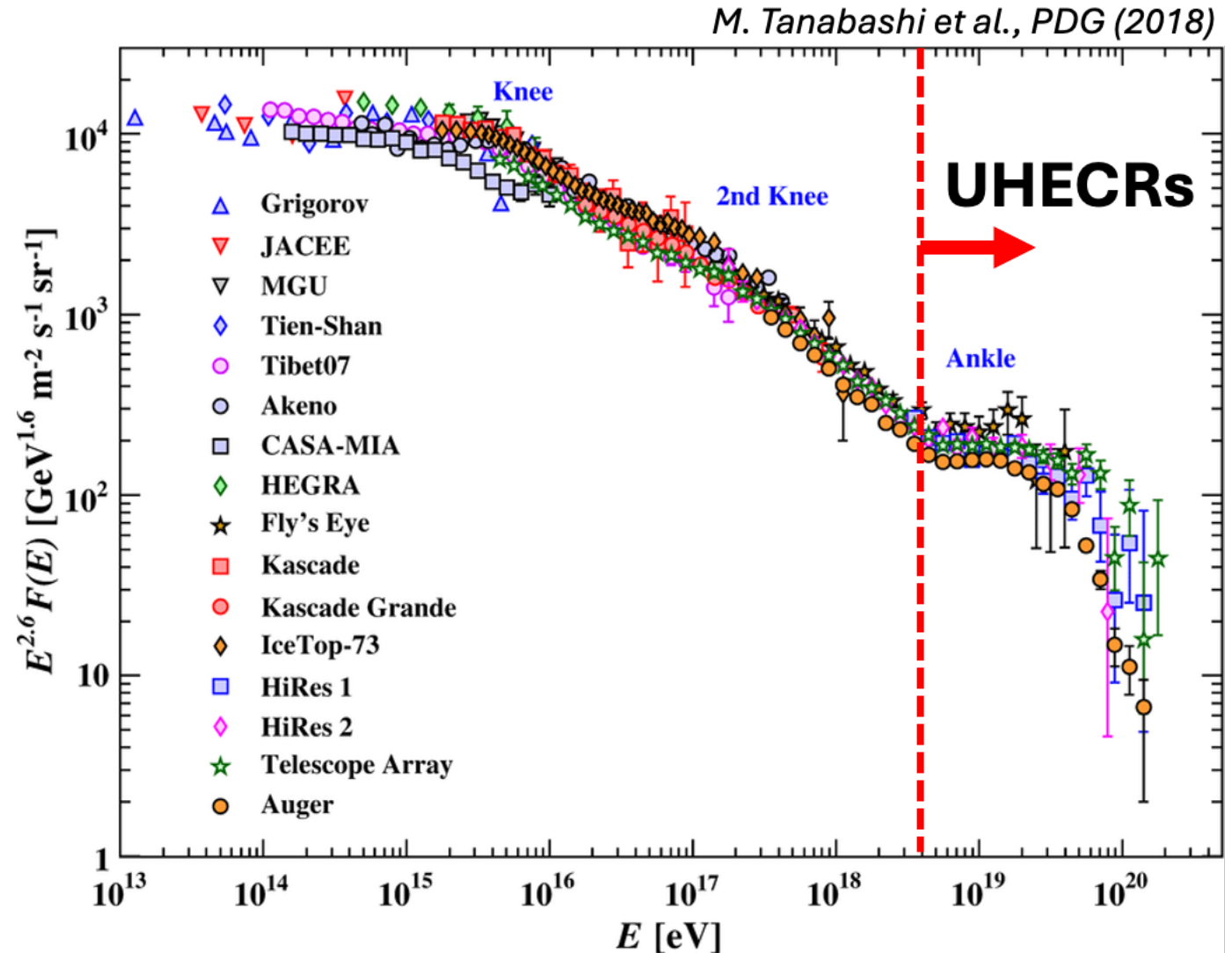
- Where do they come from?
- How are they accelerated to such high energies?
- What types of nuclei are they?

Cosmic Rays: Energy spectrum

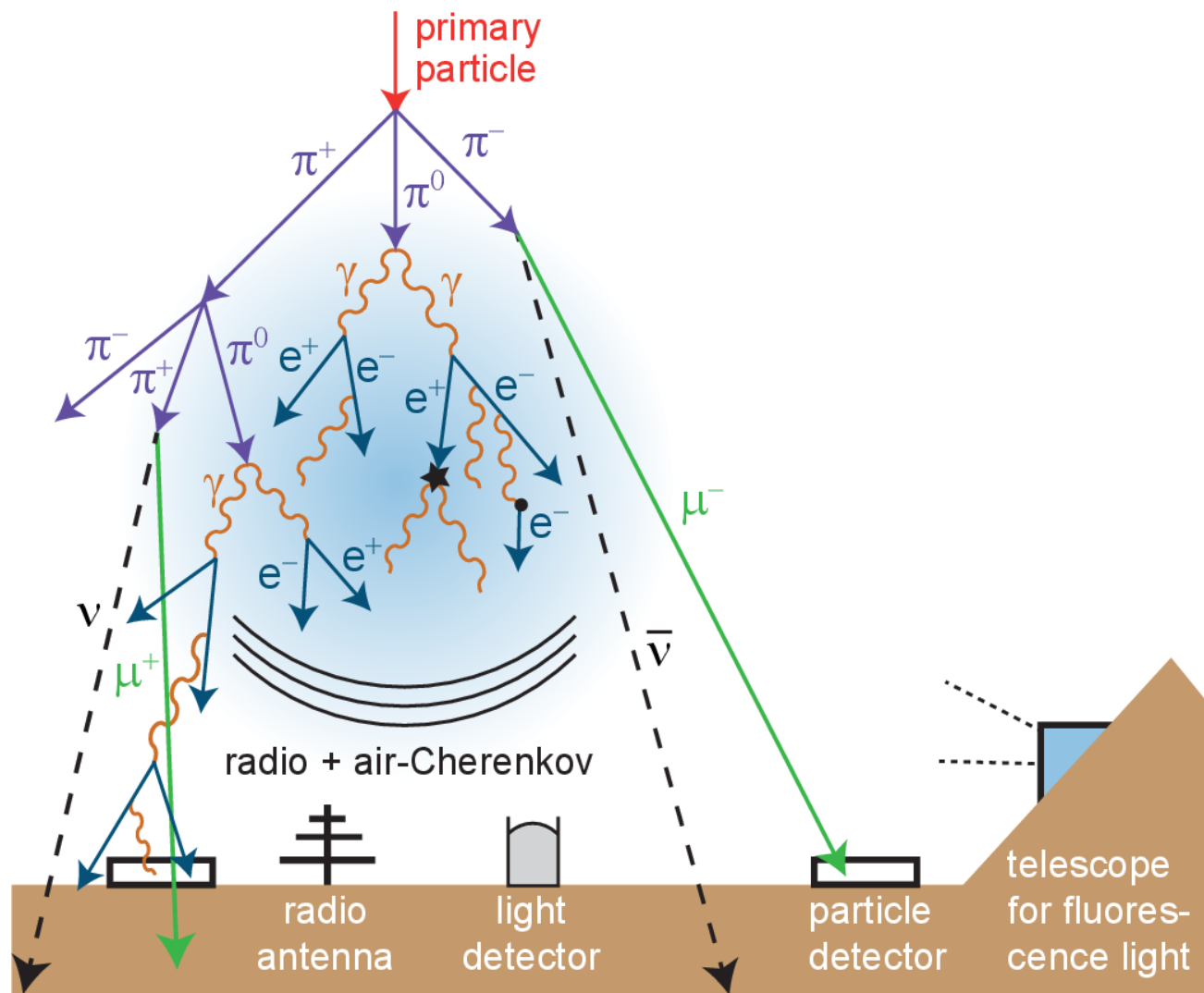
- Flux of cosmic rays arriving at Earth follows a **power law**

$$\frac{dN}{dE} \propto E^{-\gamma}$$

- For UHECRs, **less than 1 particle / km² / year**
- At FAST, we want to measure the energy spectrum above 10²⁰ eV with large statistics



Extensive Air Showers: Overview

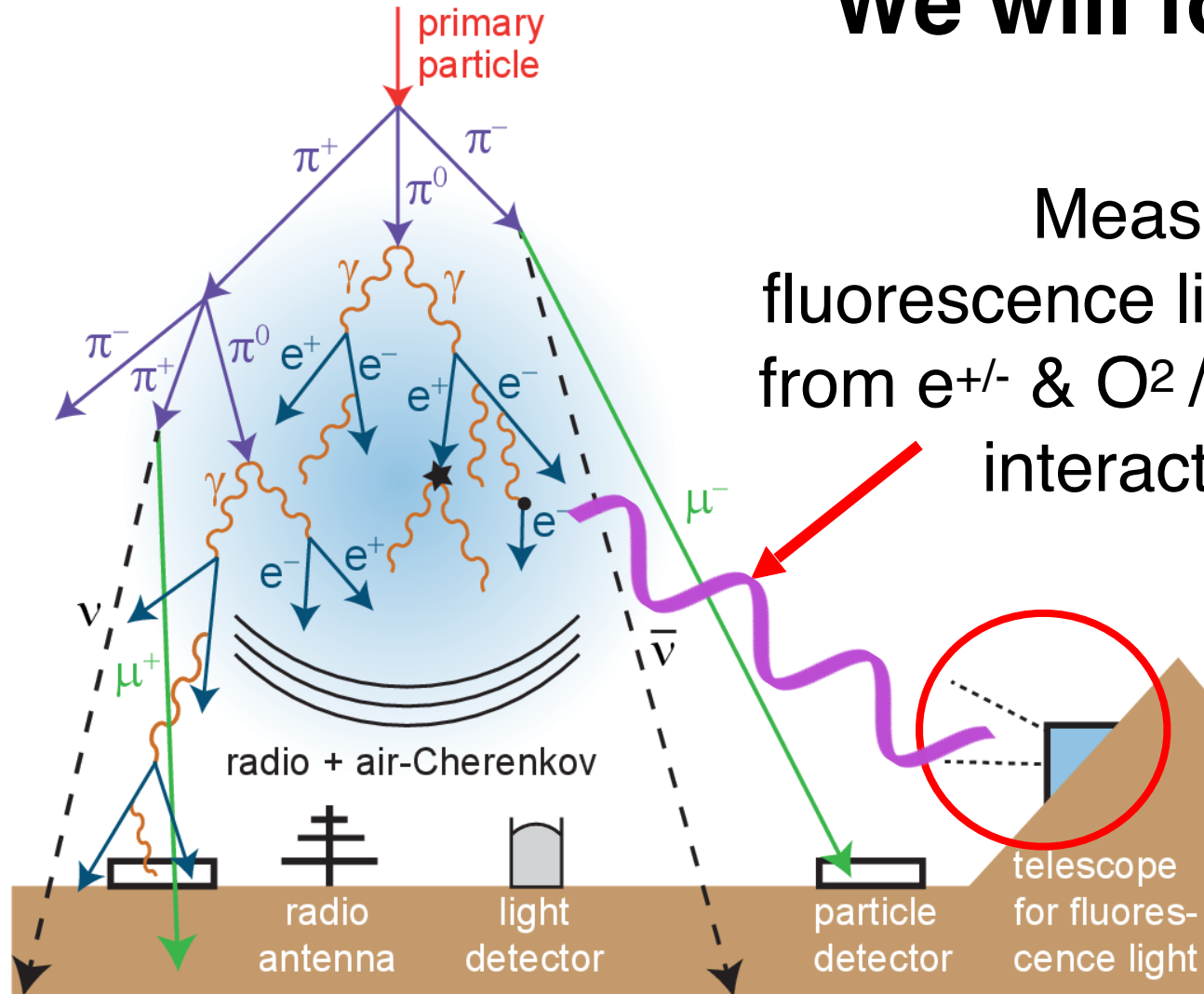


Frank Schröder (2017)

- Cascade of particles generated from a cosmic ray interacting with an air nucleus
- Three main types of particles
 - Muons & neutrinos
 - Hadrons
 - $e^\pm$, γ
- Primary method of detecting cosmic rays above $\sim 10^{15}$ eV

Extensive Air Showers: Detectors

We will focus on fluorescence detection!



Frank Schröder (2017)



Extensive Air Showers: Parameters

Geometry:

Zenith (θ)

Azimuth (ϕ)

Core location (x, y)

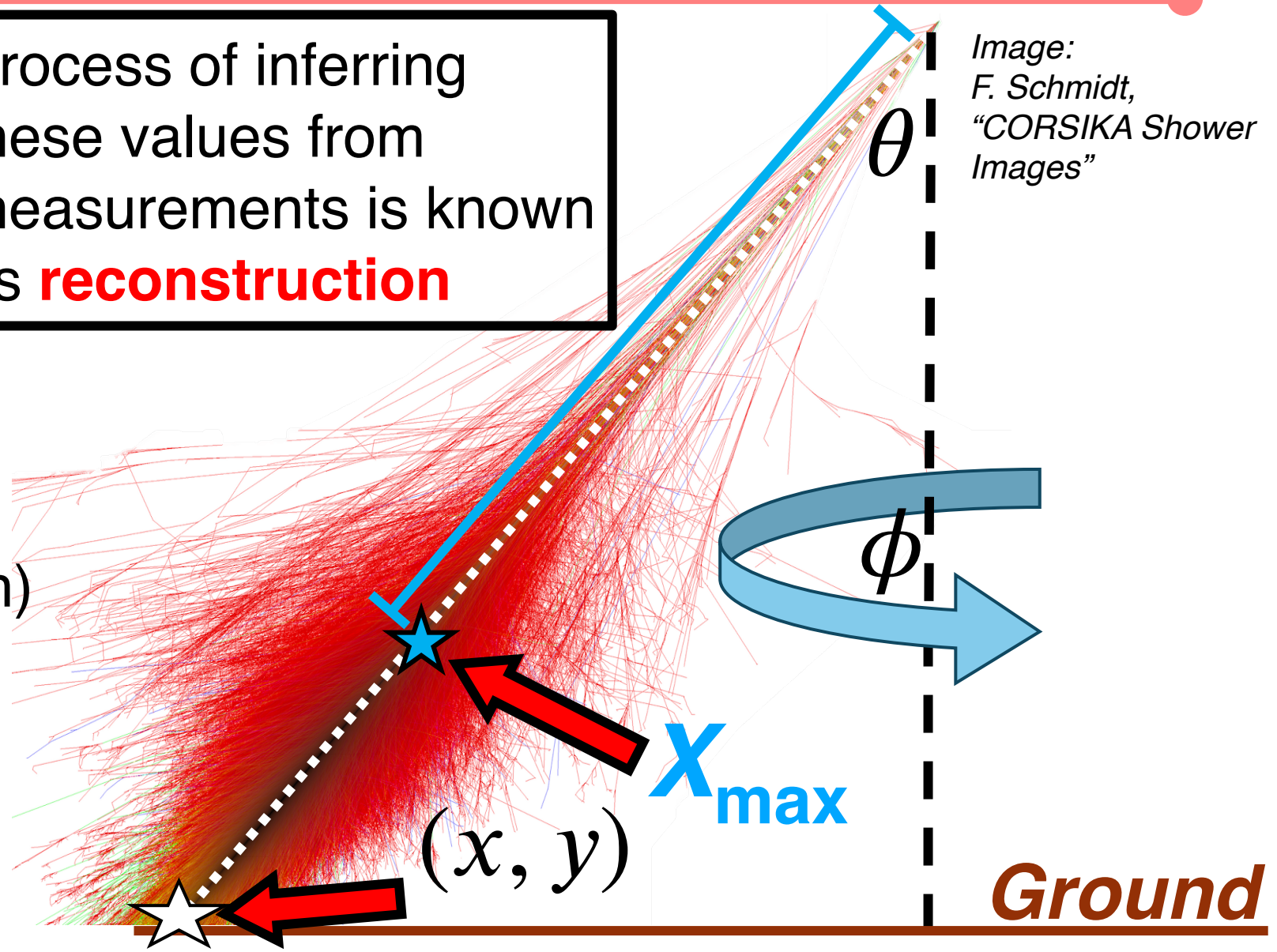
Process of inferring these values from measurements is known as **reconstruction**

$X_{\max}$:

Location of max # of particles (atmos. depth)

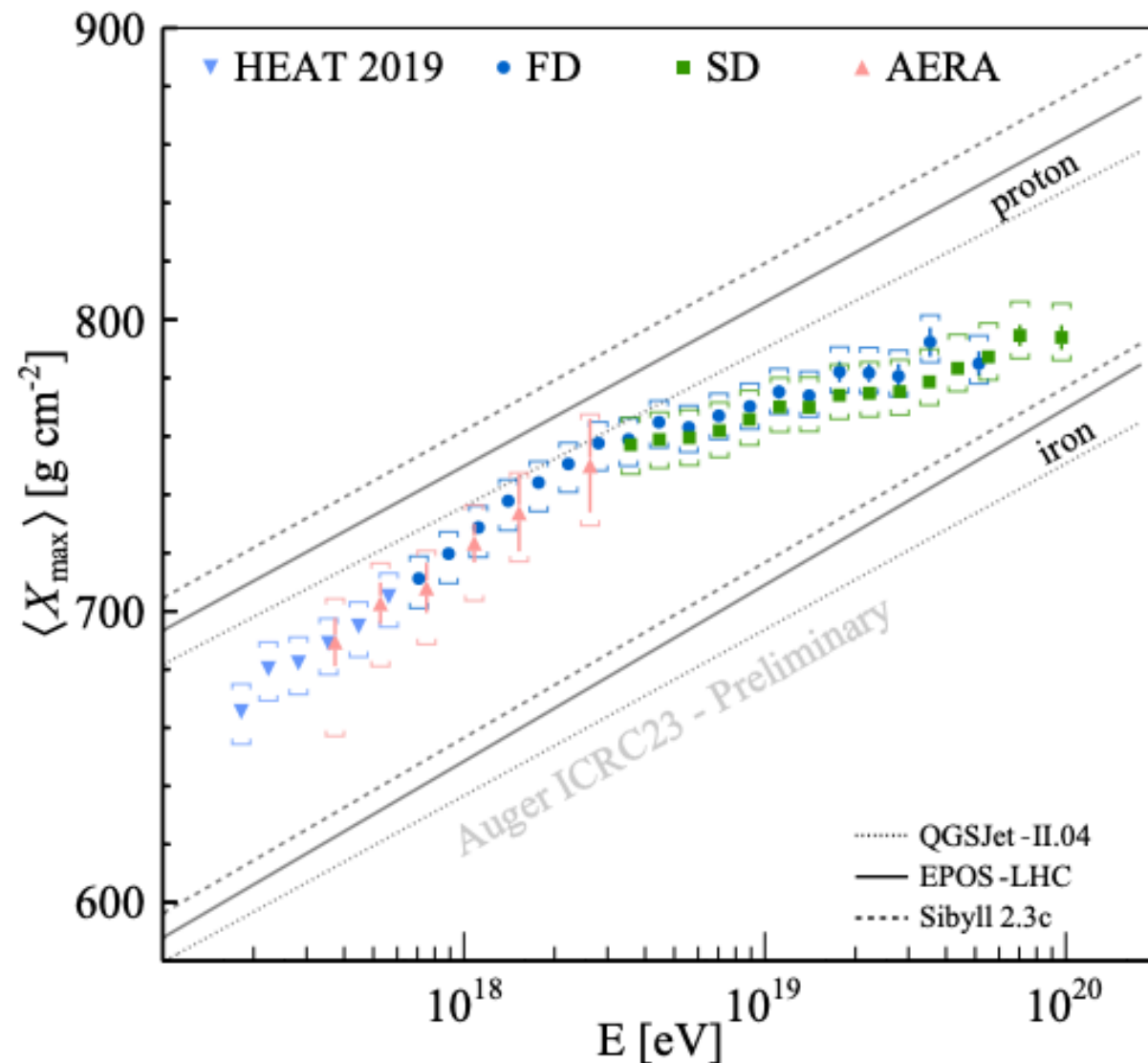
Energy:

Initial cosmic ray energy



$X_{\max}$ & Mass Composition

- “Mass composition” = what nuclei make up observed cosmic rays
- $\langle X_{\max} \rangle$ is a proxy for mass composition
- $\langle X_{\max} \rangle$ is energy dependent
 - Lighter nuclei around $10^{18.5} - 10^{19}$ eV
 - Heavier nuclei at the highest energies...
- Cosmic ray trajectories are bent by magnetic fields $\propto Z$ –
 - **Important to know mass composition!**



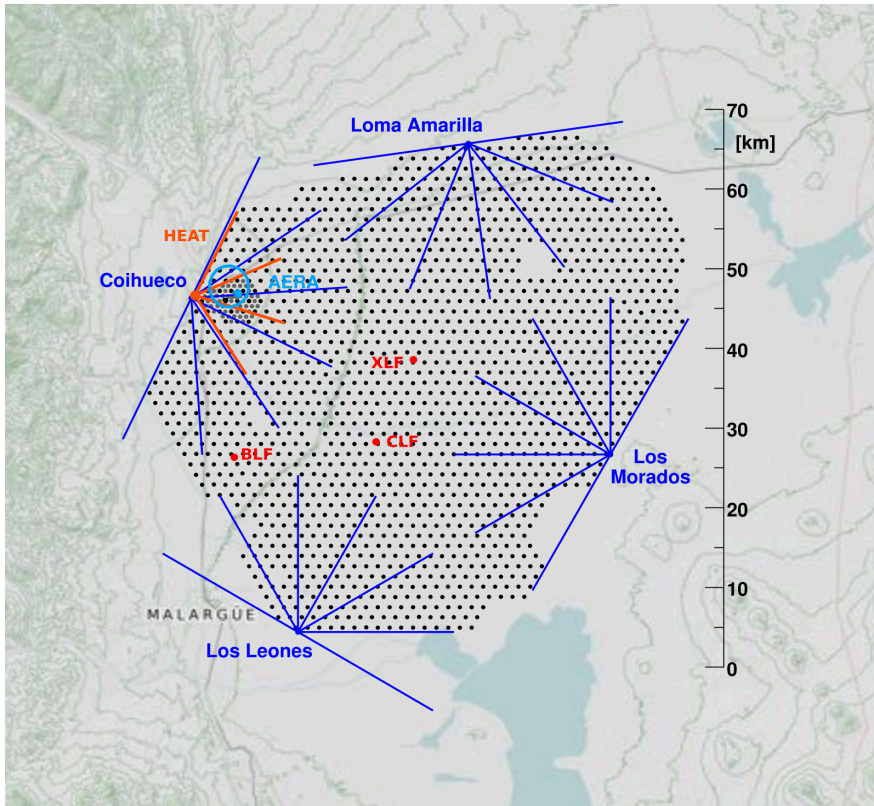
Eric Mayotte (2023)

Current Largest Experiments

Current Largest Experiments

The Pierre Auger Observatory:

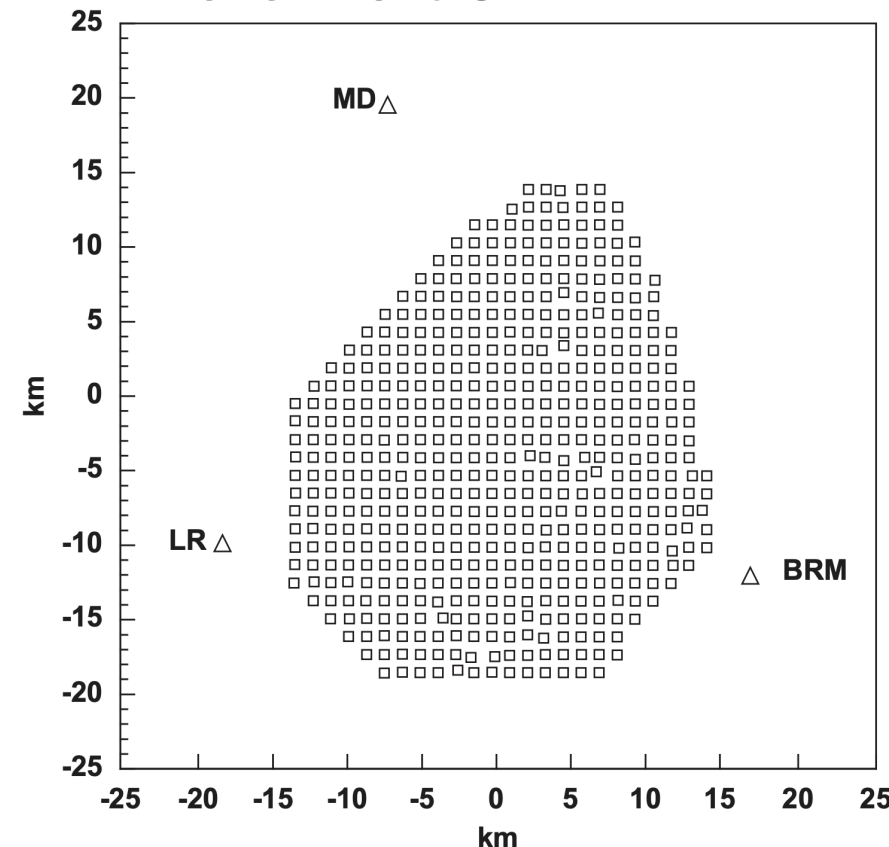
- Located in Malargüe, Argentina
- Area $\sim 3000 \text{ km}^2$
- Refer to as **Auger**



*The Pierre Auger
Collaboration*

The Telescope Array experiment:

- Located in Utah, USA
- Area $\sim 700 \text{ km}^2$
- Refer to as **TA**



*The Telescope
Array
Collaboration*

Current Largest Experiments

The Pierre Auger Observatory:

- Located in Malargüe, Argentina
- Area $\sim 3000 \text{ km}^2$
- Refer to as **Auger**

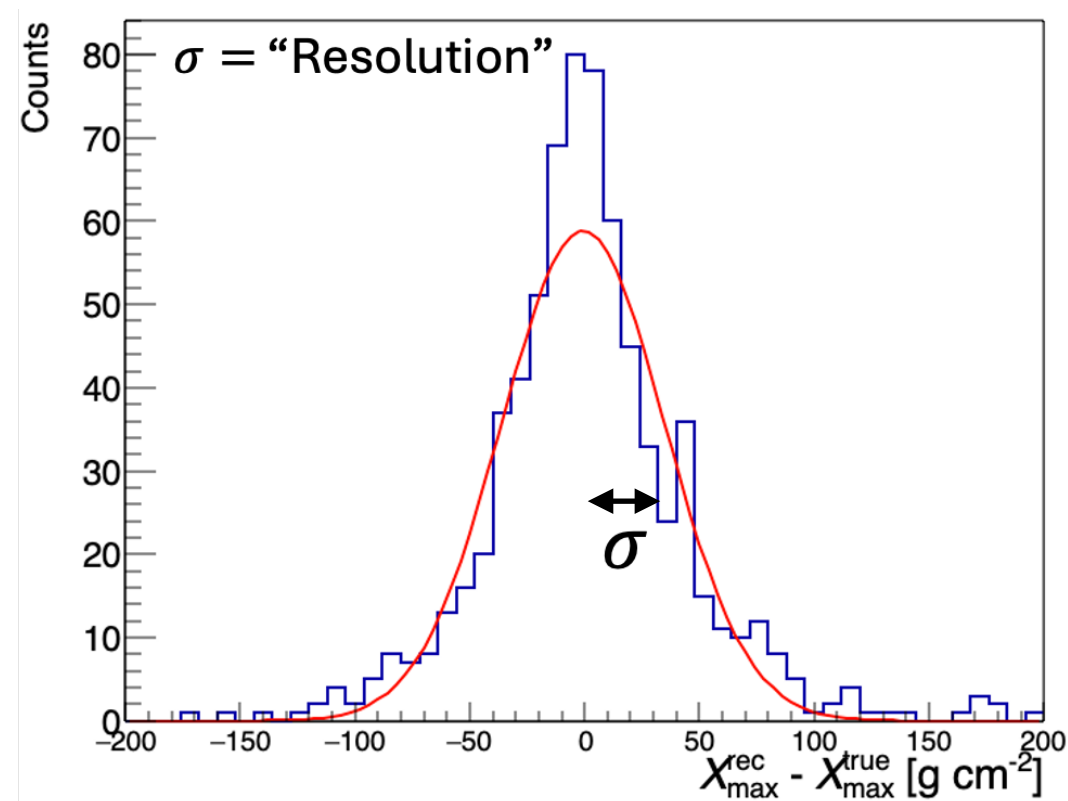
The Telescope Array experiment:

- Located in Utah, USA
- Area $\sim 700 \text{ km}^2$
- Refer to as **TA**

Auger / TA Resolutions:

$X_{\text{max}} \sim 25 \text{ g/cm}^2$

Energy $\sim 10\%$



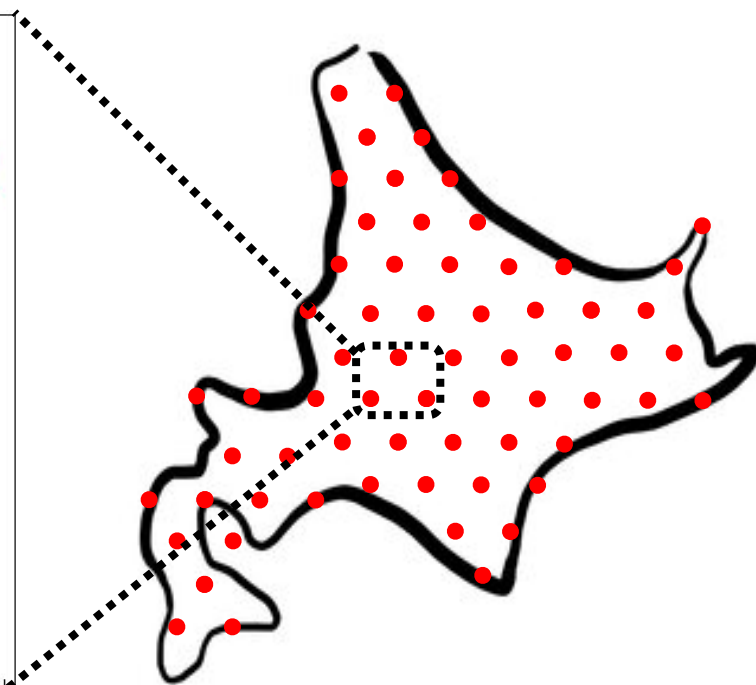
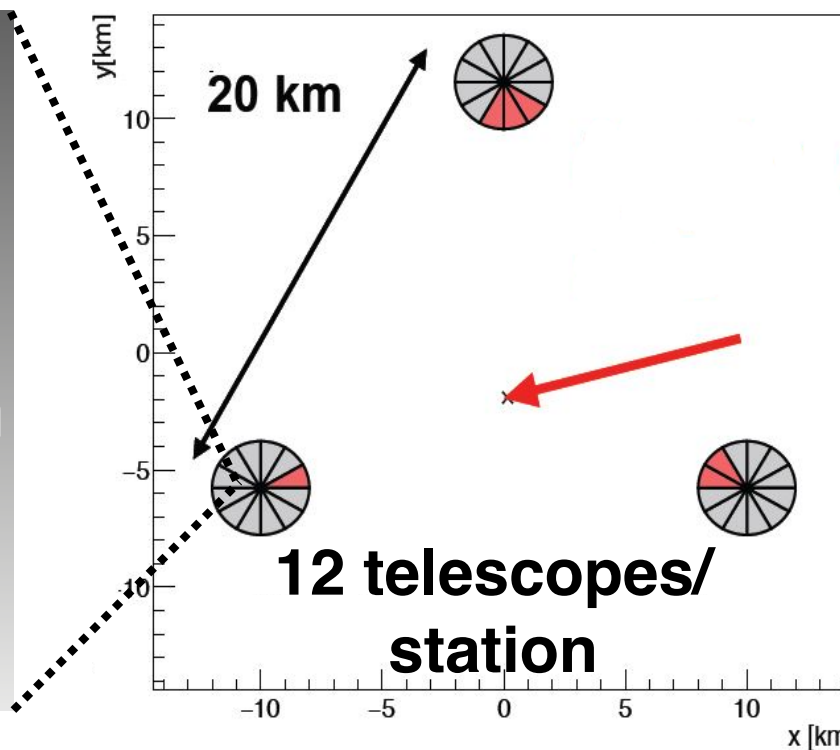
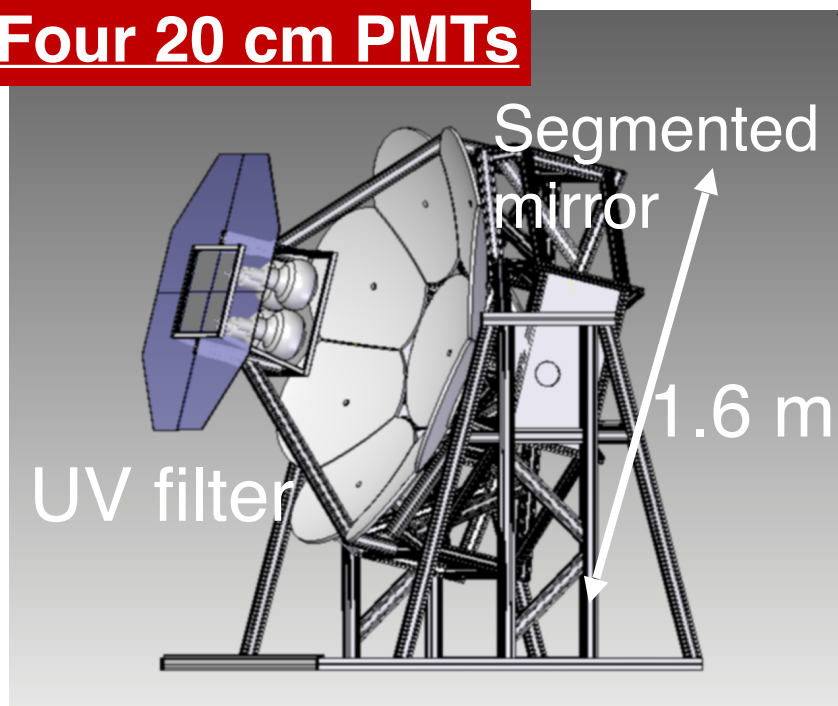
The Fluorescence detector Array of Single-pixel Telescopes

FAST

(The Fluorescence detector Array of Single-pixel Telescopes)

- Next generation cosmic ray experiment
- Sites in both hemispheres
- Array of fluorescence telescopes $\sim 60,000 \text{ km}^2$
- **Just 4 pixels!**
- **Ideally want same resolutions as Auger/TA**

Four 20 cm PMTs



FAST aims to measure an unprecedented number of UHECRs above 10^{20} eV

FAST: Top-Down Reconstruction (TDR)

Waveform matching:

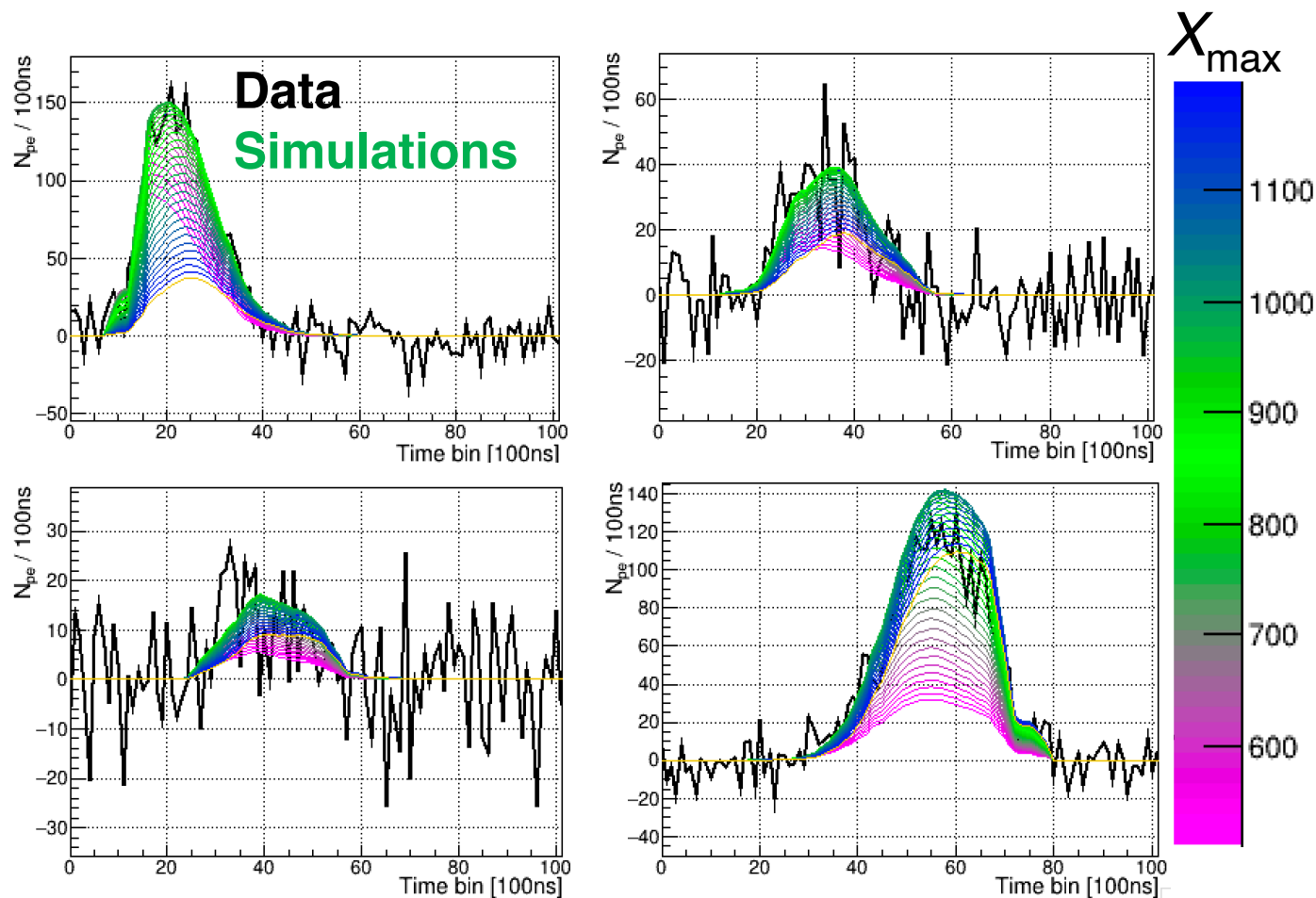
- Compare data directly to simulations
- Use **minimizer** to select the simulated parameters to test

Maximize log-likelihood:

$$\ln \mathcal{L}(\vec{x}|\vec{a}) = \sum_k \sum_i \ln (P_k(x_i|\vec{a}))$$

Probability of observing signal x_i in bin i of PMT k given shower parameters

$$\vec{a} = (E, X_{\max}, \theta, \phi, x, y)$$

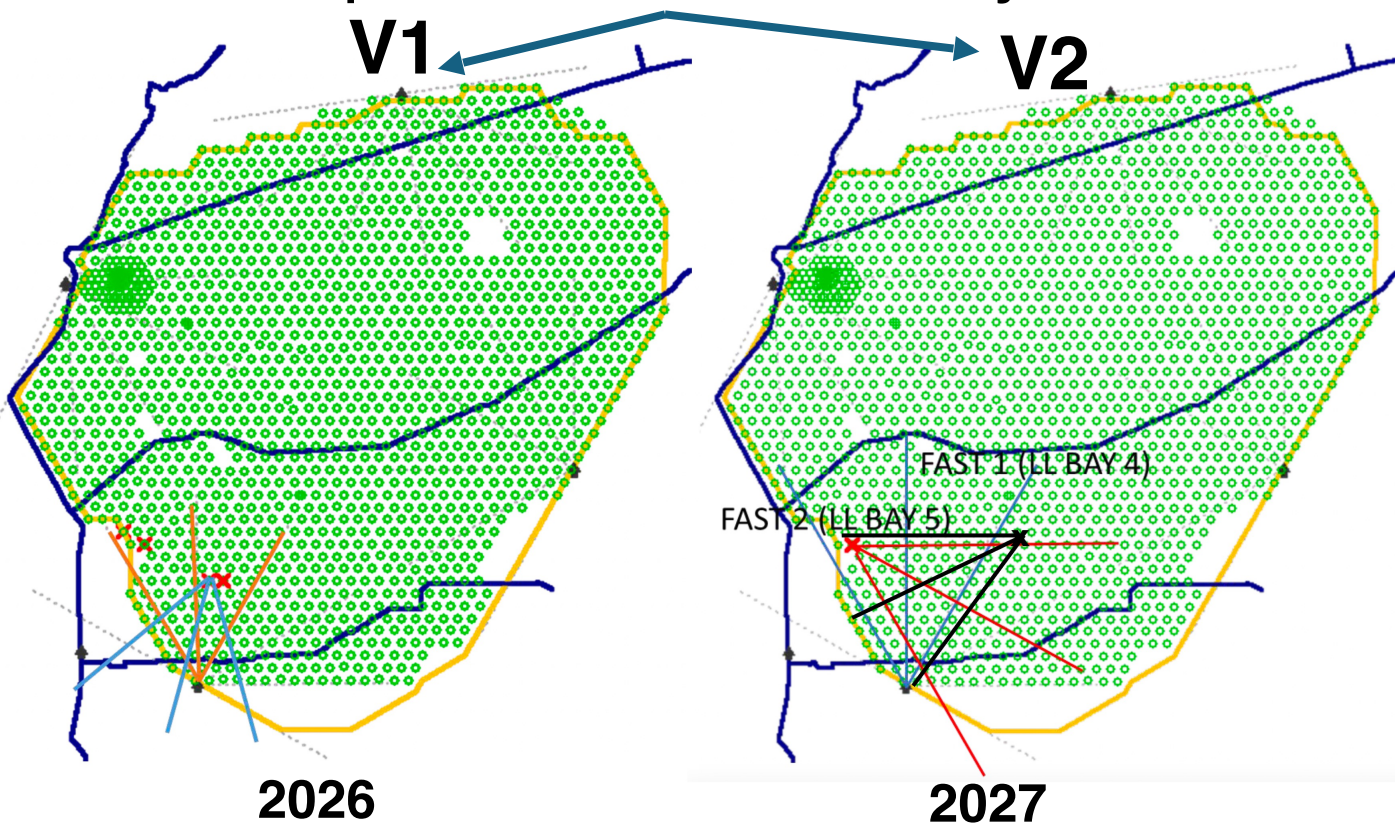


The result of the TDR depends heavily on the “first guess”

FAST (The Fluorescence detector Array of Single-pixel Telescopes)

Two sets of prototypes currently in operation

Next step – “FAST mini-array”!



Research Motivation

Research motivation

WANT TO → Elucidate sources of UHECRs

by

Performing UHECR astronomy (with lighter nuclei)

which requires

Measuring many UHECRs $> 10^{20}$ eV (with FAST)

so we need to be able to

reconstruct shower parameters reliably with FAST

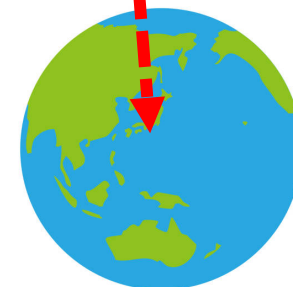
(which will also allow)

cross calibration between TA / Auger



10^{20} eV

$\vec{B}$



Research motivation

To reconstruct shower parameters reliably with FAST, need to

1. Address issues with the current top-down reconstruction
2. Produce a robust first guess method
3. Check the performance of the full reconstruction chain
4. Apply full reconstruction to prototype data and check results against expectations and measurements from other experiments

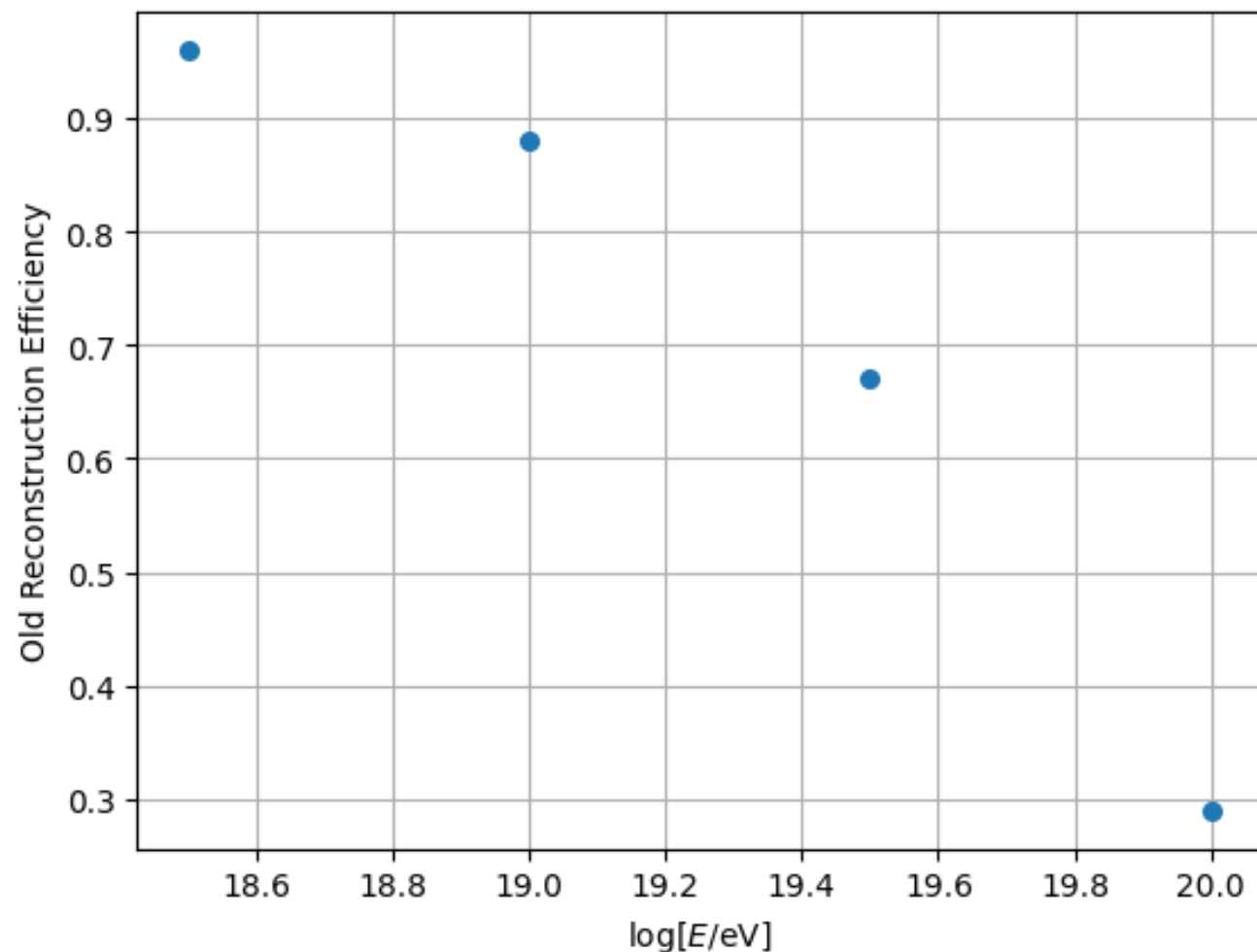
Top-Down Reconstruction Improvements

Reconstruction Efficiency

- Biggest issue with current reconstruction is the **efficiency**, first identified by Albury
- Efficiency defined as
$$N_{\text{rec}}/N_{\text{trig}}$$

N_{rec} = # showers successfully reconstructed
 N_{trig} = # showers triggering ≥ 1 PMT
- Reconstruction is **“successful”** if minimization is successful

Adapted from Albury (2020)



Why does efficiency drop?

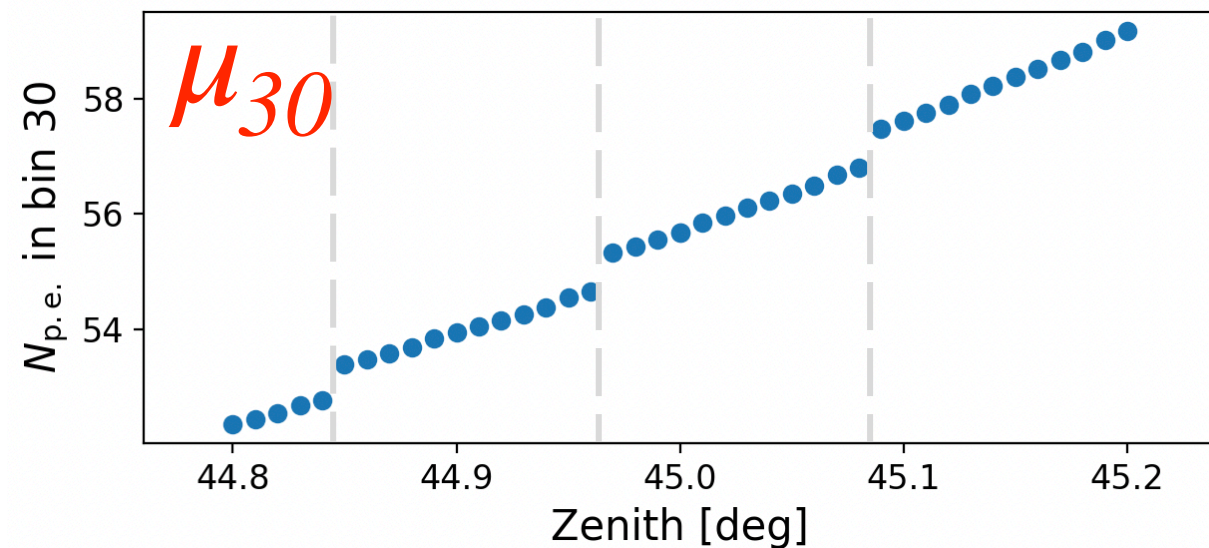
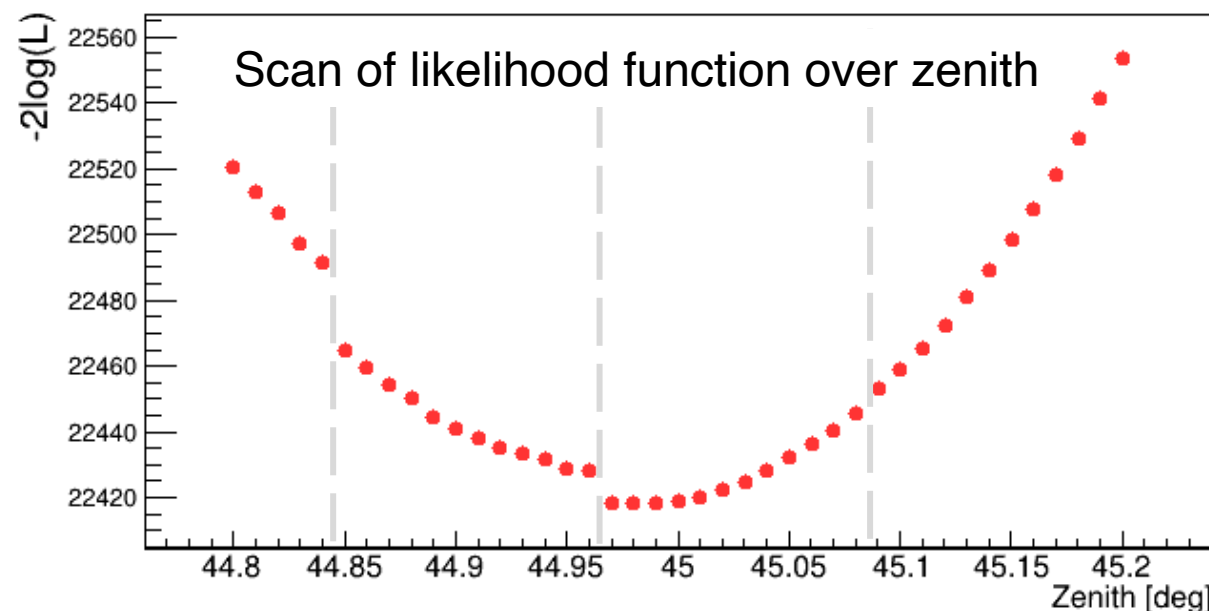
Main reason:

Discontinuities in likelihood function

$$\ln \mathcal{L}(\vec{x}|\vec{a}) = \sum_k^{N_{\text{pix}}} \sum_i^{N_{\text{bins}}} \ln (P_k(x_i|\vec{a}))$$

$$P(x_i|\mu_i, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left[-\frac{(x_i - \mu_i)^2}{2\sigma^2} \right]$$

- These come from discontinuities in the simulated/expected signal



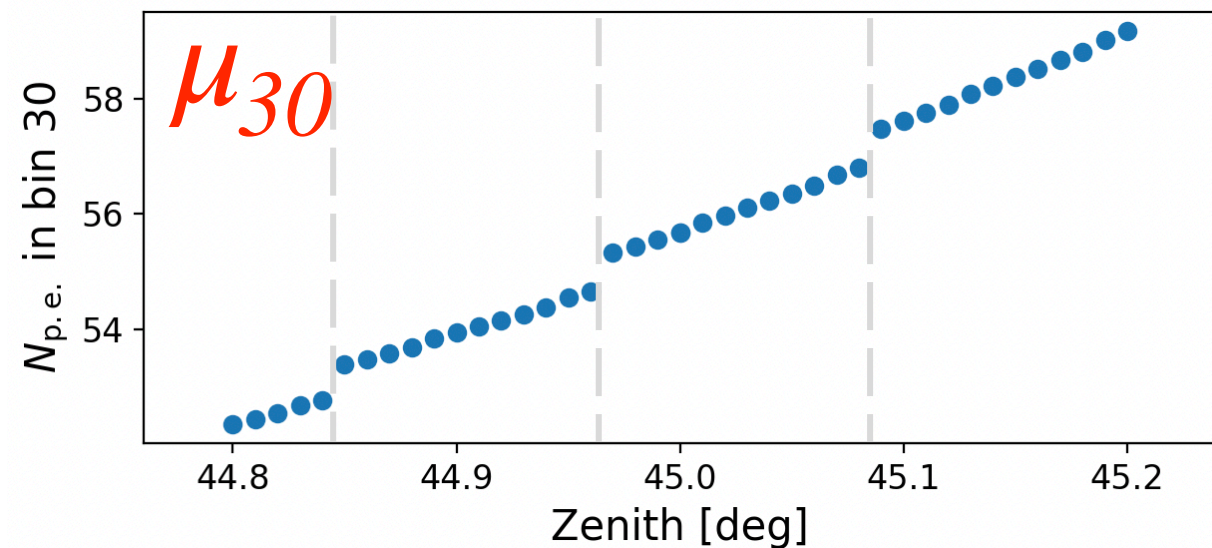
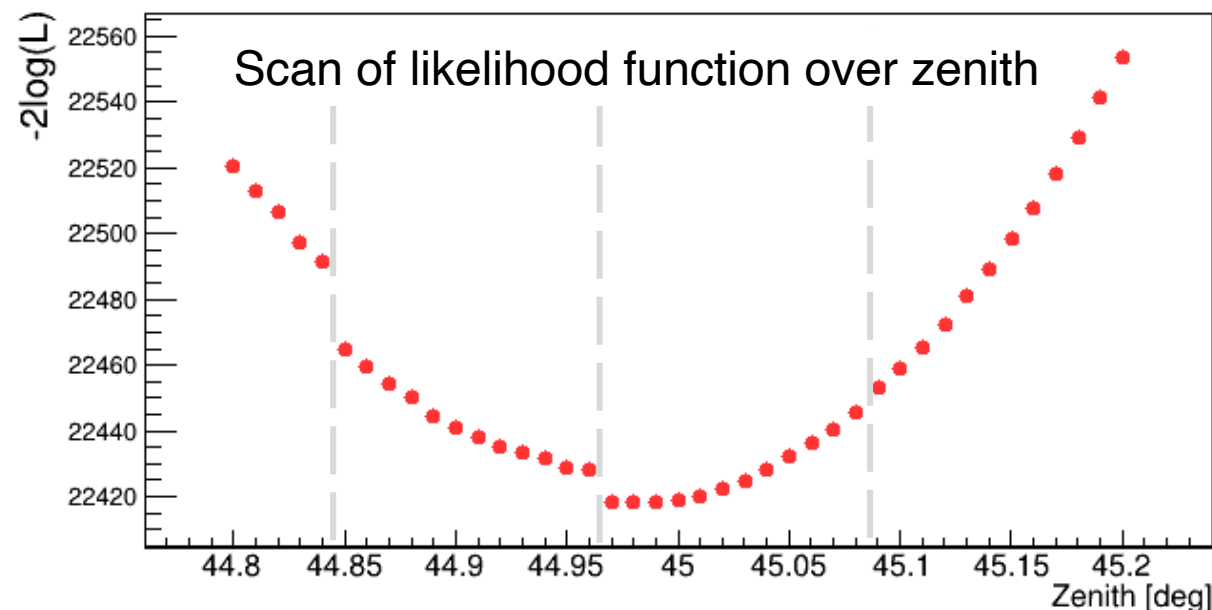
Why does efficiency drop?

Main reason:

Discontinuities in likelihood function

$$\ln \mathcal{L}(\vec{x}|\vec{a}) = \sum_k^{N_{\text{pix}}} \sum_i^{N_{\text{bins}}} \ln (P_k(x_i|\vec{a}))$$

- Discontinuities bad for minimizers
- Showers with higher energy
 - higher average signals
 - effect of discontinuities larger
 - more failed minimizations



Discontinuity Sources

Three Main Issues:

Shower Axis Binning

Azimuthal Difference

Directional Efficiency

Discontinuity Sources

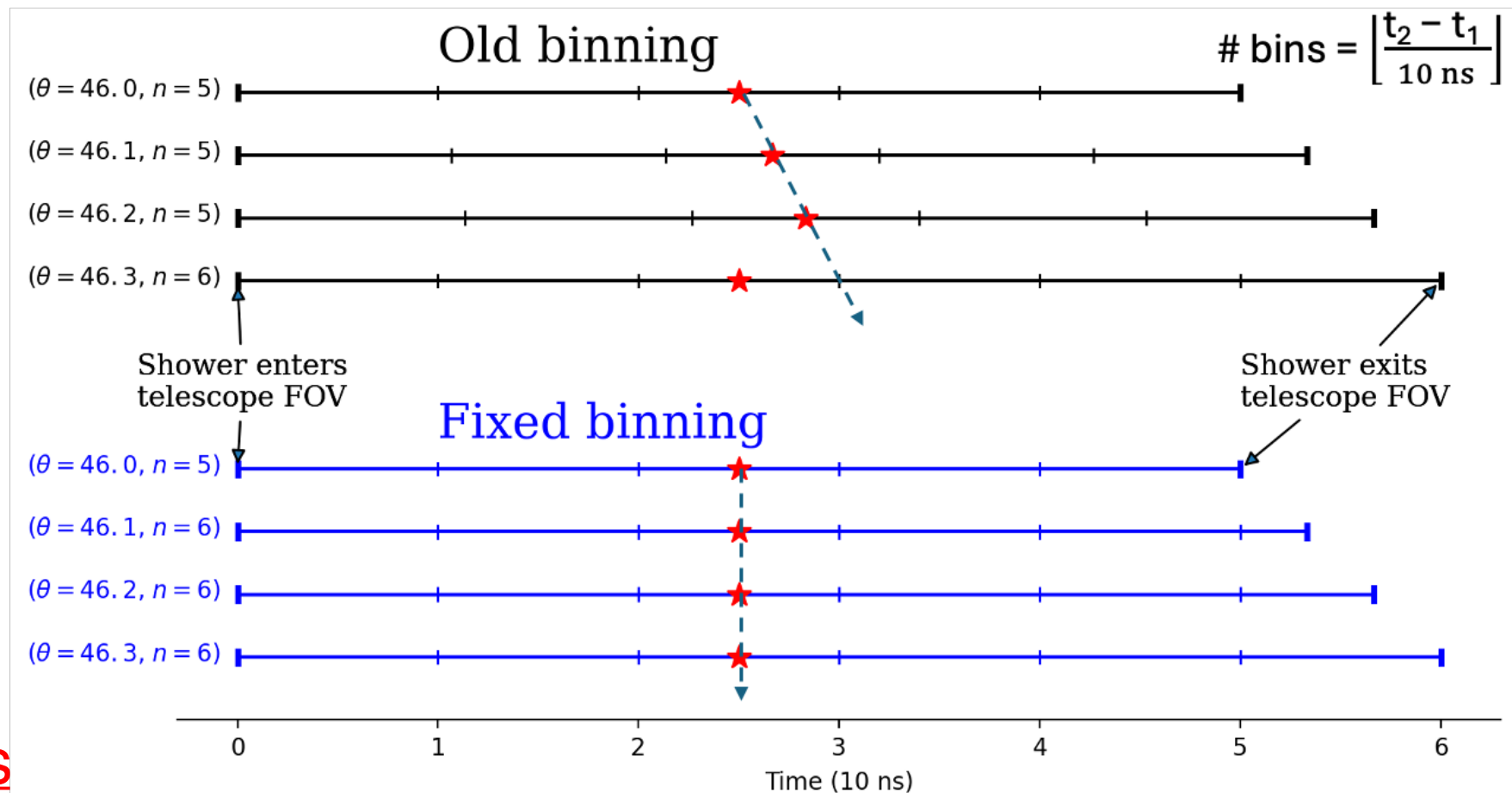
Three Main Issues:

Shower Axis Binning

Azimuthal Difference

Directional Efficiency

- Signal in a trace bin comes from a single bin on shower axis
- This signal depends on bin size and position
- Old binning method caused discontinuities



Discontinuity Sources

Three Main Issues:

Shower Axis Binning

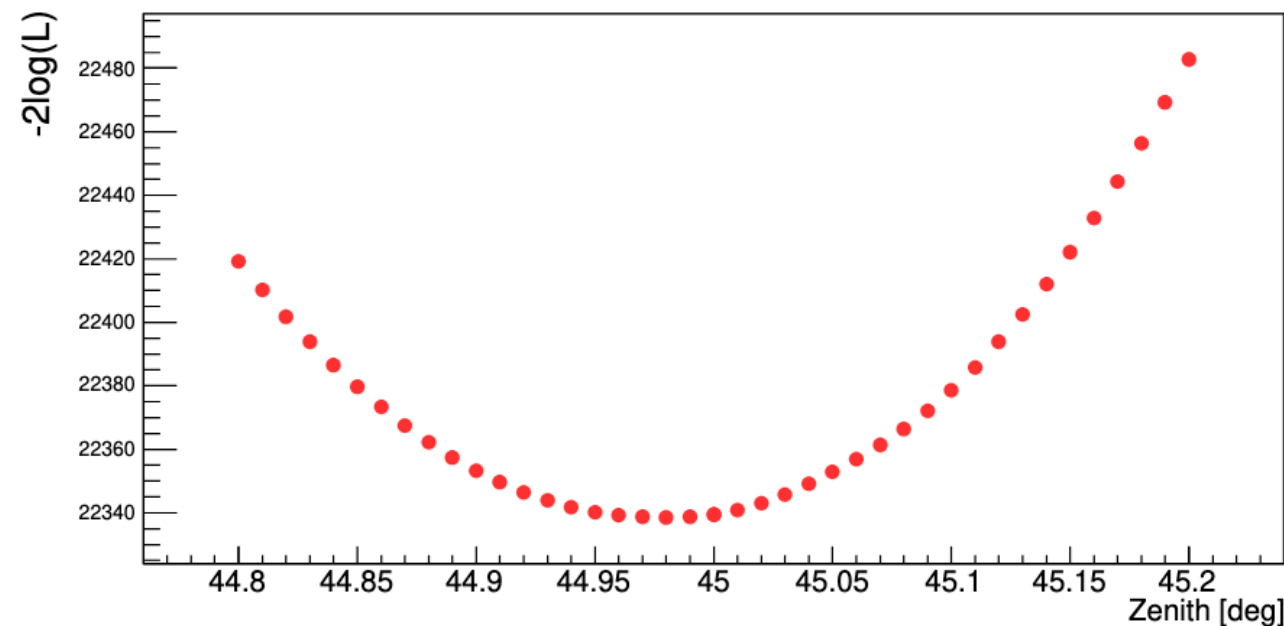
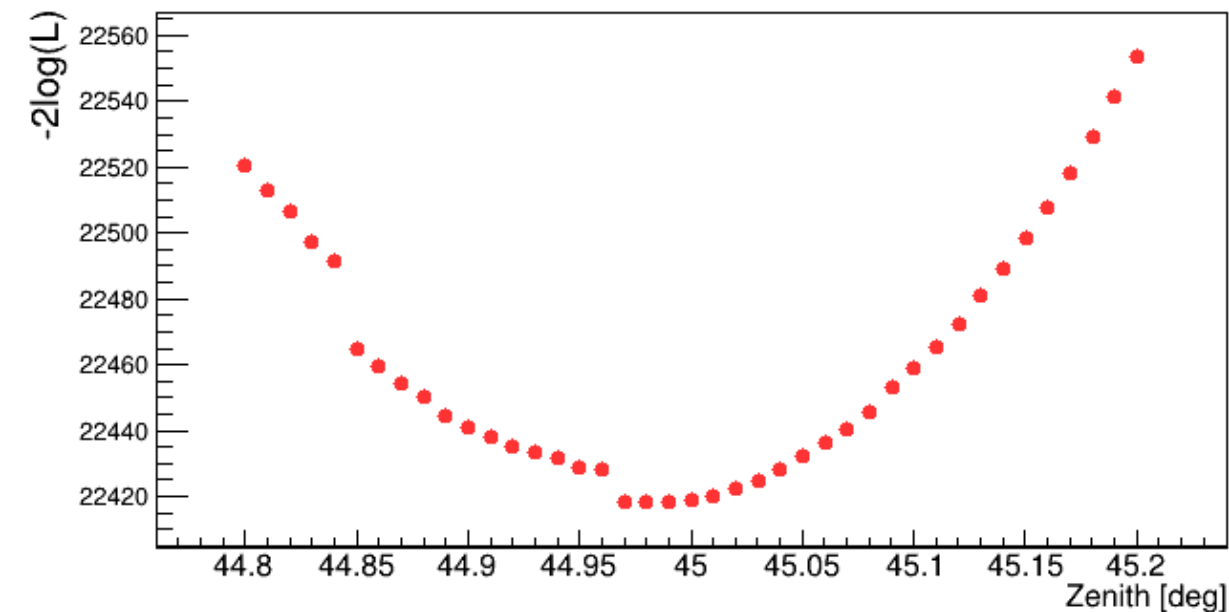
Azimuthal Difference

Directional Efficiency

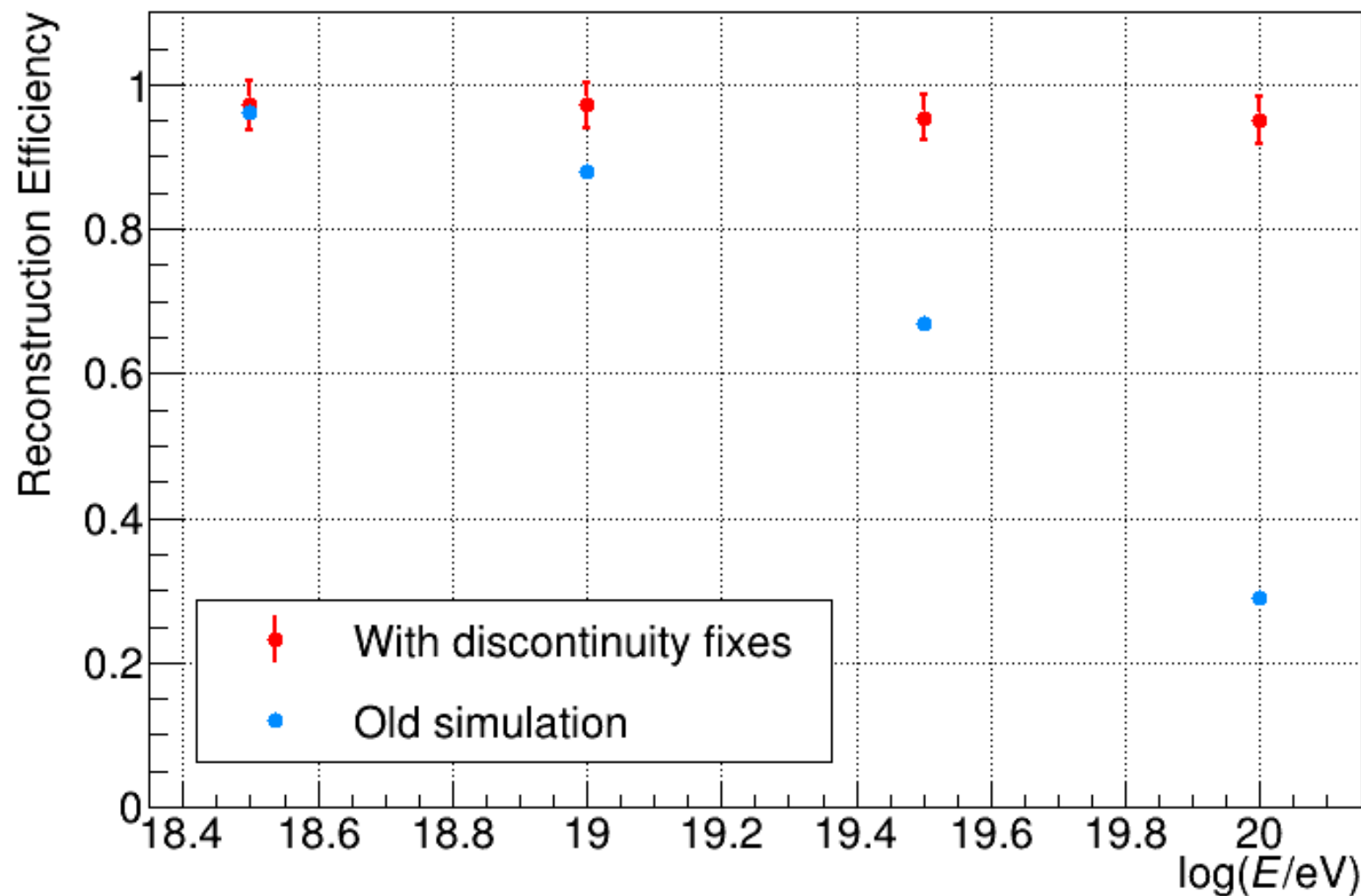
Old binning



New binning



Updated Efficiency

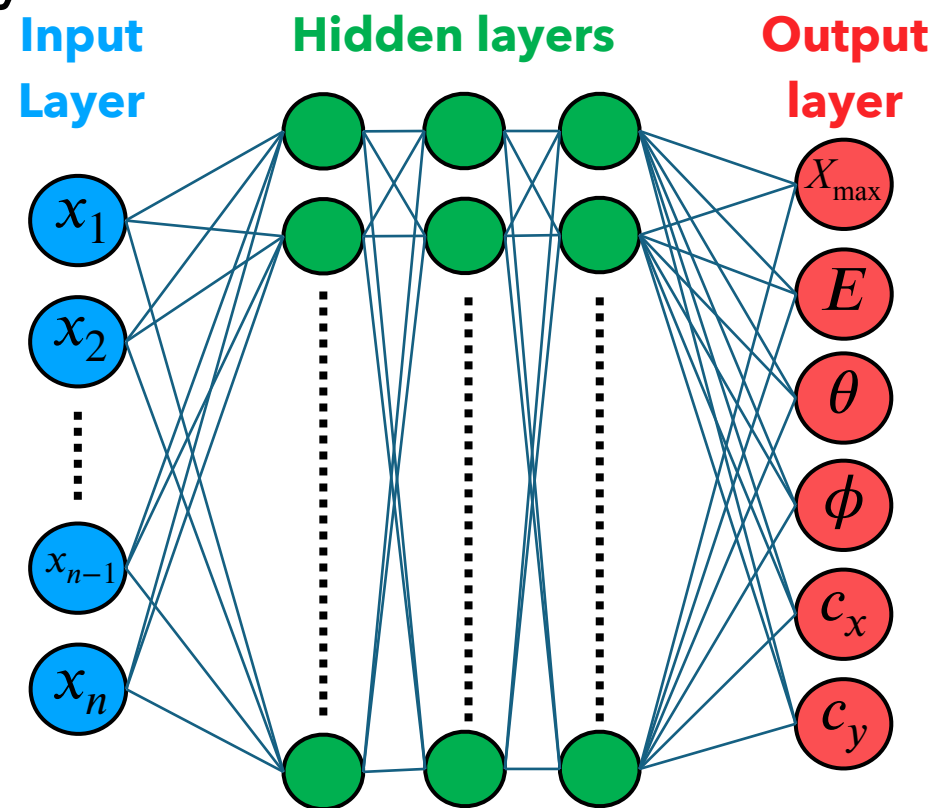
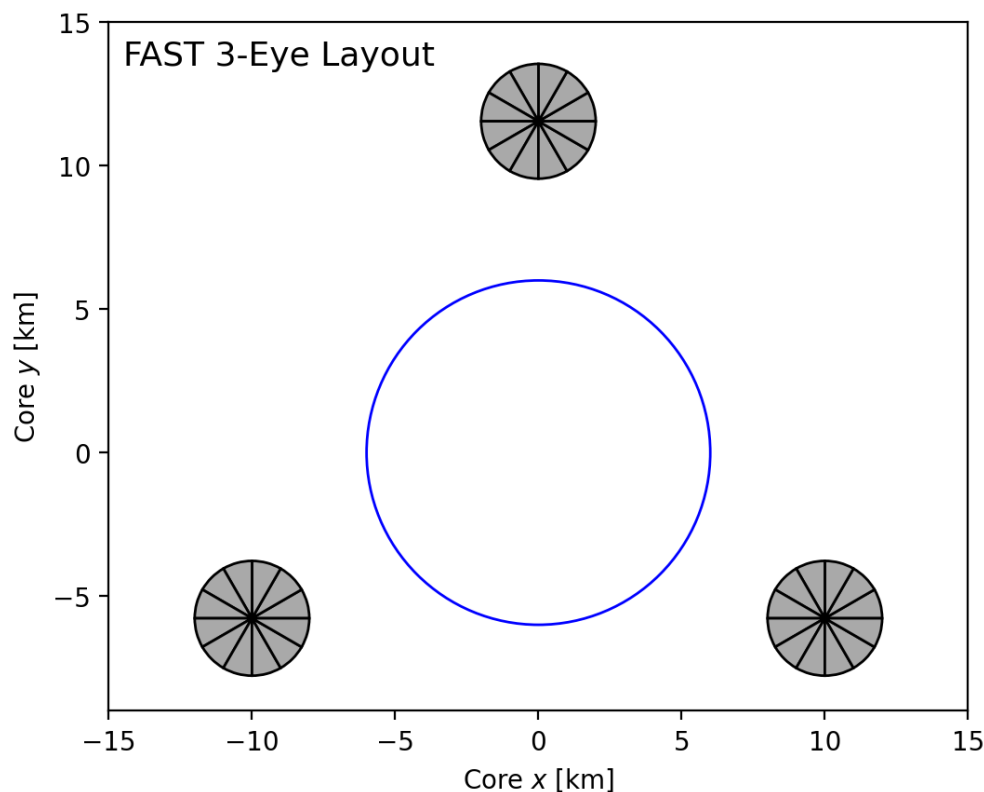


- Efficiency (i.e. minimisation success) now $\sim 95\%$ across whole energy range

First Guess I – Neural Networks

Overview

- Albury and Fujii showed that a feed-forward, multi-layer perceptron can predict the shower parameters with full sized FAST array well
 - Resolutions** - $X_{\max}$: 30 g cm⁻², Energy: 8%

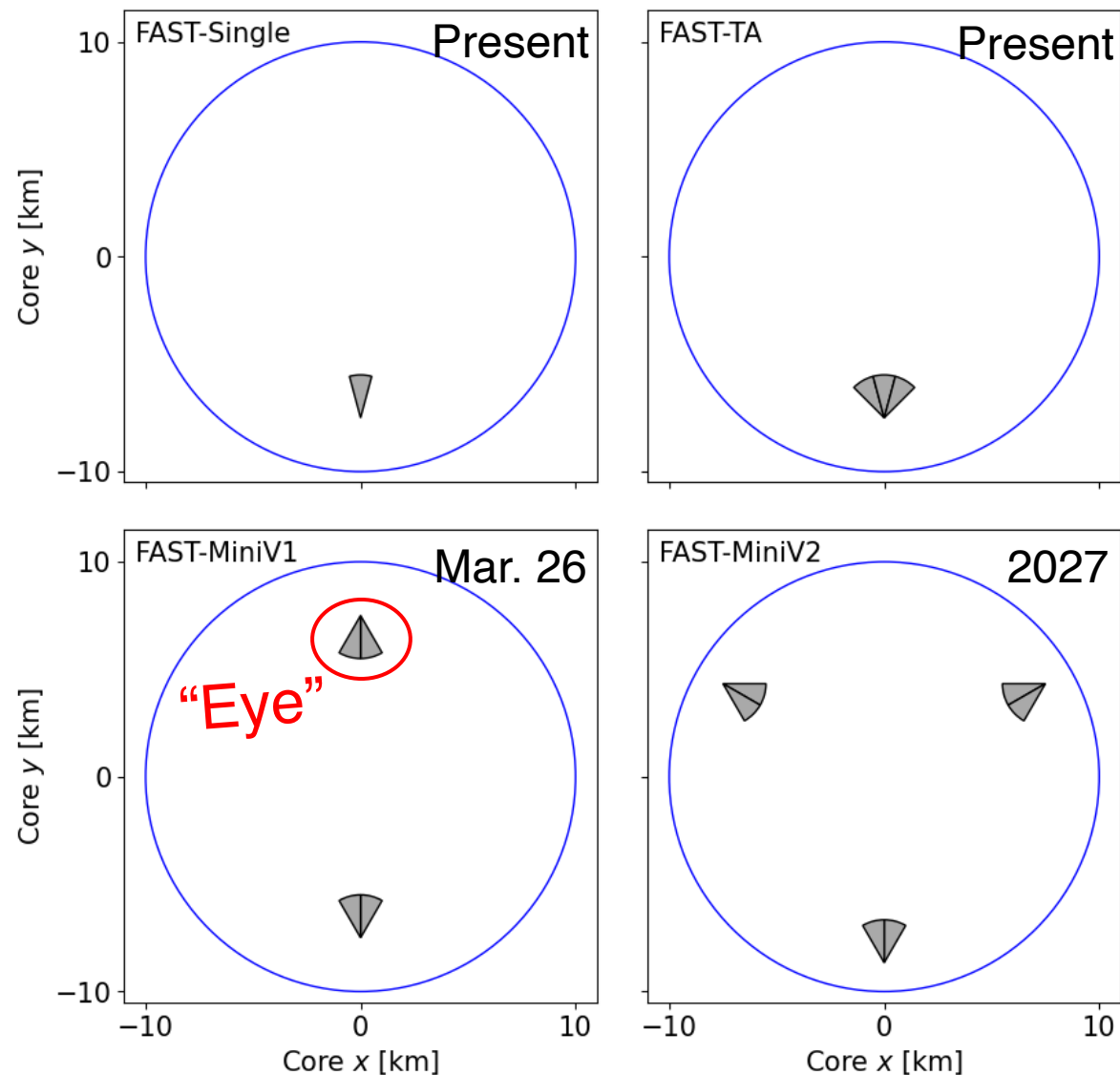


Can a similar model work for current layouts? More complex architectures?

Dataset

- 1,000,000 showers
- 4 layouts
- Showers with significant signal in ≥ 1 PMT
 - FAST-Single $\sim 2 \times 10^5$
 - FAST-TA $\sim 4.1 \times 10^5$
 - FAST-MiniV1 $\sim 4.7 \times 10^5$
 - FAST-MiniV2 $\sim 5.5 \times 10^5$

$X_{\max}$	Uniform (500 - 1200 g cm ⁻²)
Energy	Uniform in log(E/eV) (17.3 - 20)
θ	$\sin \theta \cos \theta$ (0 - 80°)
ϕ	Uniform (0 - 360°)
Core	Uniform (in circle centred at (0,0), $r = 10$ km)



Models

Models Tested:

• Basic DNN:

- Identical architecture to previous studies
- Use **height of PMT pulse**, **total signal** and **timing** from each PMT as inputs

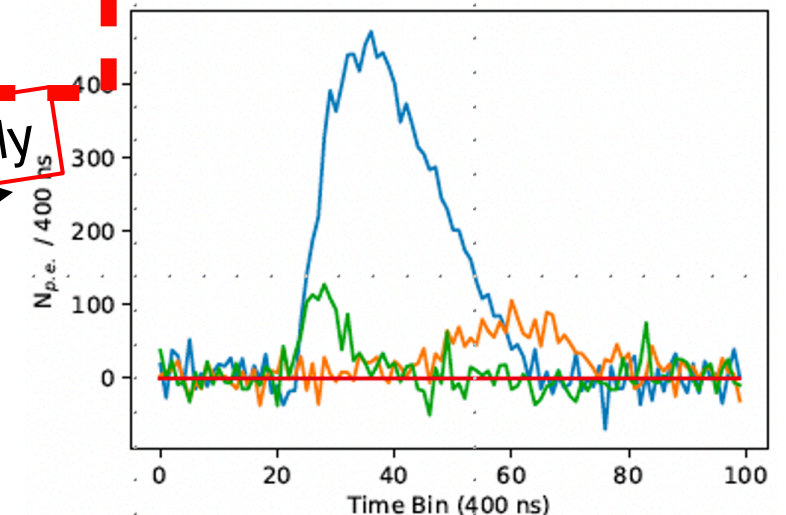
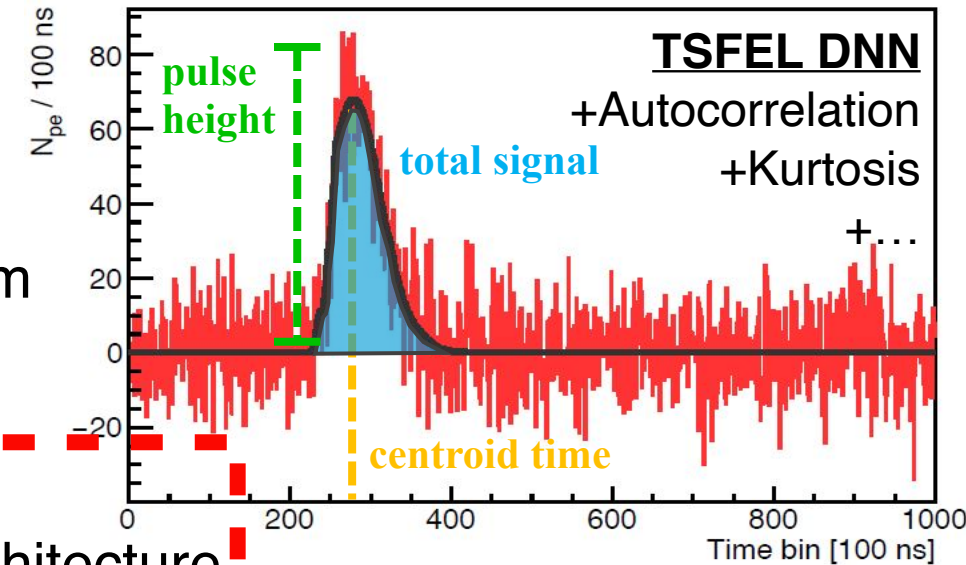
• TSFEL DNN:

- Extension of the Basic DNN - same feedforward architecture
- Use TSFEL python library to extract additional trace features
- Total of 11 inputs per PMT

• LSTM (Long-Short Term Memory):

- Type of recurrent neural network
- Often used for analysing time series data
- 64 features extracted per PMT

Pass in traces directly



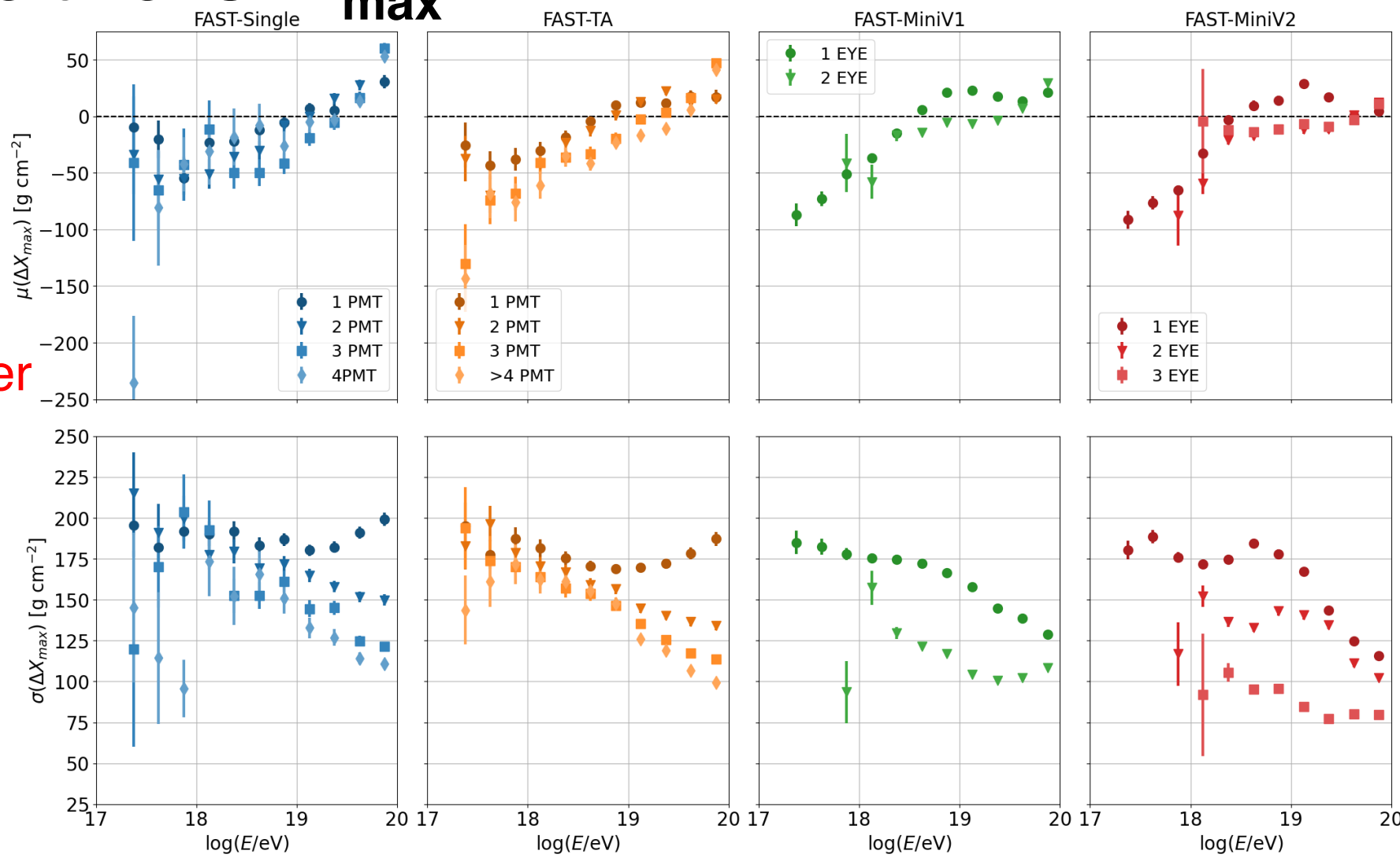
TSFEL DNN Performance

Biases and resolutions: $X_{\max}$

Biases

More PMTs & eyes is better

Resolutions



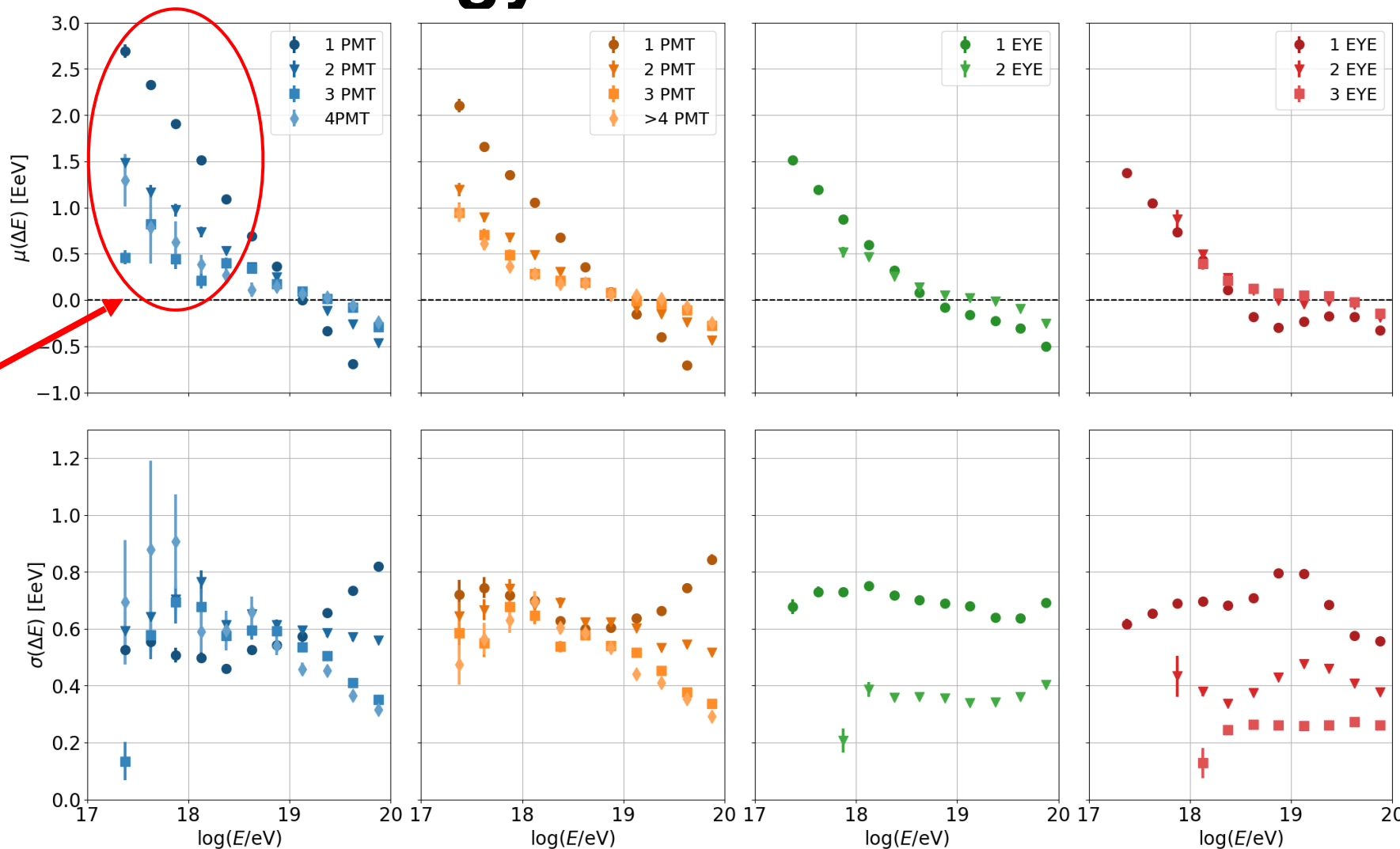
TSFEL DNN Performance

Biases and resolutions: Energy

Biases

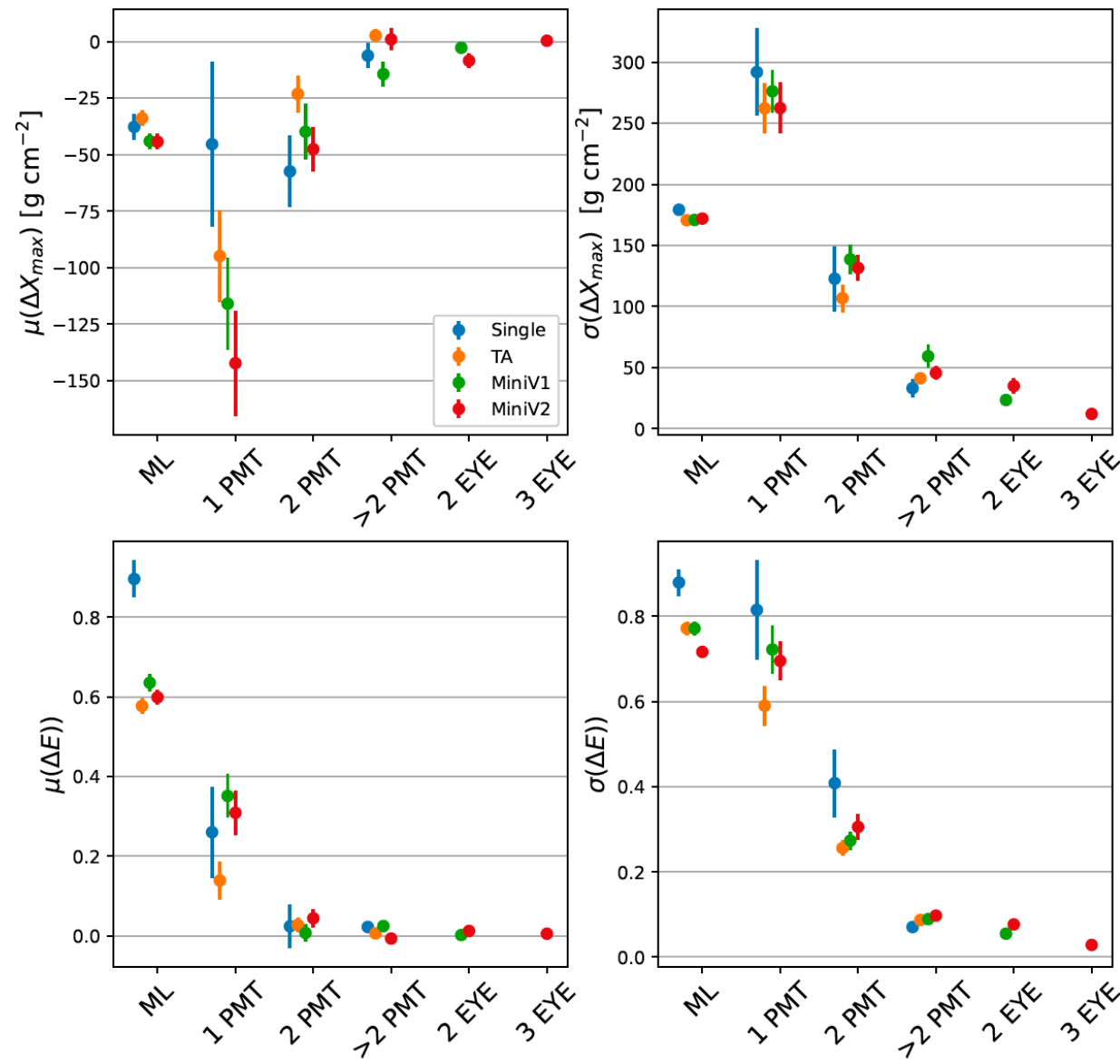
Degeneracy between
energy and core position

Resolutions



Full Reconstruction: Summary

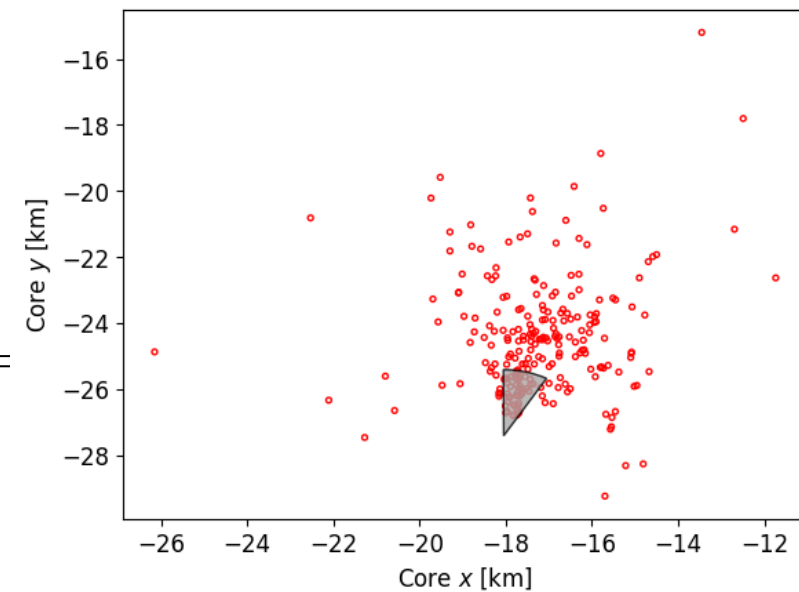
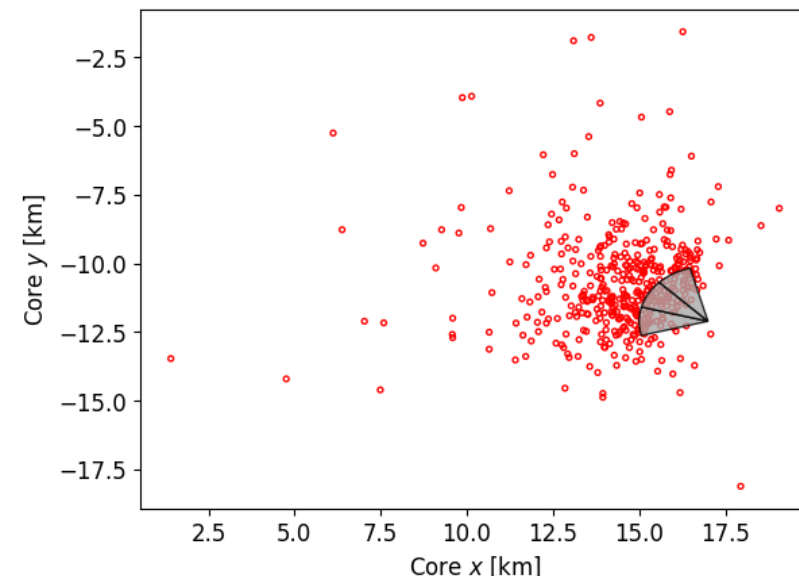
- Use TSFEL DNN output as first guess for TDR (simulations)
- 1 or 2 PMTs - poor resolutions
- 3 or more PMTs
 - $X_{\max} \sim 40 - 50 \text{ g cm}^{-2}$, Energy $\sim 10\%$
- Stereo performs even better!
 - Slightly unrealistic results...
 - No shower-to-shower fluctuations
 - No atmospheric/calibration uncertainties
 - No simulation of electronic response (e.g. saturated signals)



Coincidence event data analysis

Overview

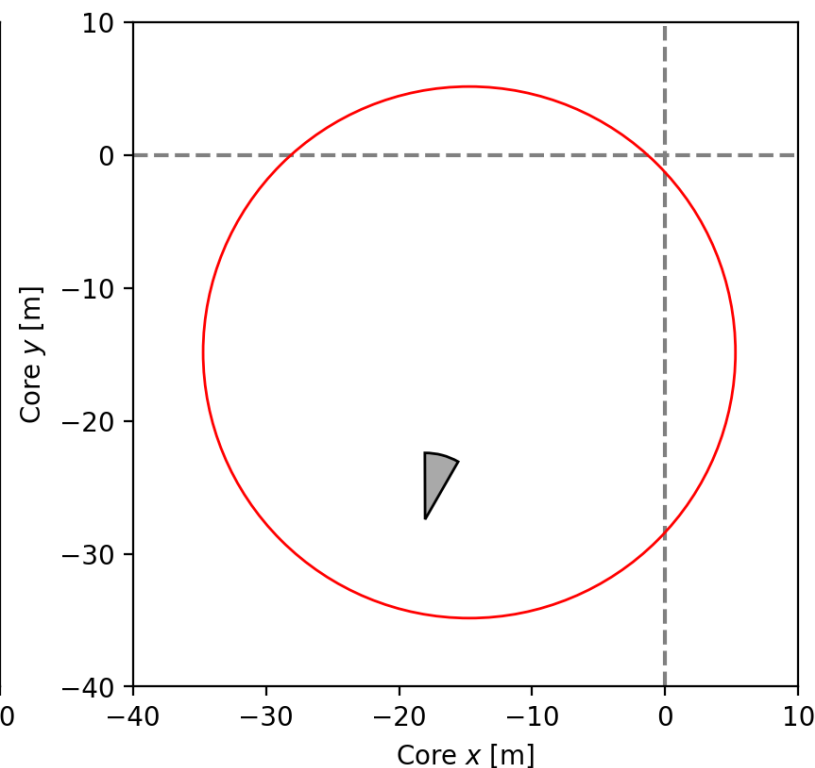
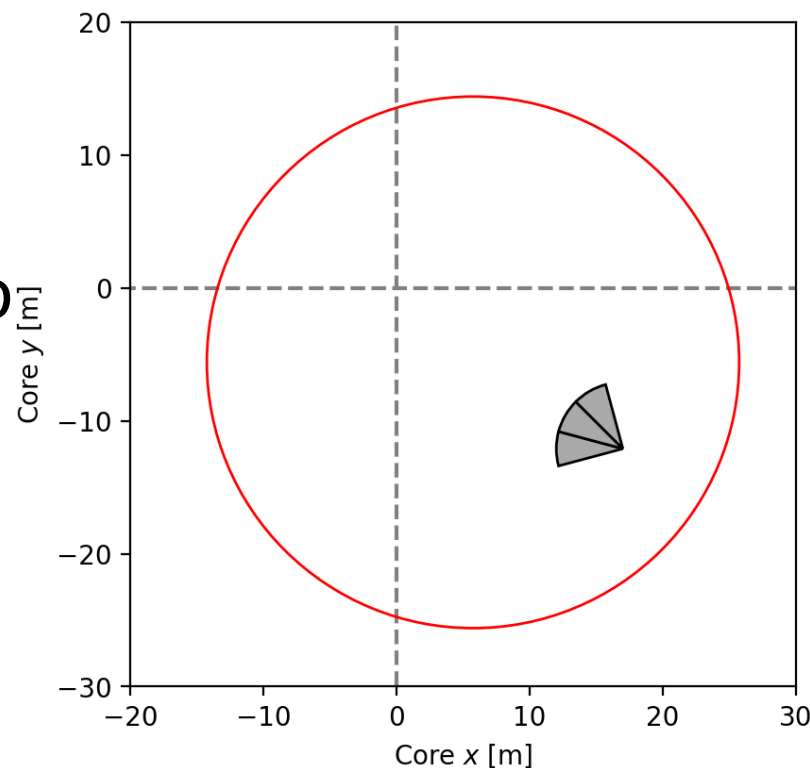
- Want to reconstruct real events observed by FAST@TA and FAST@Auger, ideally using developed First Guess Methods + TDR
- Current prototypes receive external triggers from TA/Auger to collect data
- Use events seen by both TA/Auger and FAST → **coincidence events**



	FAST@TA	FAST@Auger
Analysis period	2018/03 – 2018/10 (2018/10 – 2023/02)	2022/07 – 2022/10
Observation time	65 hrs (182 hrs)	122 hrs
Coincidence events	438	235

MC/Data Comparison

- Check whether telescopes are performing as expected by comparing simulations to data
- “Data” are TA/Auger reconstructed parameters
- Simulation details shown on right



$X_{\max}$	EPOS-LHC (500 - 1200 g cm ⁻²)
Energy	$E^{-1.5}$ (1 - 100 EeV)
Zenith	$\sin \theta \cos \theta$ (0 - 80°)
Azimuth	Uniform (0 - 360°)
Core position	Uniformly distributed in circle, $r = 20$ km, centred at (5741 m, -5060 m) (TA) & (-14726 m, -14846 m) (Auger)

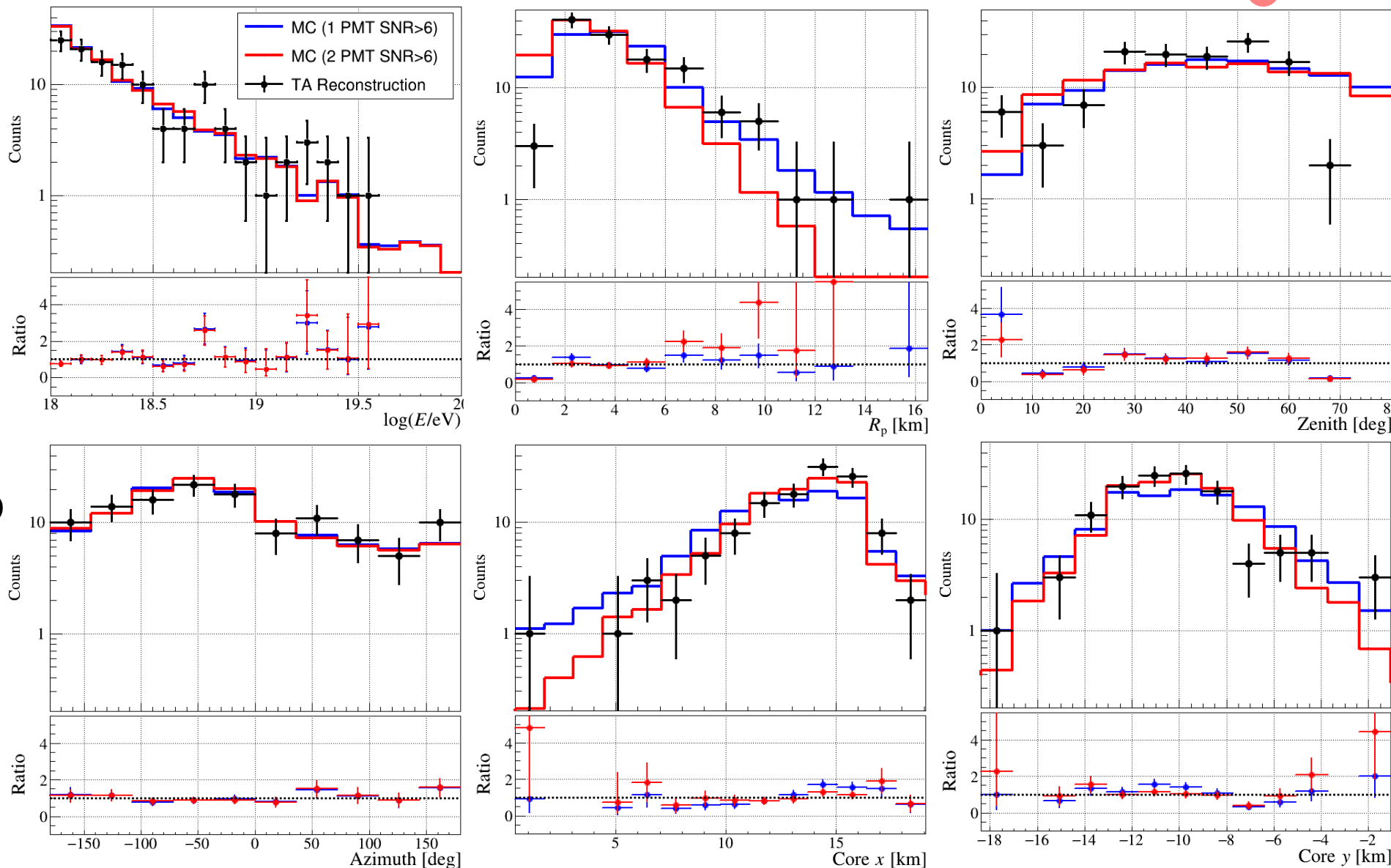
MC/Data Comparison: FAST@TA

- To produce MC histograms, calculate “exposure”

- Related to number of events expected to be *detected*

- Can’t replicate *detection* process, so try two simple trigger conditions instead

- ≥ 1 PMT triggered
- ≥ 2 PMTs triggered



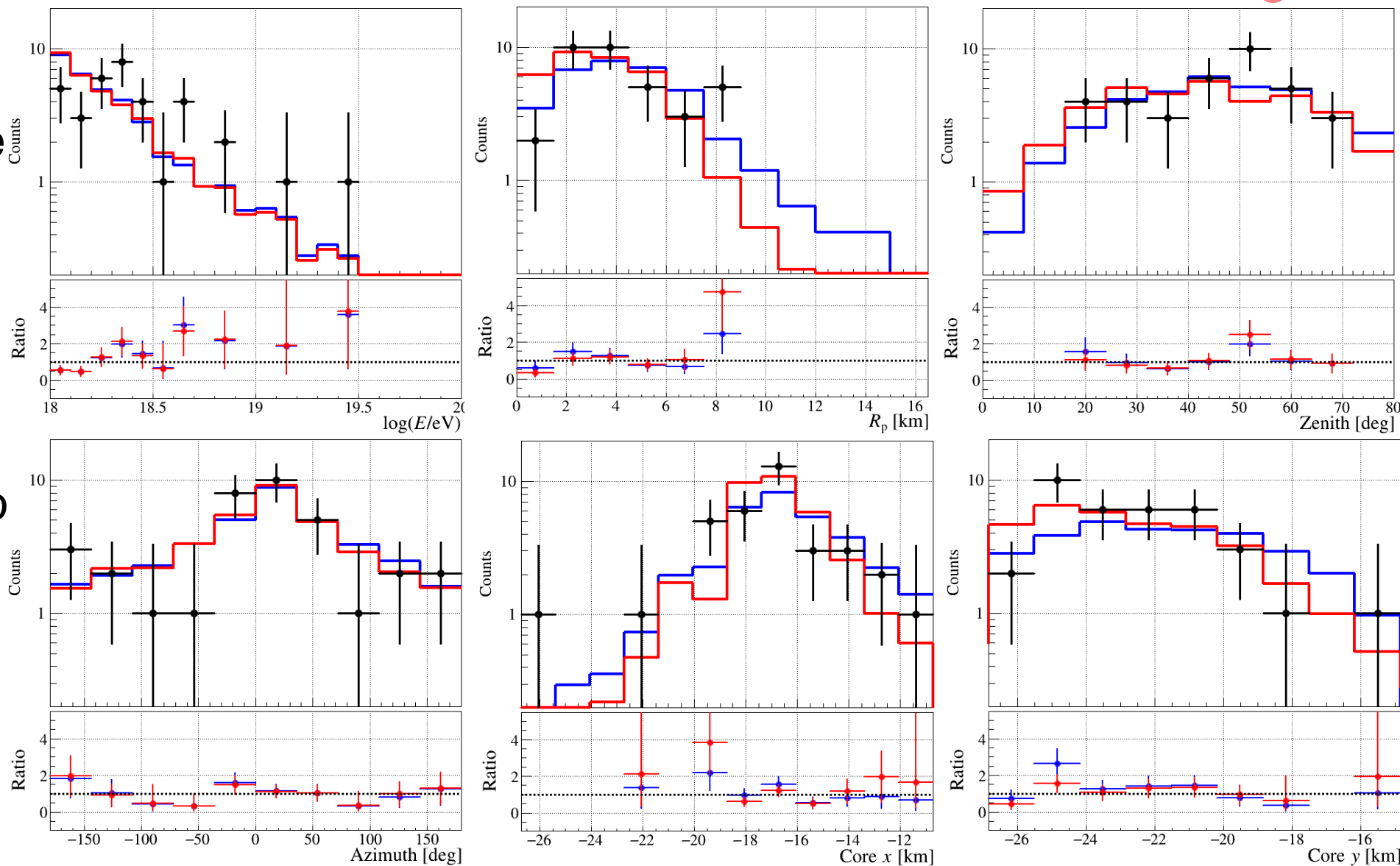
MC/Data Comparison: FAST@Auger

- To produce MC histograms, calculate “exposure”

- Related to number of events expected to be *detected*

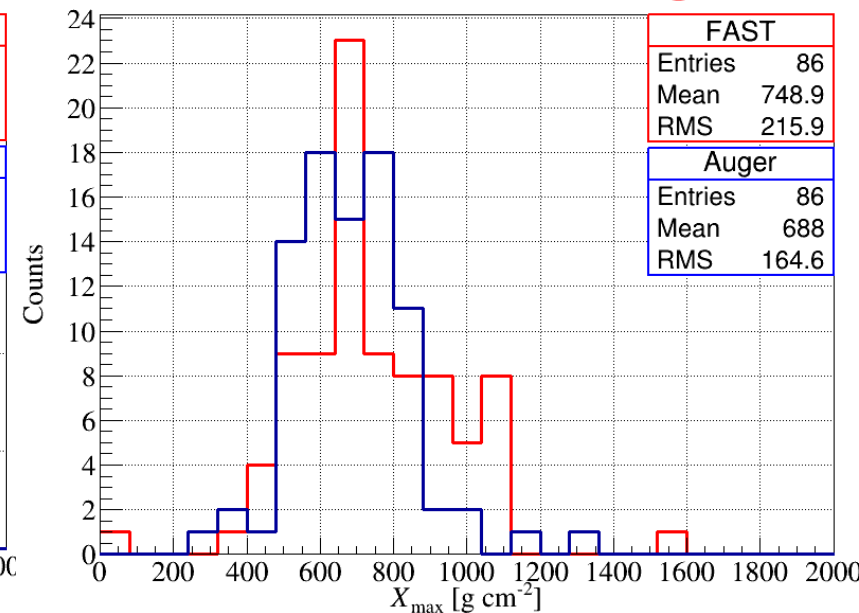
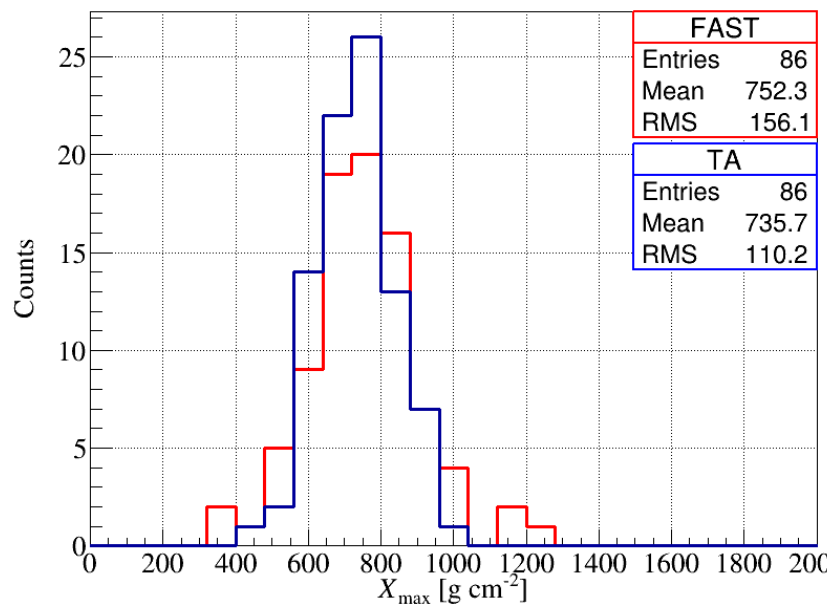
- Can't replicate *detection* process, so try two simple trigger conditions instead

- ≥ 1 PMT triggered
- ≥ 2 PMTs triggered

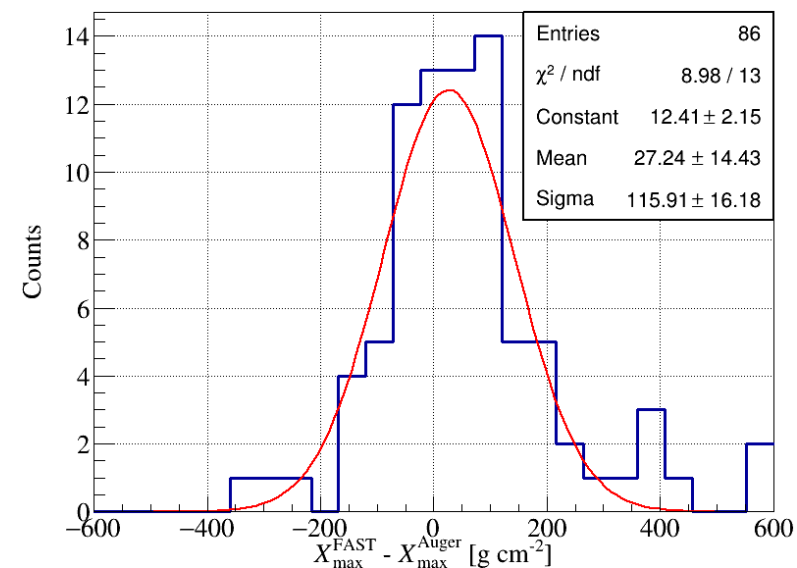
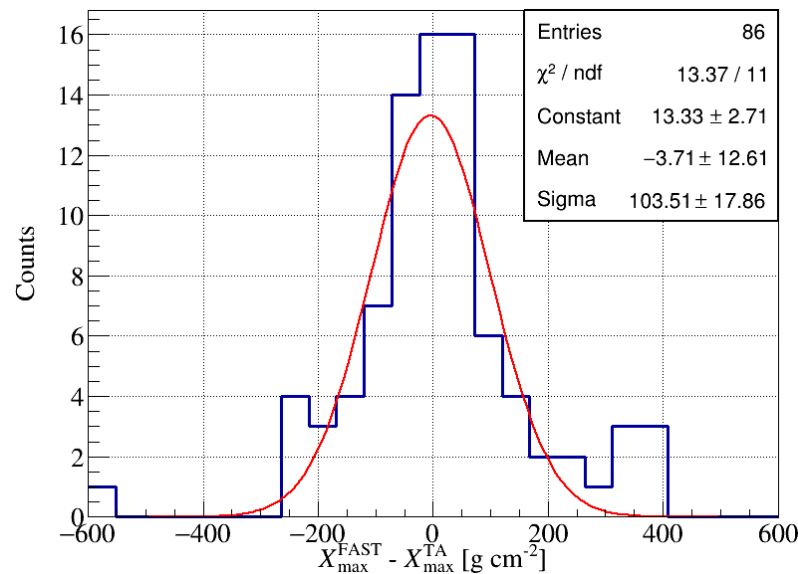


Initial Reconstruction: $X_{\max}$

- Reconstruct all six shower parameters simultaneously with TDR
- Use TA / Auger reconstructed values as first guess

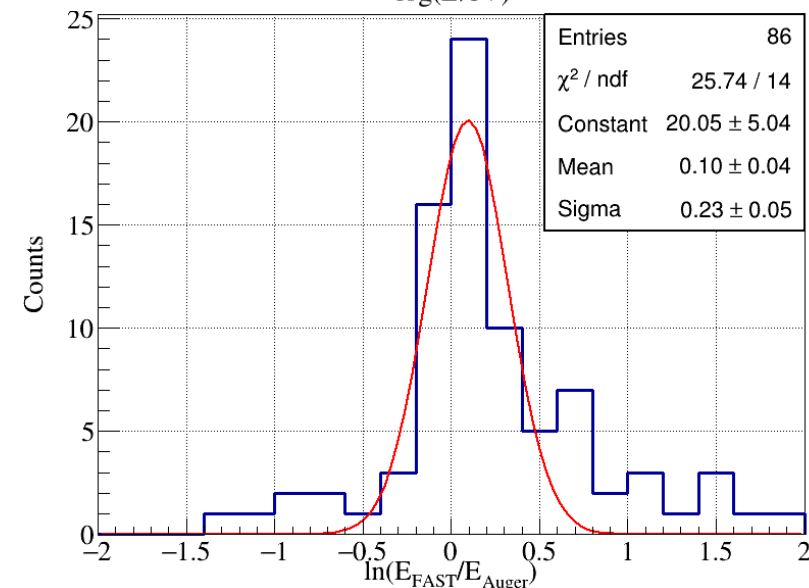
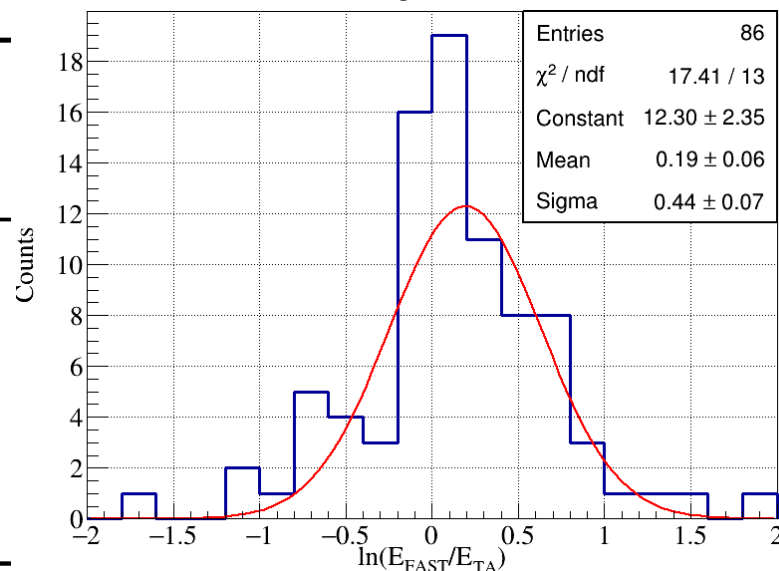
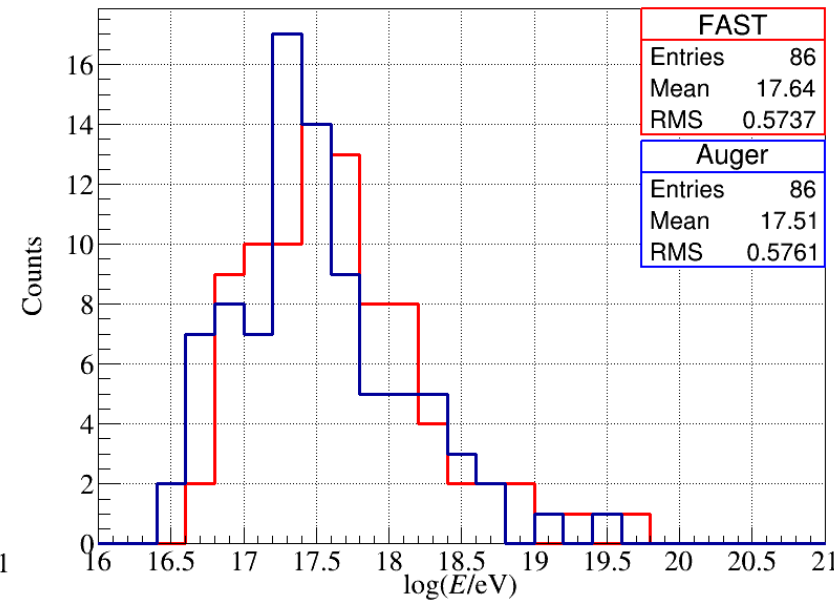
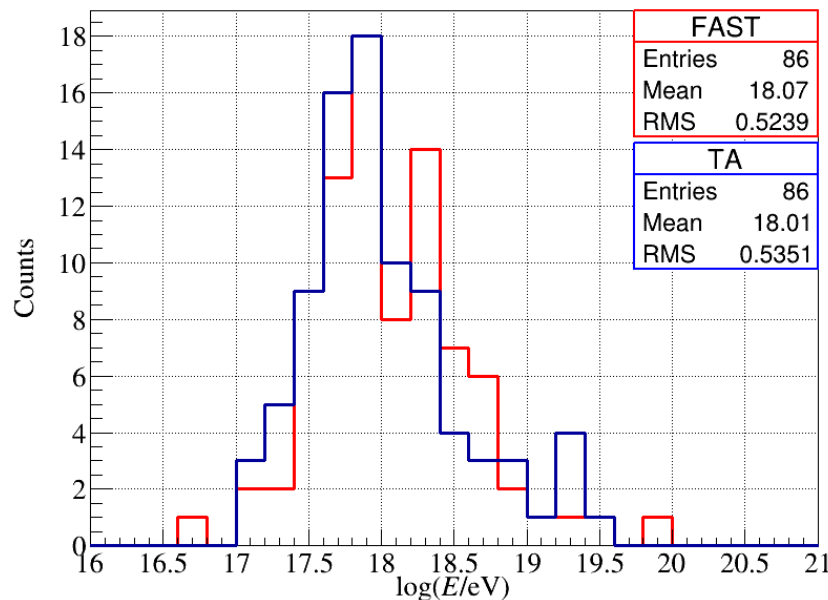


Cut	Events Remaining	
	TA	Auger
No cuts	438	235
Minimisation	412	215
Time Offset	405	211
$X_{\max}$ in FOV	103	116
Relative Error	86	86



Initial Reconstruction: Energy

- Reconstruct all six shower parameters simultaneously with TDR
- Use TA / Auger reconstructed values as first guess



Cut	Events Remaining	
	TA	Auger
No cuts	438	235
Minimisation	412	215
Time Offset	405	211
X_{max} in FOV	103	116
Relative Error	86	86

TSFEL DNN + TDR?

Machine learning first guess

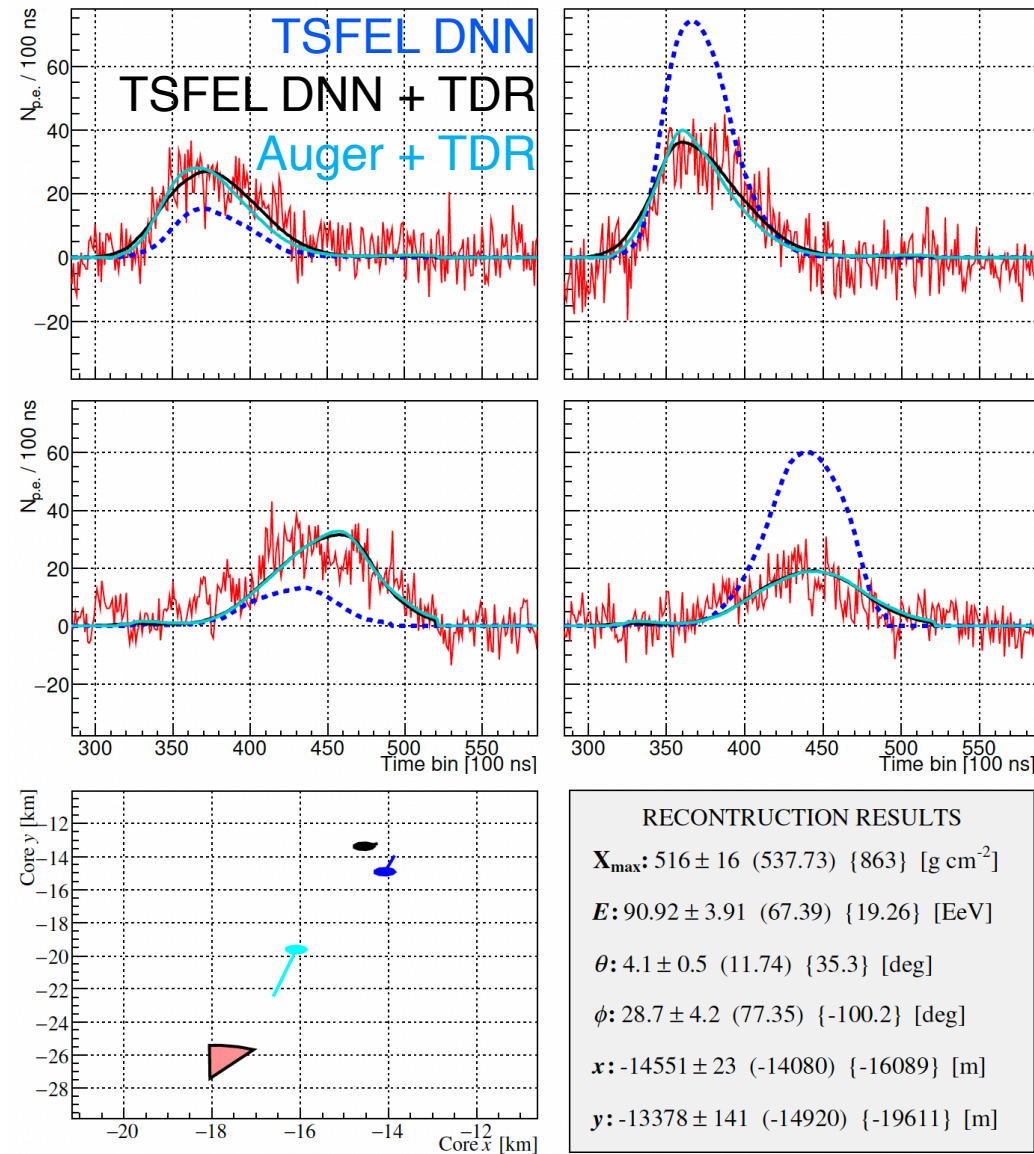
- From previous analysis, need > 2 triggered pixels for reasonable guess

- Number of events with > 2 triggered pixels

- **FAST@TA:** 70
- **FAST@Auger:** 62

- After cuts, number of fits which “reasonably” match data

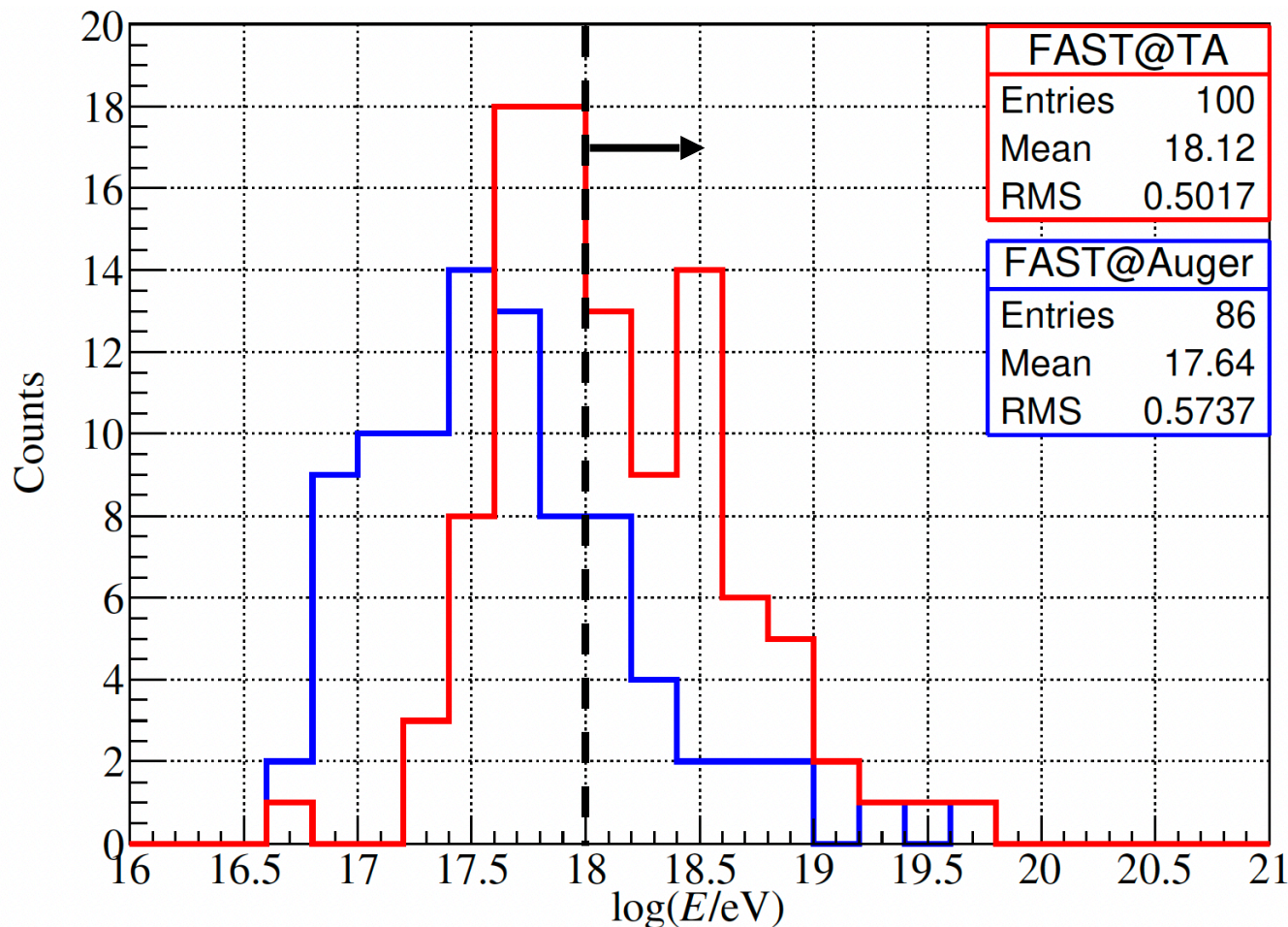
- **FAST@TA:** 30 events **Not enough statistics!**
- **FAST@Auger:** 20 events



Energy spectrum and mass composition with FAST

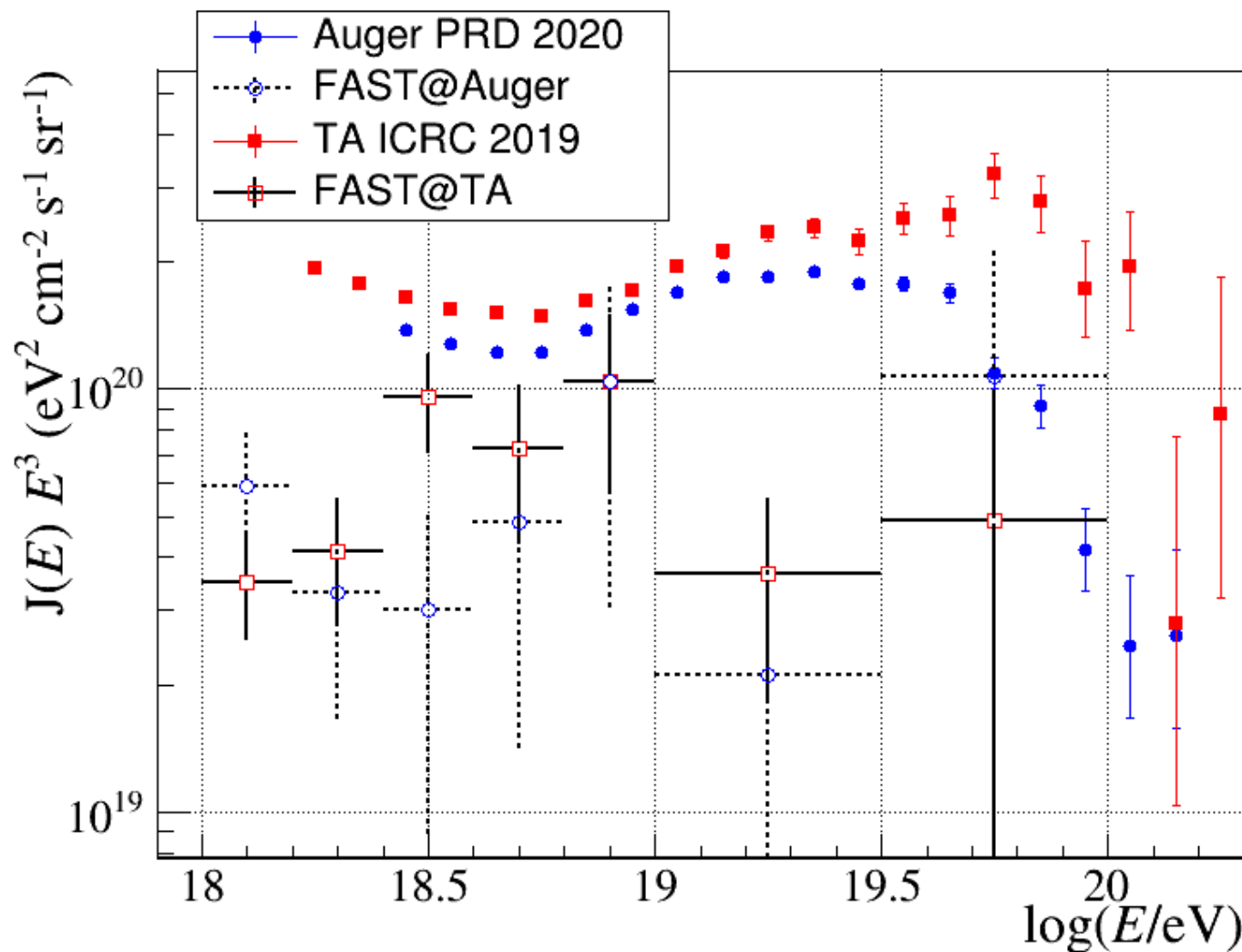
Energy Spectrum with FAST

- Biases in FAST only reconstruction + low statistics
→ use results with TA/Auger first guess
- Only events with energy $>10^{18}$ eV



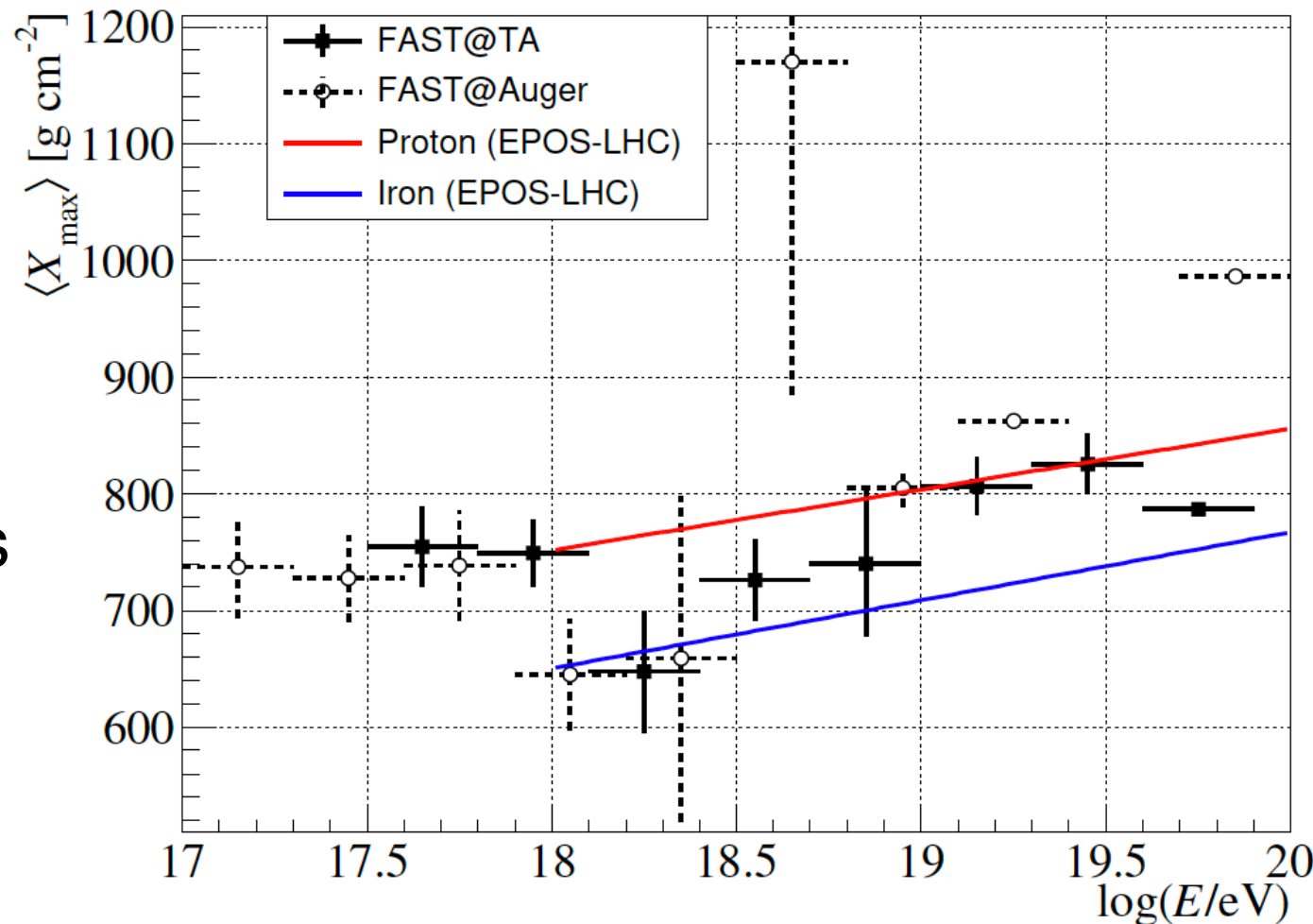
Energy Spectrum with FAST

- Use exposure calculated with ≥ 2 PMTs triggered condition
 - Recalculate FAST@TA exposure with new maps
- Factor of 2 ~ 10 lower depending on energy
- Primary reason for discrepancy is exposure calculation
- More statistics and accuracy expected with upcoming FAST mini-array



Mass Composition with FAST

- Use reconstructed $X_{\max}$ and energy from same set of showers
- Predictions for **proton** / **iron** come from simulations
- Low statistics $\rightarrow$ no conclusions to be drawn currently
- Need stereo observation for quality $X_{\max}$ / energy reconstruction



Summary

- Want to find out the sources of UHECRs using FAST
- **Need to ensure an accurate and precise reconstruction procedure!**

Results:

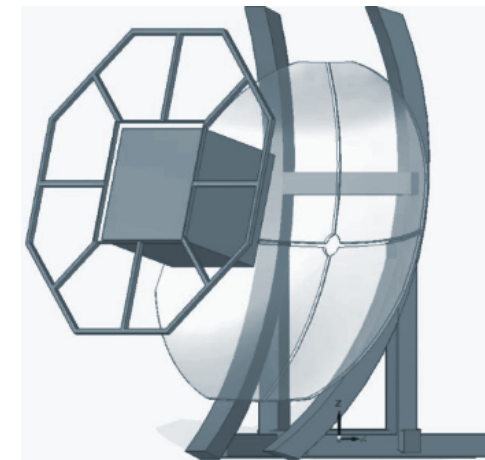
- Fixed recon. efficiency issue (30% → 95% @ 10^{20} eV)
- Developed two first guess methods, TSFEL DNN and Template Method (see backup)
 - **With TSFEL DNN + TDR, resolutions in $X_{\text{max}} \sim 30 \text{ g cm}^{-2}$ and energy $\sim 10\%$ achievable (FAST mini array)**
- Reconstructed coincidence events
 - Ok agreement between FAST results and TA/Auger results
 - First guess methods favour large energy / far core position → **need stereo observation**
- **First measurements** of cosmic ray energy spectrum and mass composition with FAST

Future

- **Stereo observation** with FAST Mini Array will verify viability of FAST design
- New prototype telescopes with reduced cost, autonomous (solar power), reduced electronics noise

Analysis:

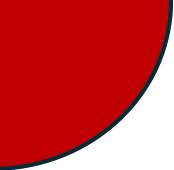
- Focus should now be to **improve on first guess methods** here so that they can be applied to upcoming FAST mini-array data!





Thank you for listening!

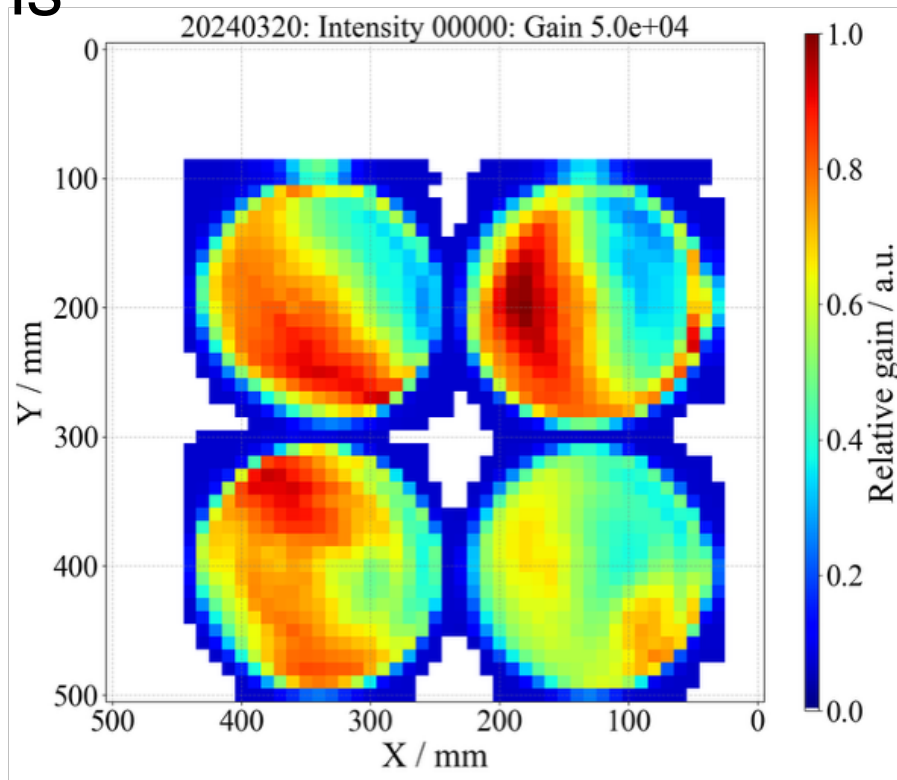




Backup

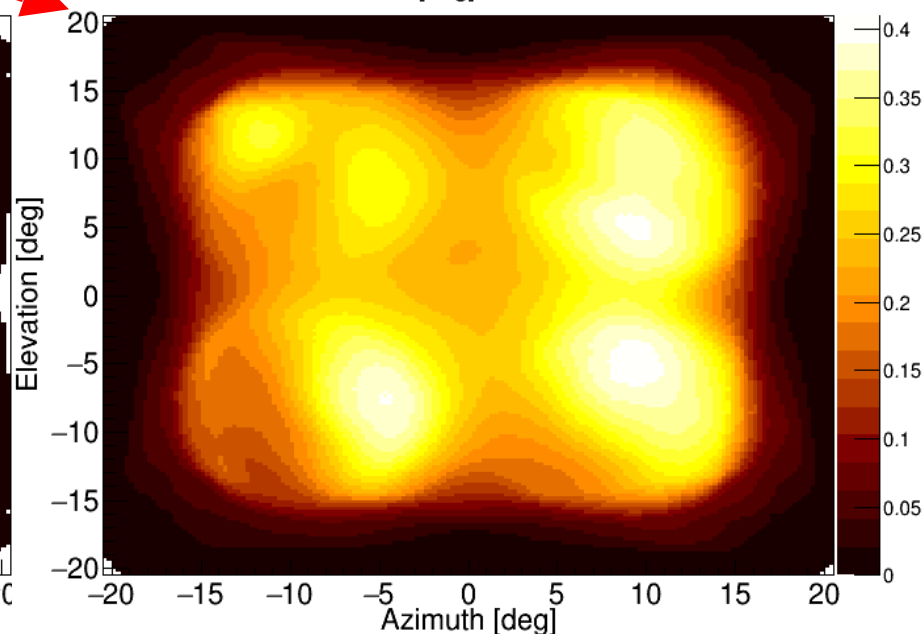
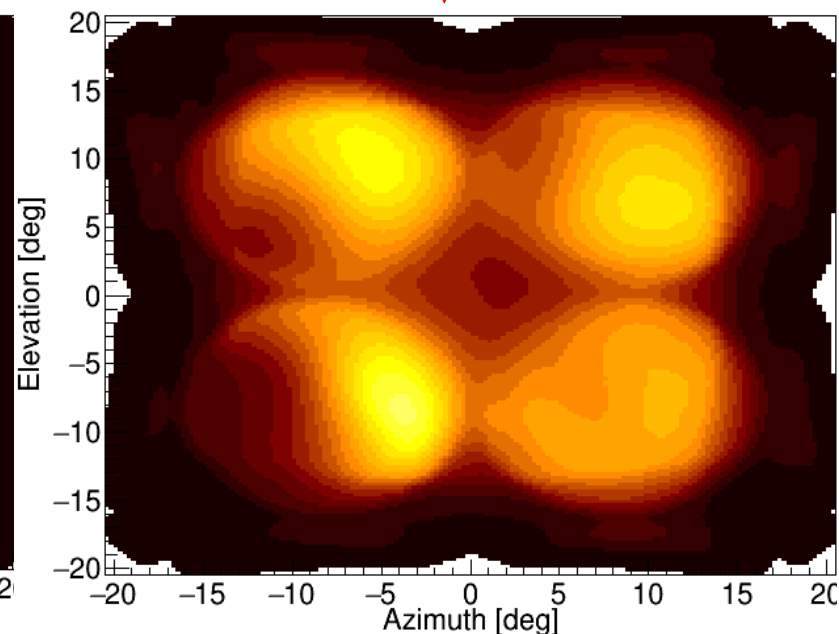
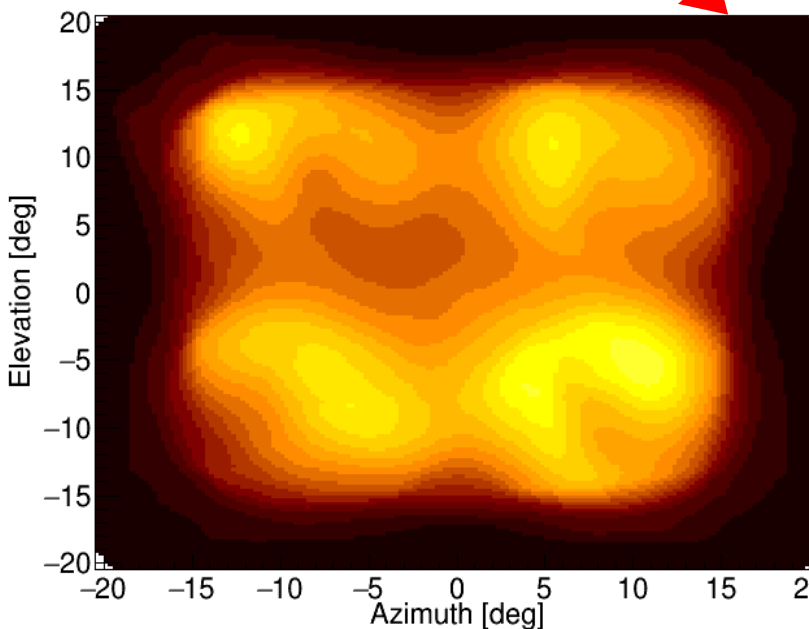
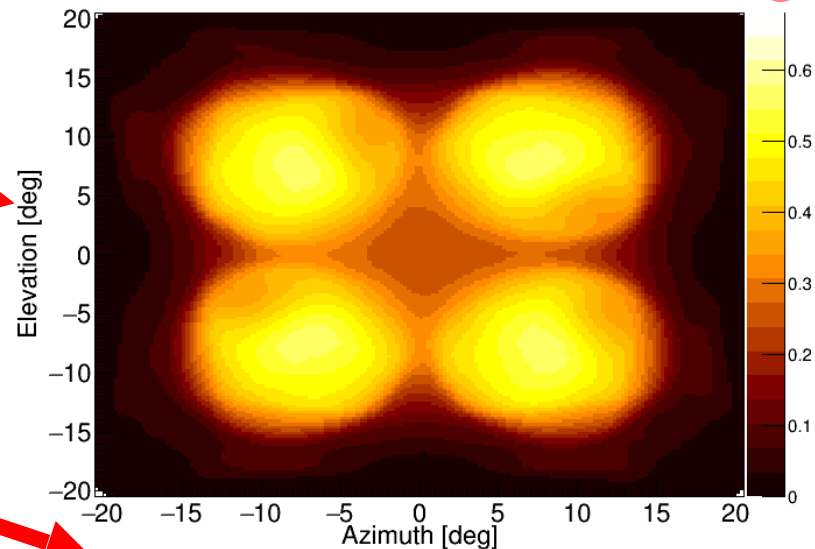
Possible sources of bias

- One possible cause of bias / large widths in the event-by-event difference distributions is **non-uniformity in the PMT gain response across PMT surface**
- Previous measurements taken in lab but no in situ measurements
 - Geo-magnetic field
- Measurements of FAST@TA telescope PMT's spatial efficiency in 2024



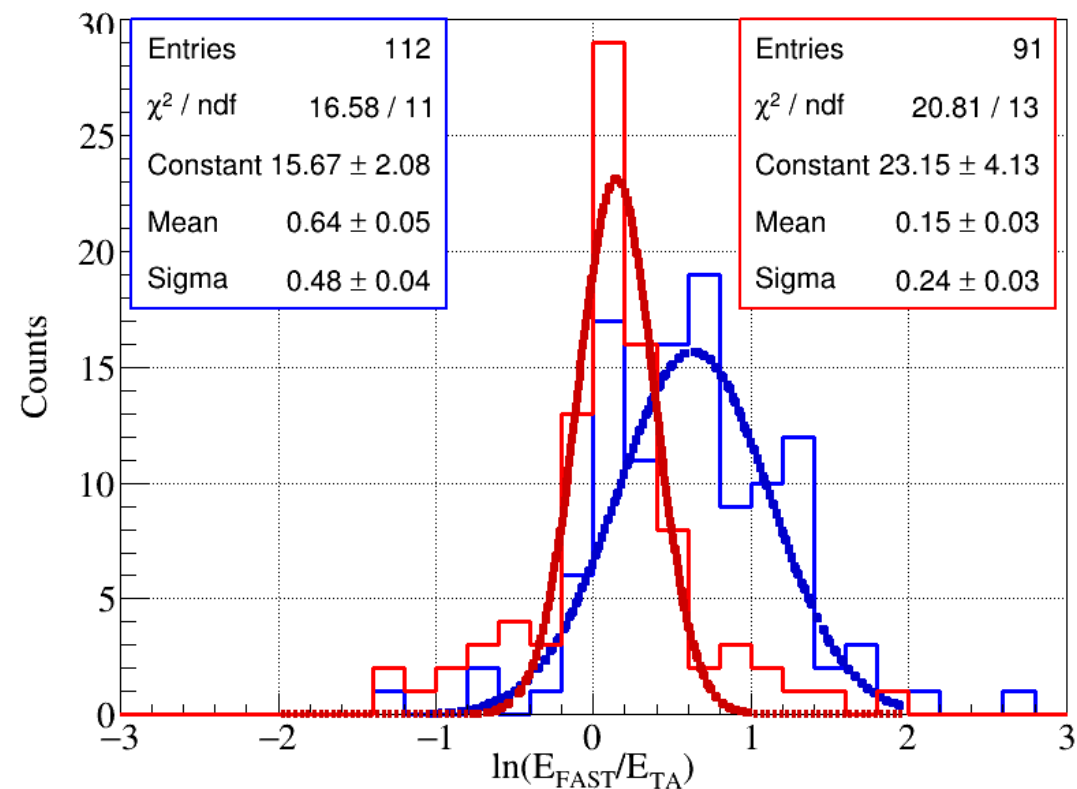
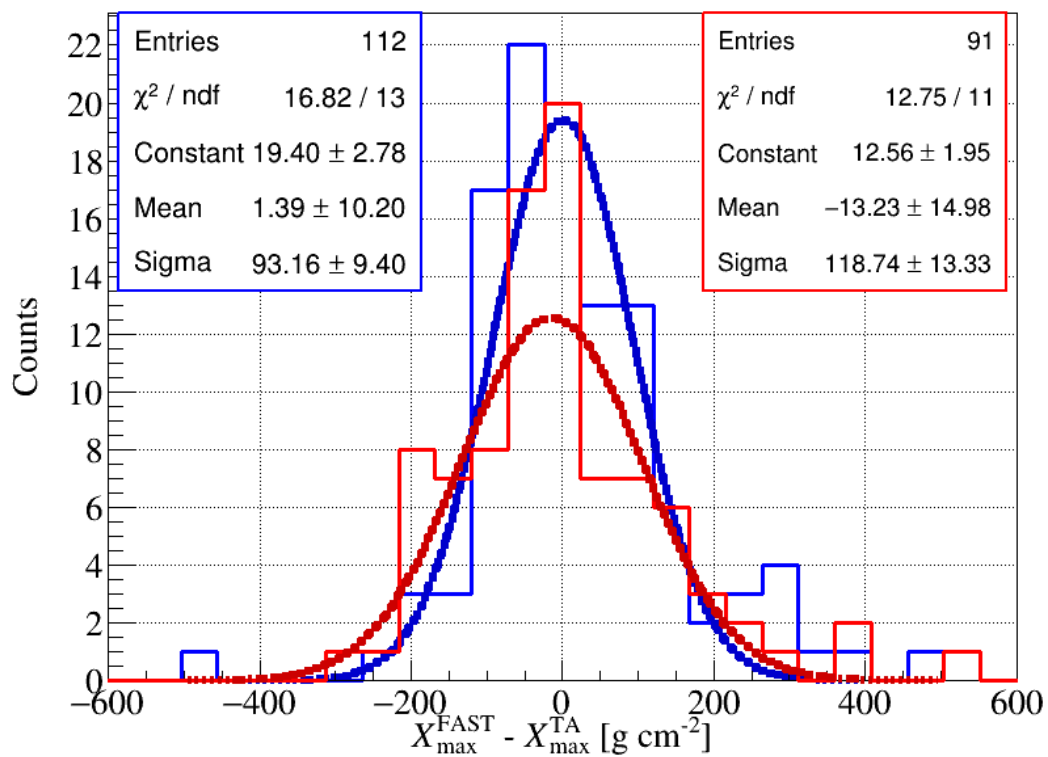
New directional efficiency maps

- Old Directional Efficiency Map
- New maps with updated PMT spatial efficiency measurements



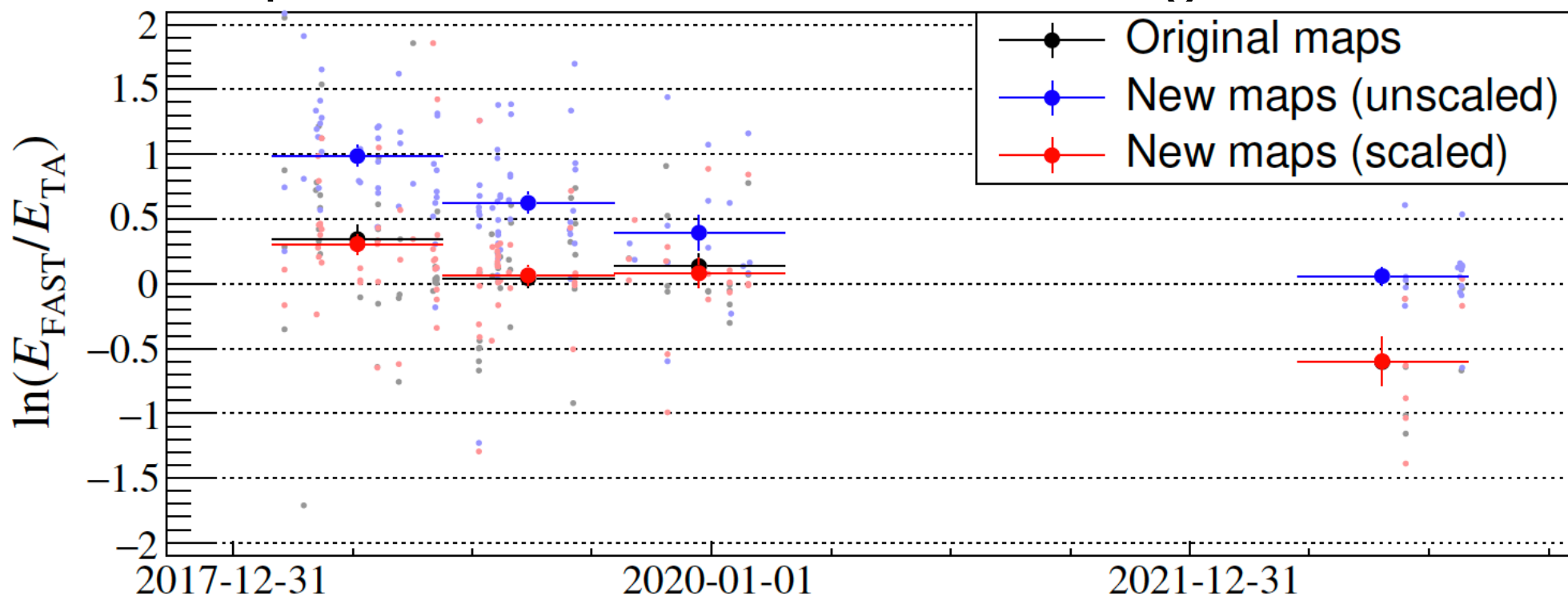
Reconstructing with new maps

- **Blue** = Reconstruction results using new maps (no scaling)
 - Large bias in energy
- **Red** = Reconstruction results using new maps with scaling
 - Match total integral of new maps to total integral of old maps ($\times 1.7$)



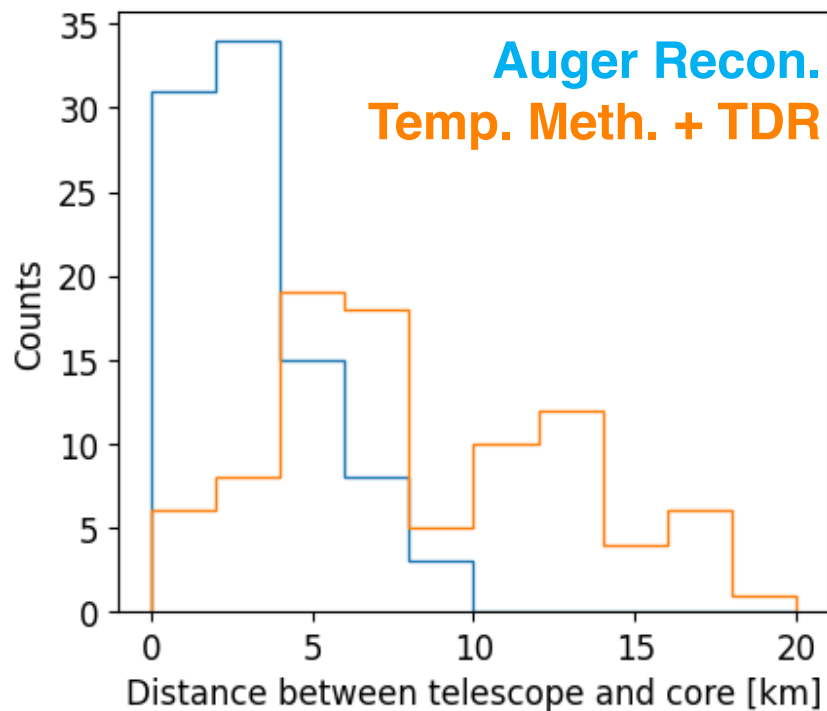
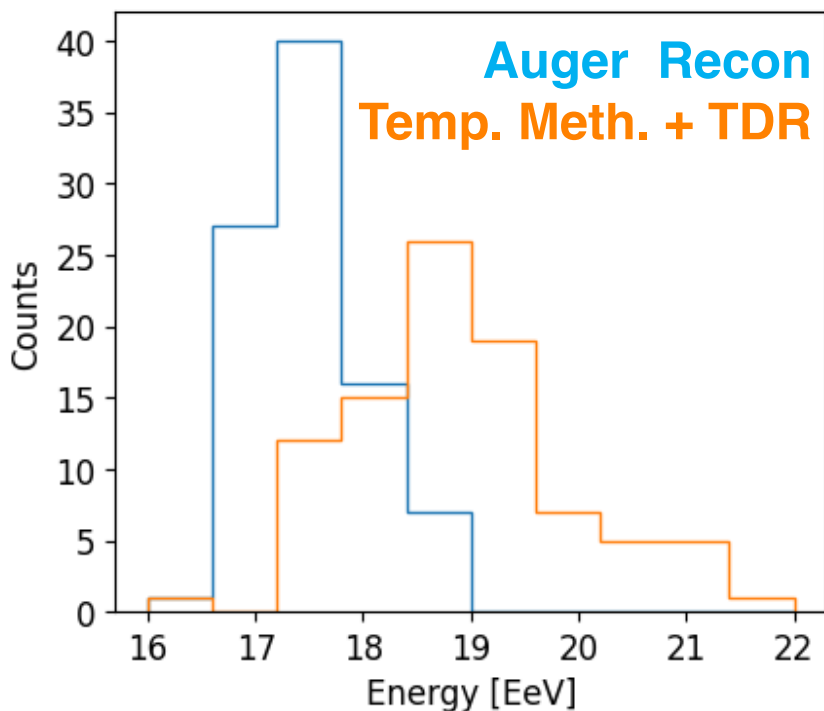
A time dependent energy bias?

- $\ln(E_{\text{FAST}} / E_{\text{TA}})$ decreasing with time
 - Has the structure of the PMT spatial efficiency changed? (unlikely...)
 - Time dependent calibration factor not being accounted for?

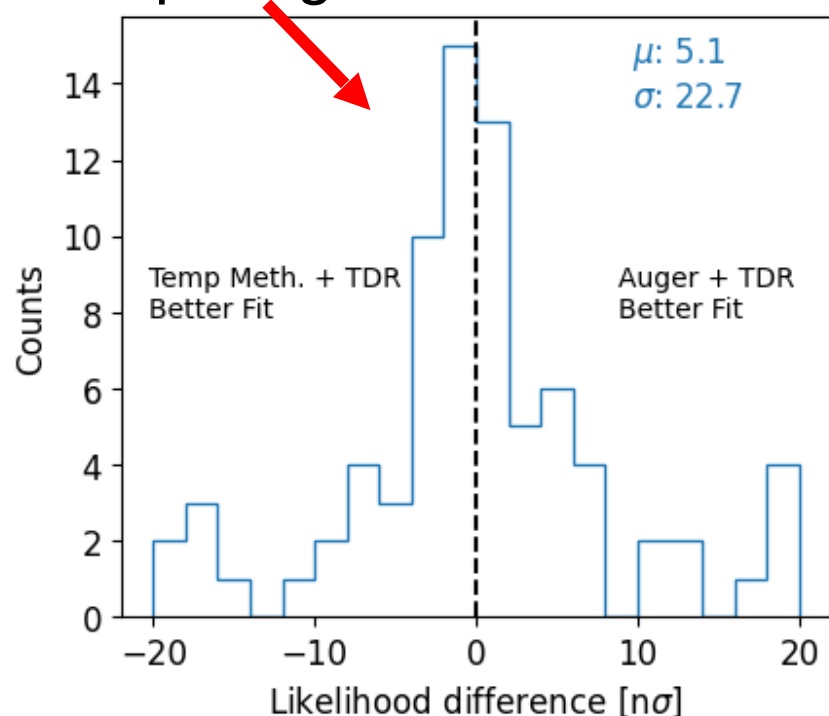


Template Method+ TDR?

- Try events with ≥ 2 triggered pixels. After recon. cuts (no $X_{\max}$ in FOV cut)
 - **FAST@TA:** ~ 170 events, **FAST@Auger:** ~ 90 events
- Find “good fits” but tendency to guess larger energy / further away core
 → **need stereo observation**



Comparing which fit is better

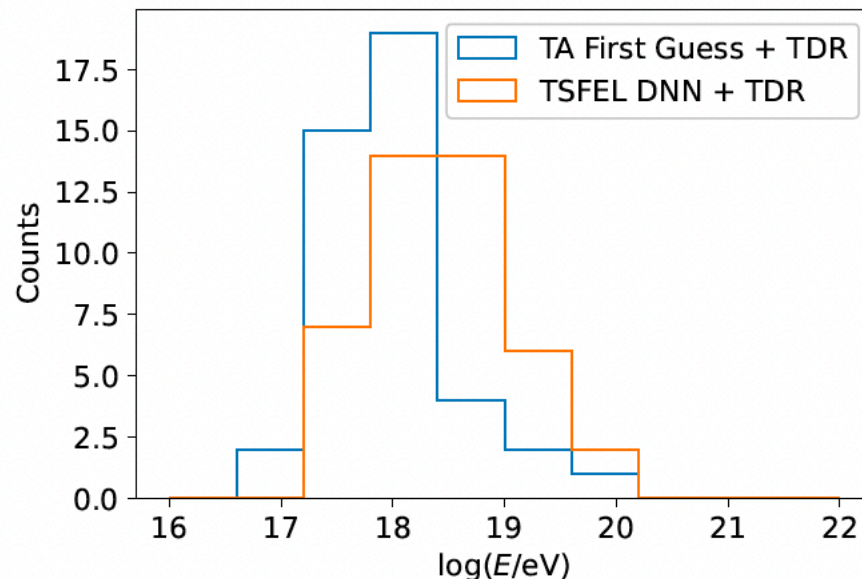
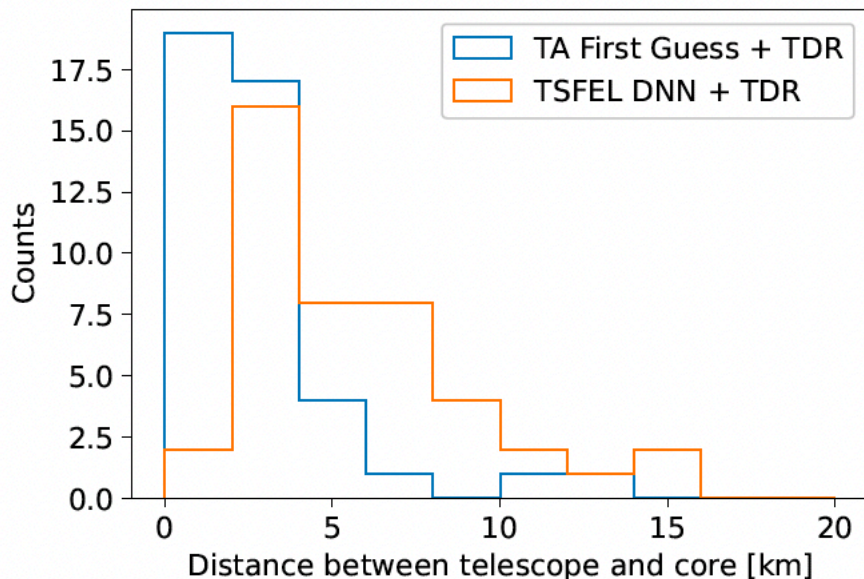


Full Reconstruction: Setup

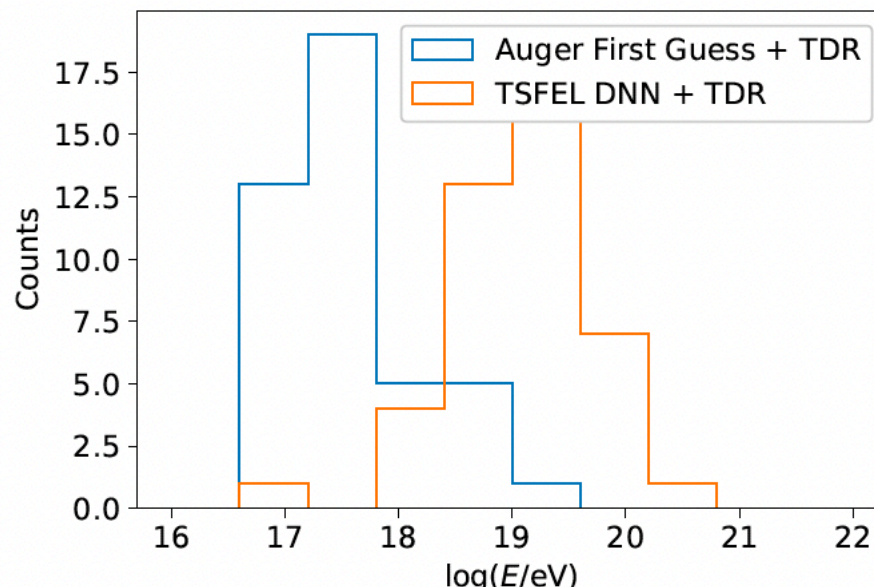
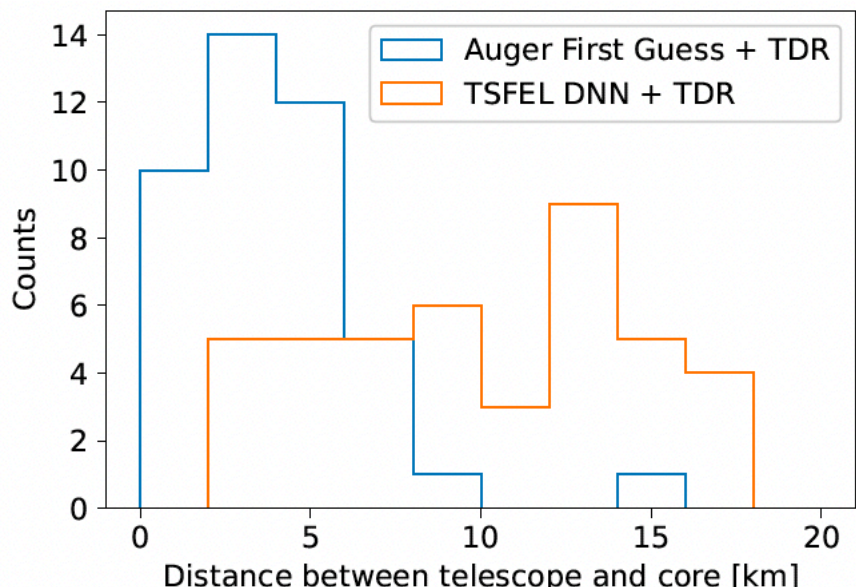
TSFEL DNN First guess + TDR

- Use first guess from TSFEL DNN as input to TDR
 - Only use first guesses with values inside the range of the training data
- TDR cuts
 - Successful minimization
 - $X_{\max}$ in FOV
 - Relative uncertainty in $X_{\max}$ and energy both < 0.5
 - Absolute uncertainty in core x and core y both < 1000 m

TSFEL DNN + TDR?



TA



Auger

- Tend to guess further away shower with higher energy...

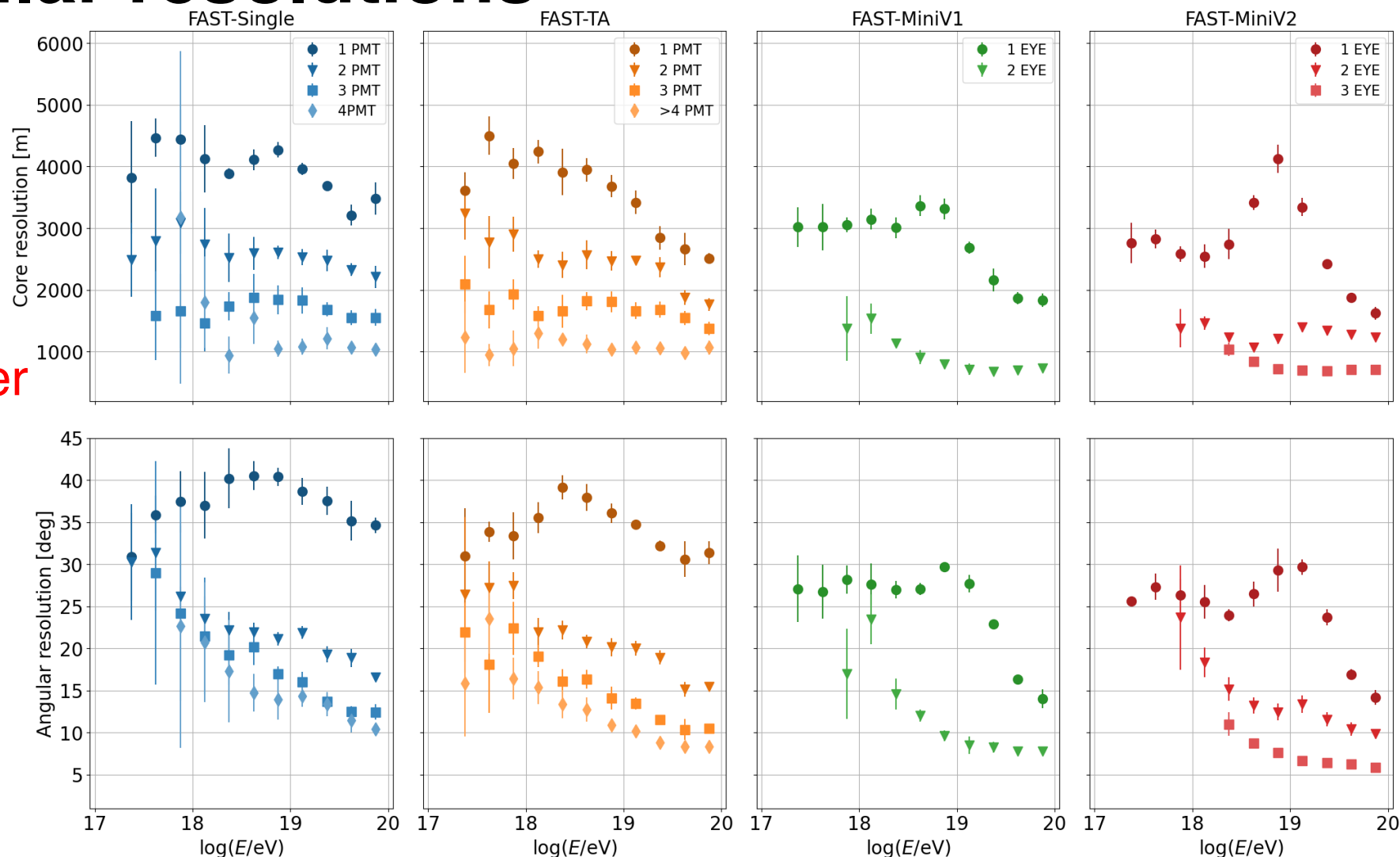
TSFEL DNN Performance

Core and angular resolutions*

Core

More PMTs & eyes is better

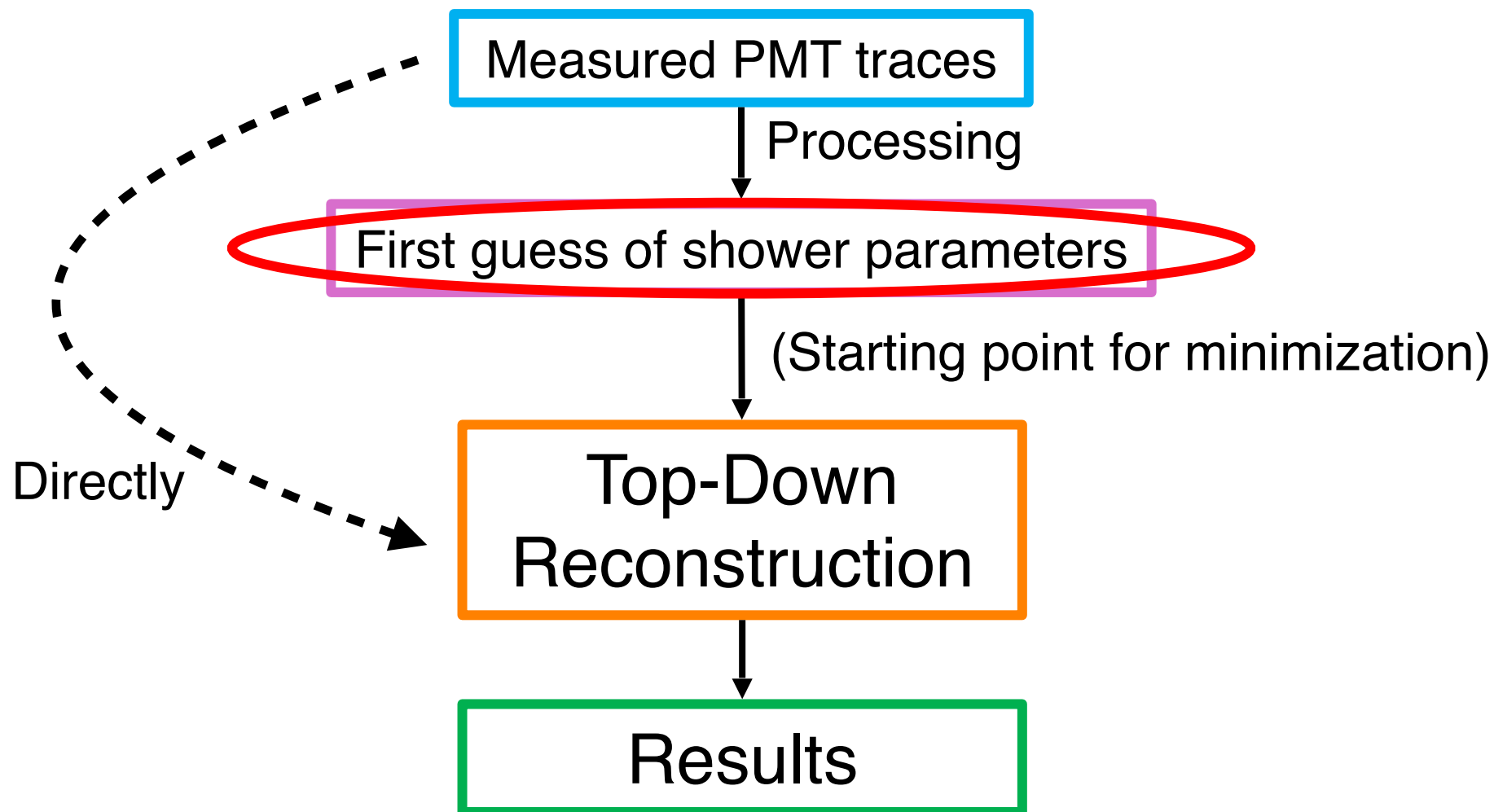
Angular



*Core distance / opening angle within which 68% of reconstructed events lie

Overview

- Flow of the FAST reconstruction



Discontinuity Sources

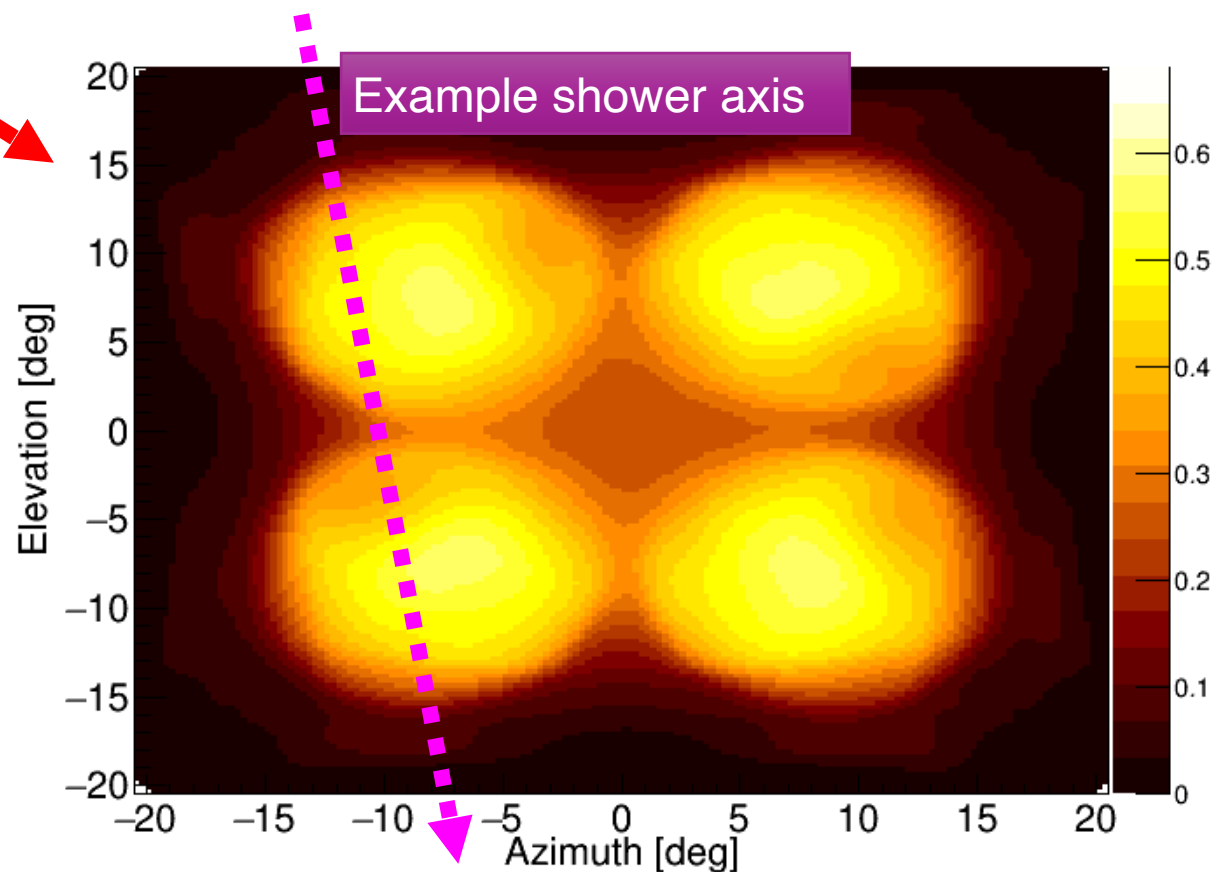
Three Main Issues:

Shower Axis Binning

Azimuthal Difference

Directional Efficiency

- Directional efficiency map (amount to scale signal based on position of shower axis bin in telescope FOV)
 - Only defined up to $\pm 20^\circ$



Discontinuity Sources

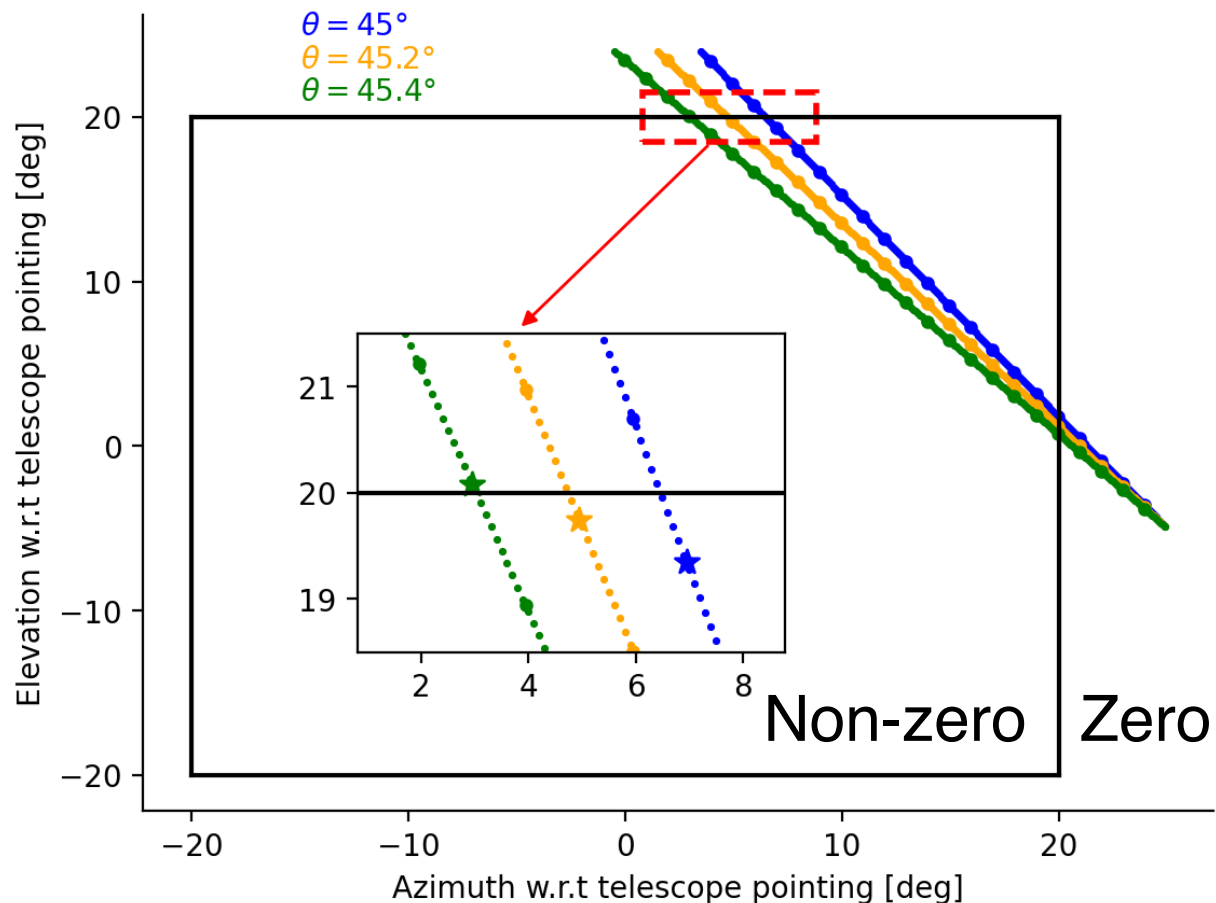
Three Main Issues:

Shower Axis Binning

Azimuthal Difference

Directional Efficiency

- Directional efficiency map (amount to scale signal based on position of shower axis bin in telescope FOV)
 - Only defined up to $\pm 20^\circ$
- Small change in geometry can cause bin to go from inside to outside map range
 - signal drops to 0
 - discontinuity



Discontinuity Sources

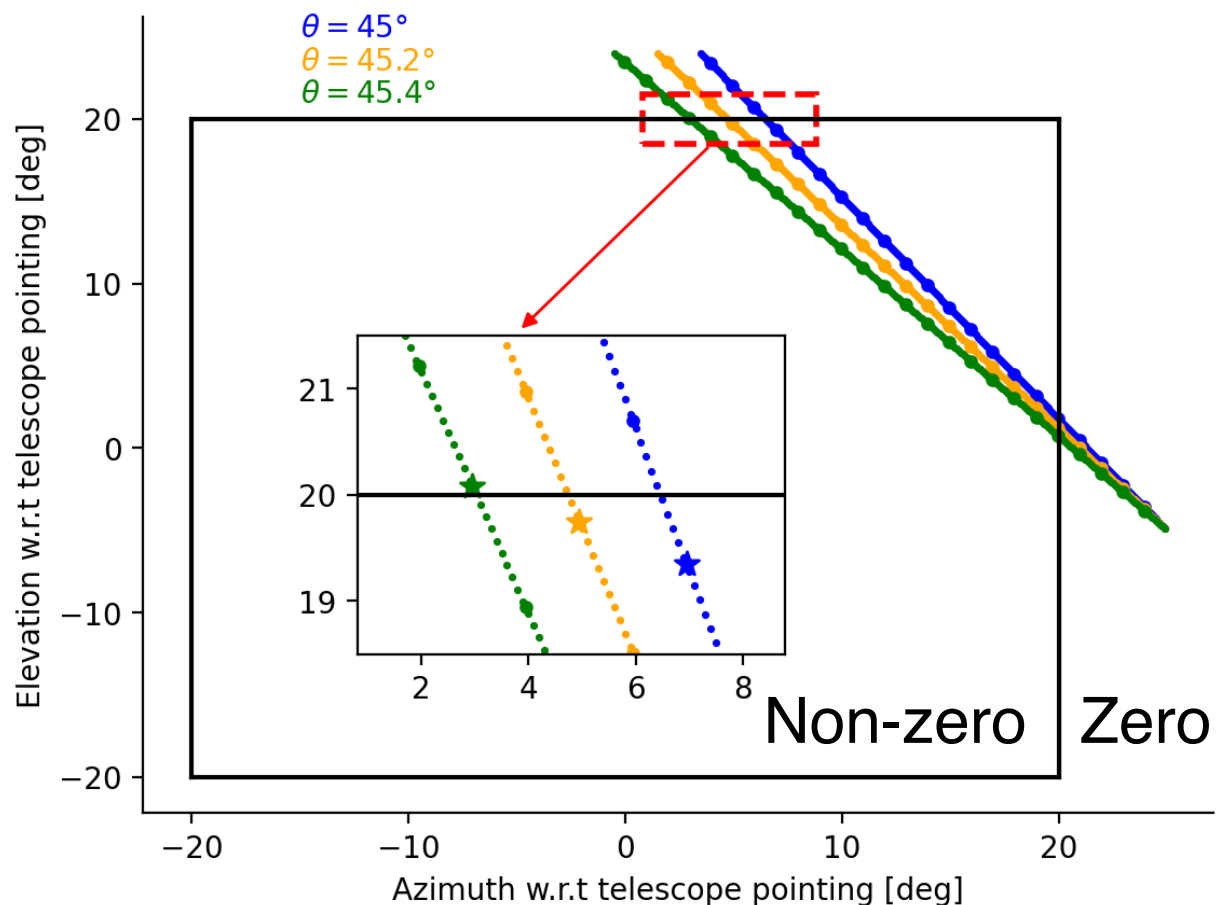
Three Main Issues:

Shower Axis Binning

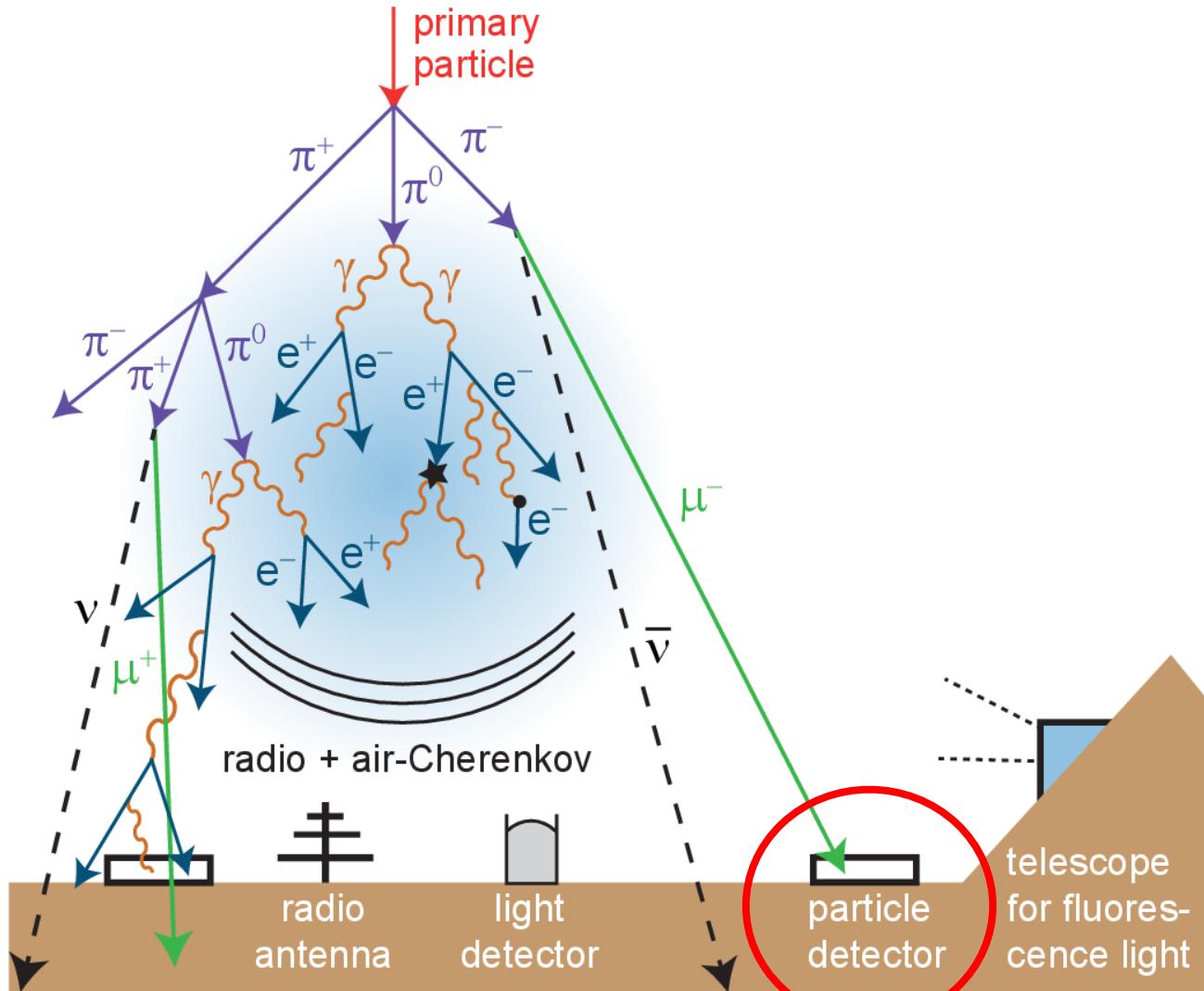
Azimuthal Difference

Directional Efficiency

- Fix by smoothing map and using finer time binning
→ reduces size of discontinuities



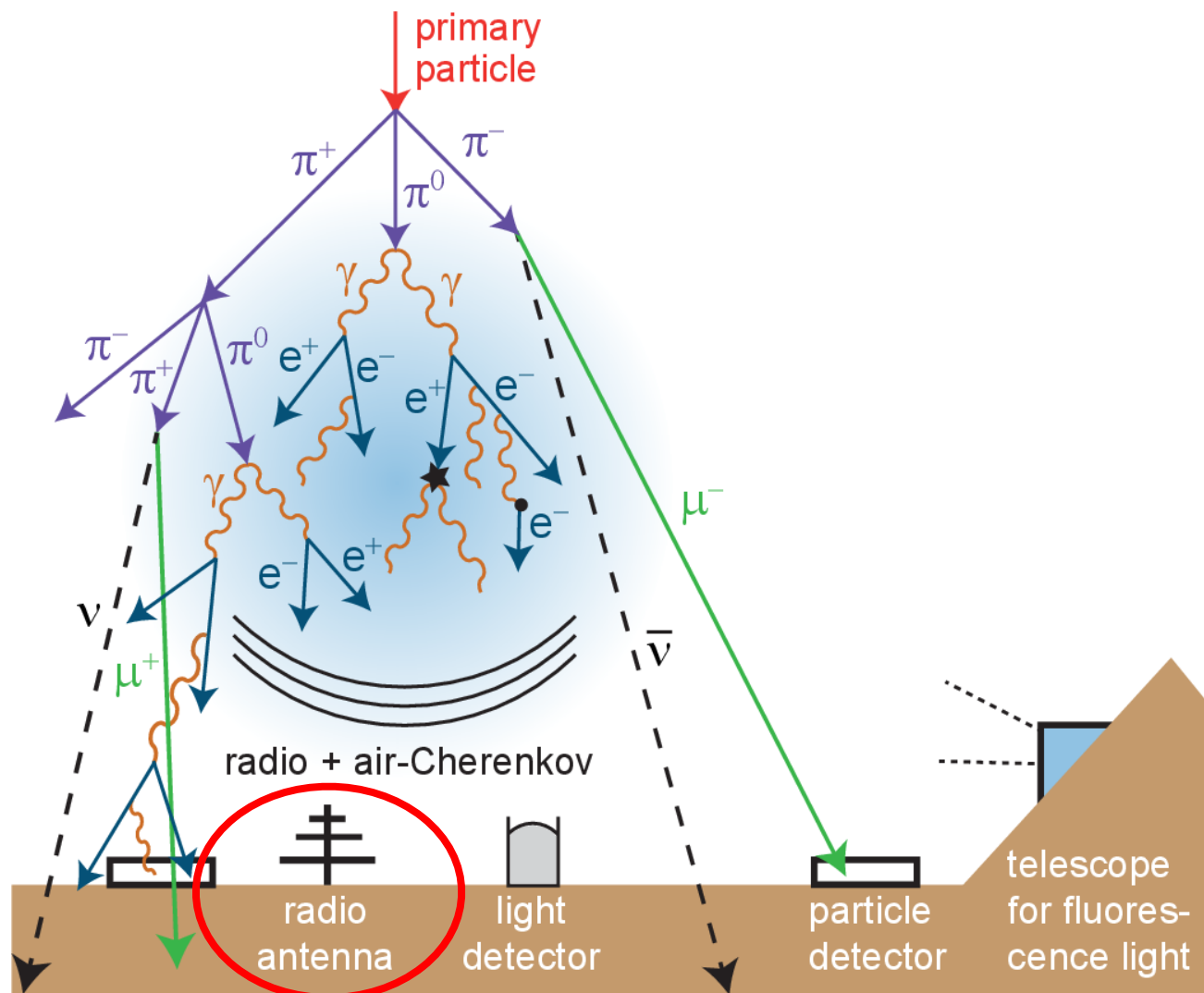
Extensive Air Showers: Detectors



Frank Schröder (2017)



Extensive Air Showers: Detectors

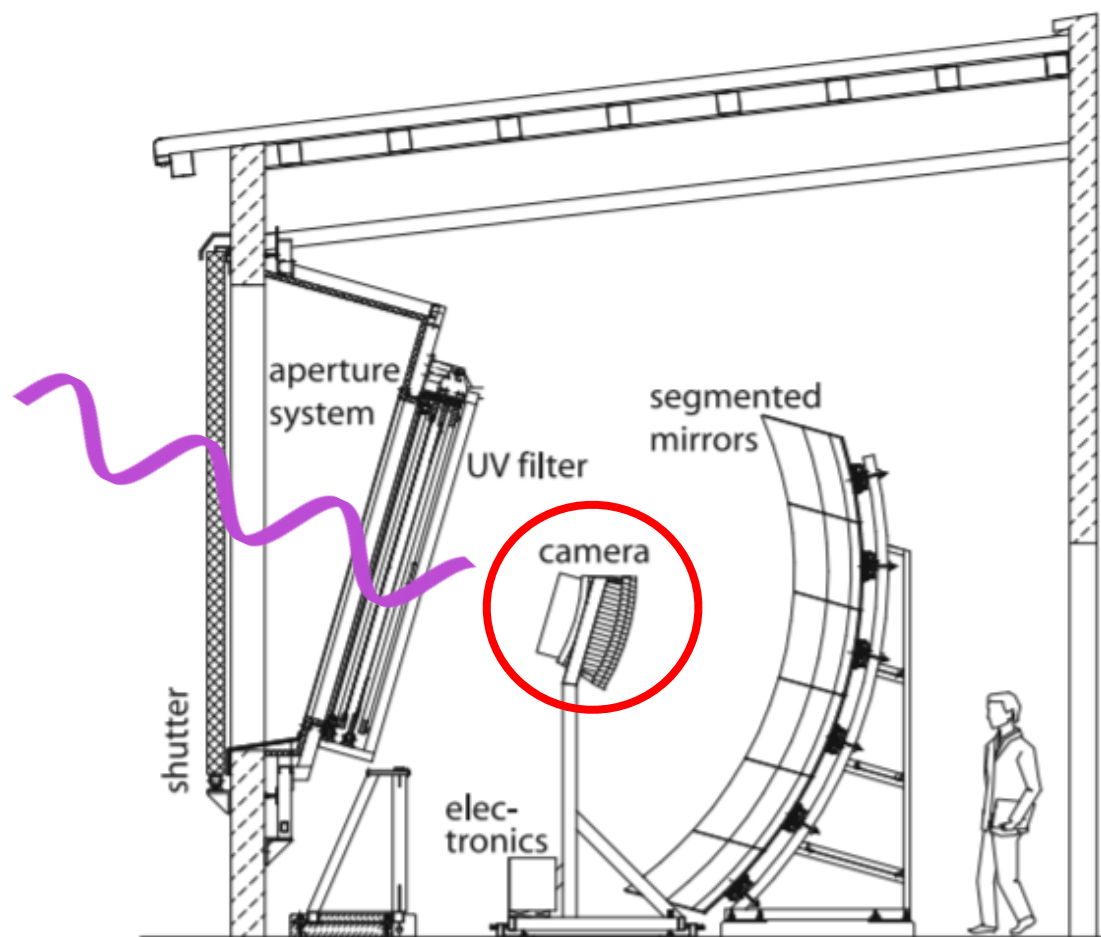


Frank Schröder (2017)



Current Largest Experiments

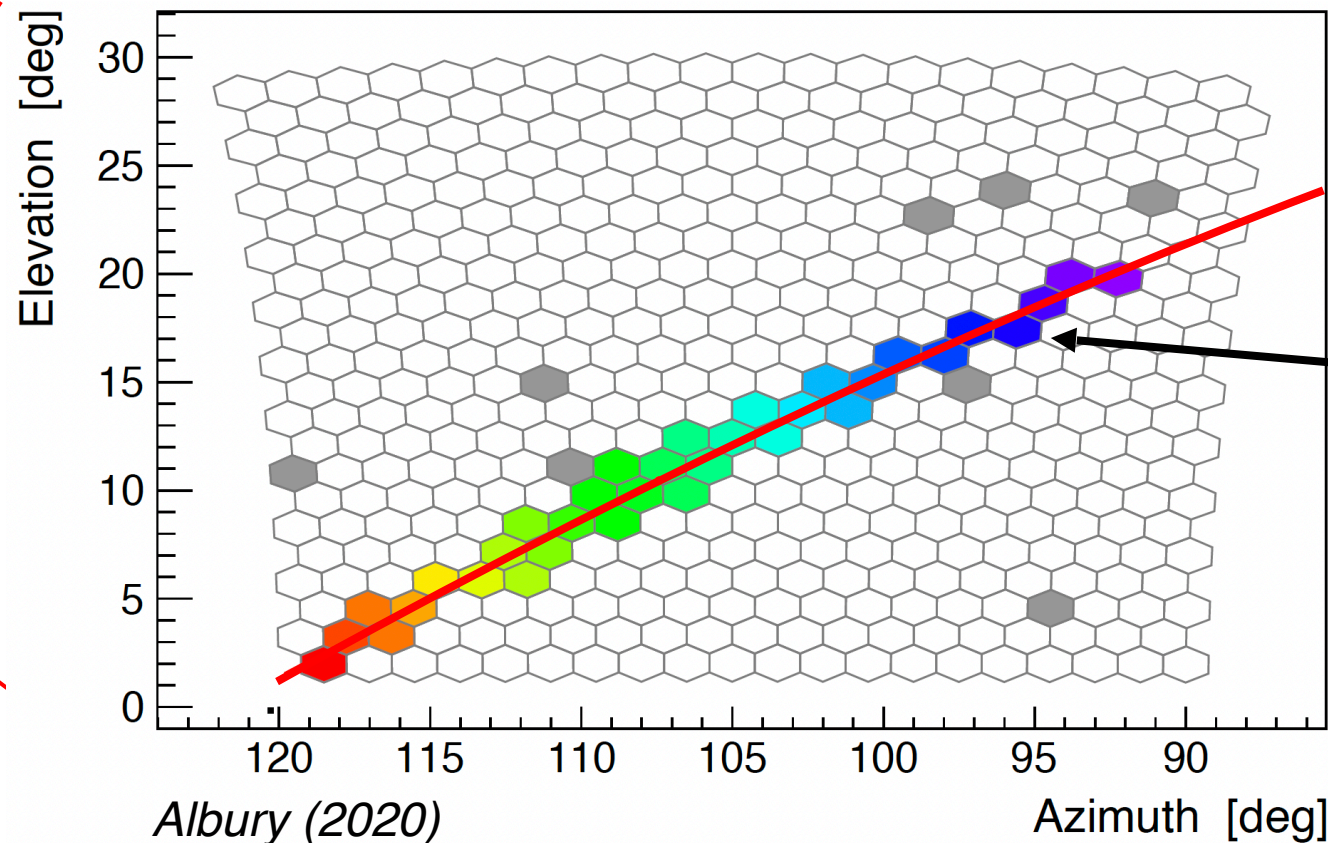
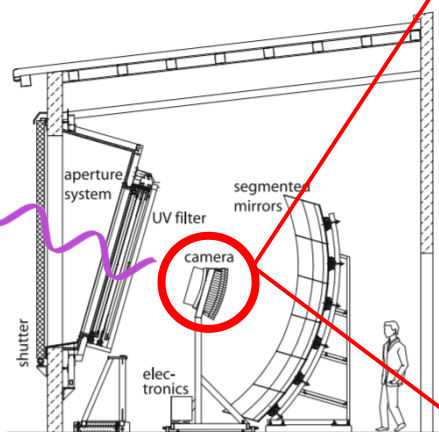
How do the Auger and TA fluorescence telescopes reconstruct the shower geometry?



Abraham (2010)

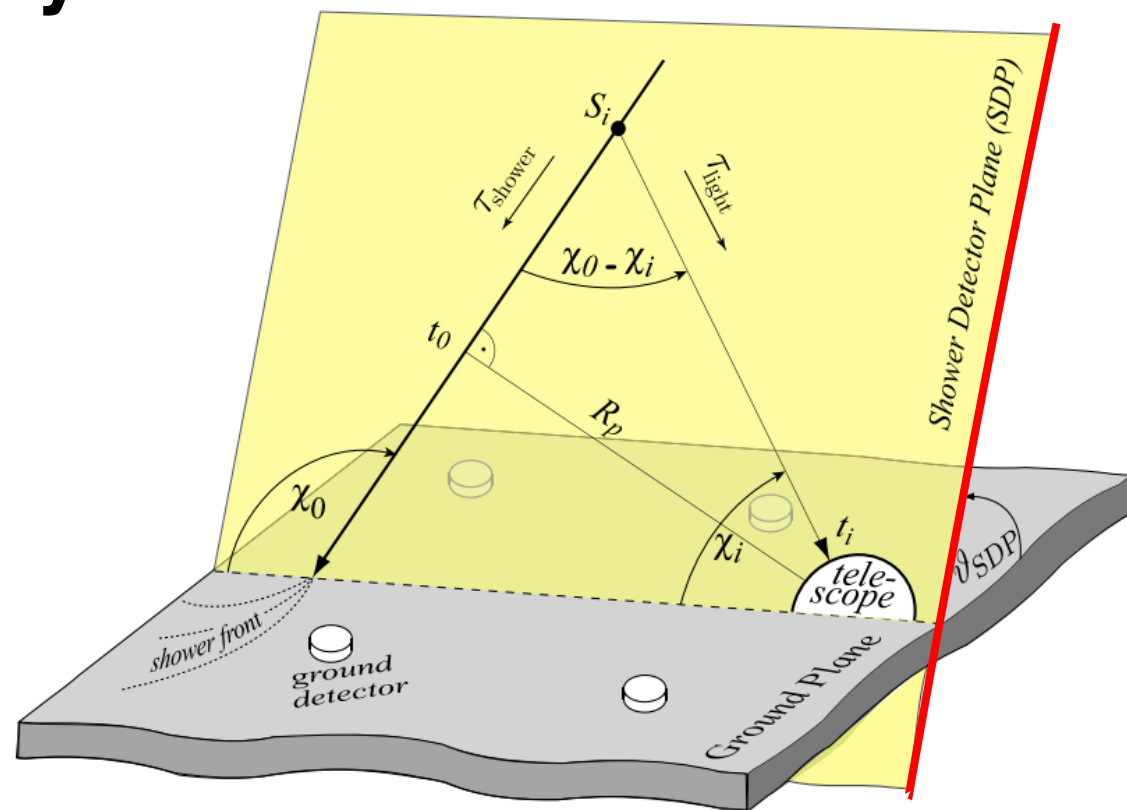
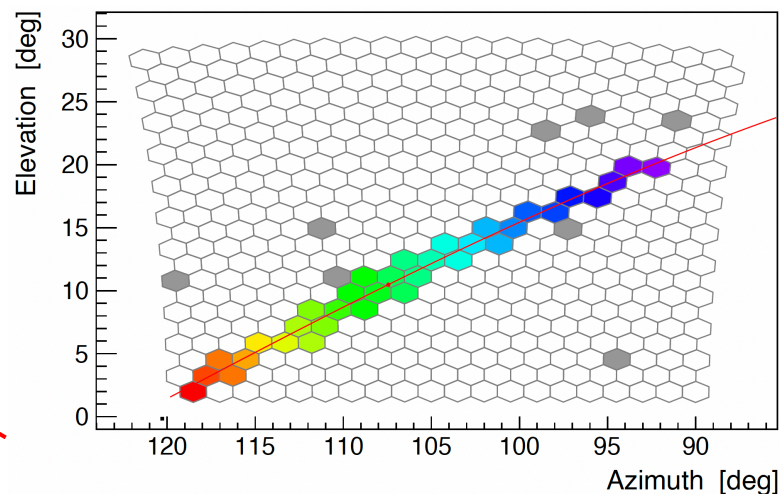
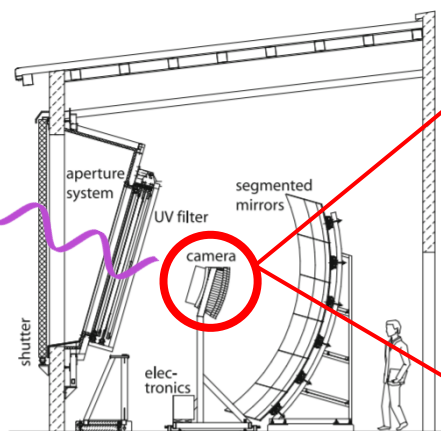
Current Largest Experiments

How do the Auger and TA fluorescence telescopes reconstruct the shower geometry?



Current Largest Experiments

How do the Auger and TA fluorescence telescopes reconstruct the shower geometry?



Kuempel (2008)

Possible because of finely pixelated cameras!

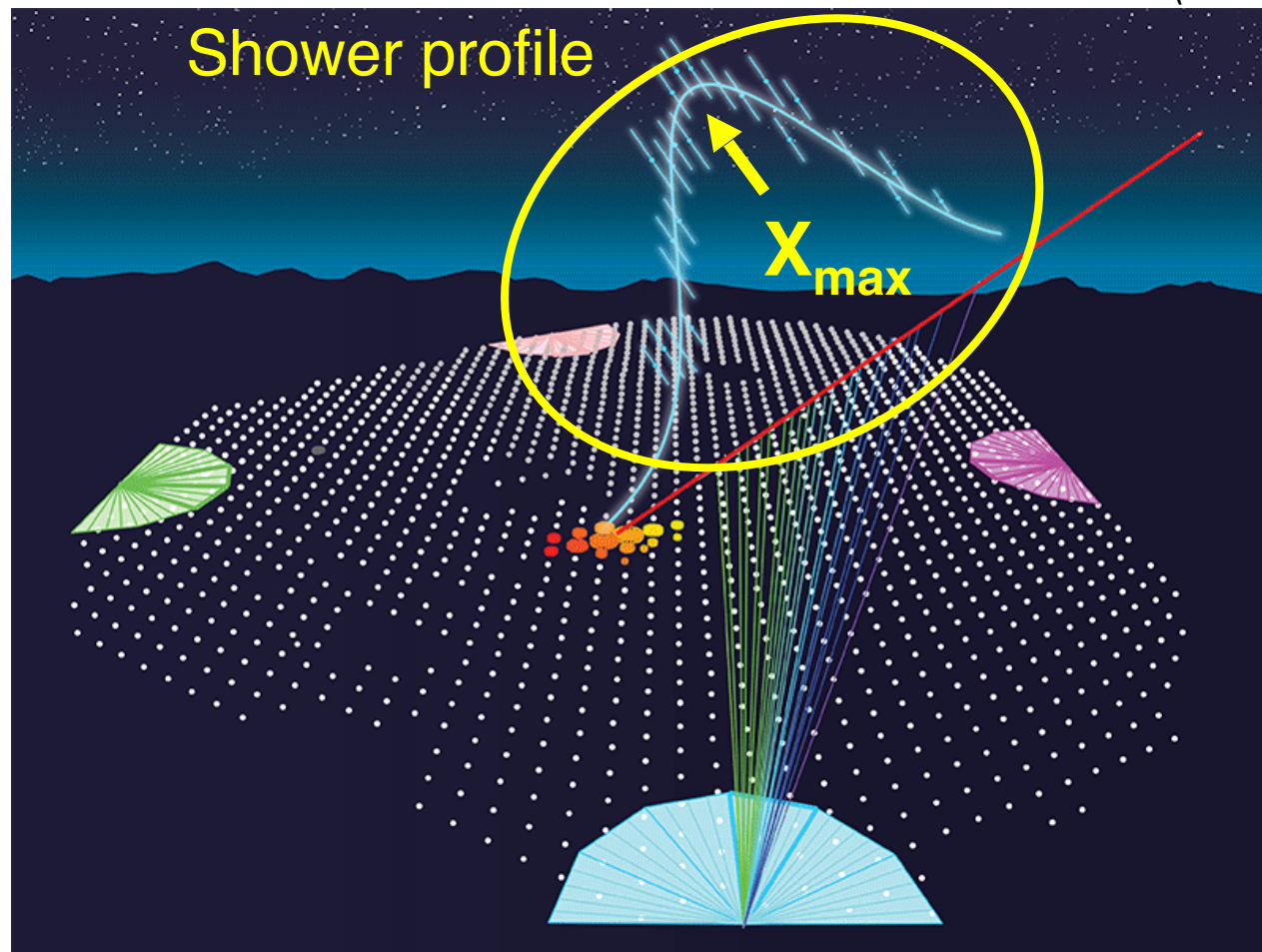
Cameras at: Auger – 440 pixels, TA – 256

Current Largest Experiments

How do the Auger and TA fluorescence telescopes reconstruct $X_{\max}$ and energy?

Thomas Gaisser (2016)

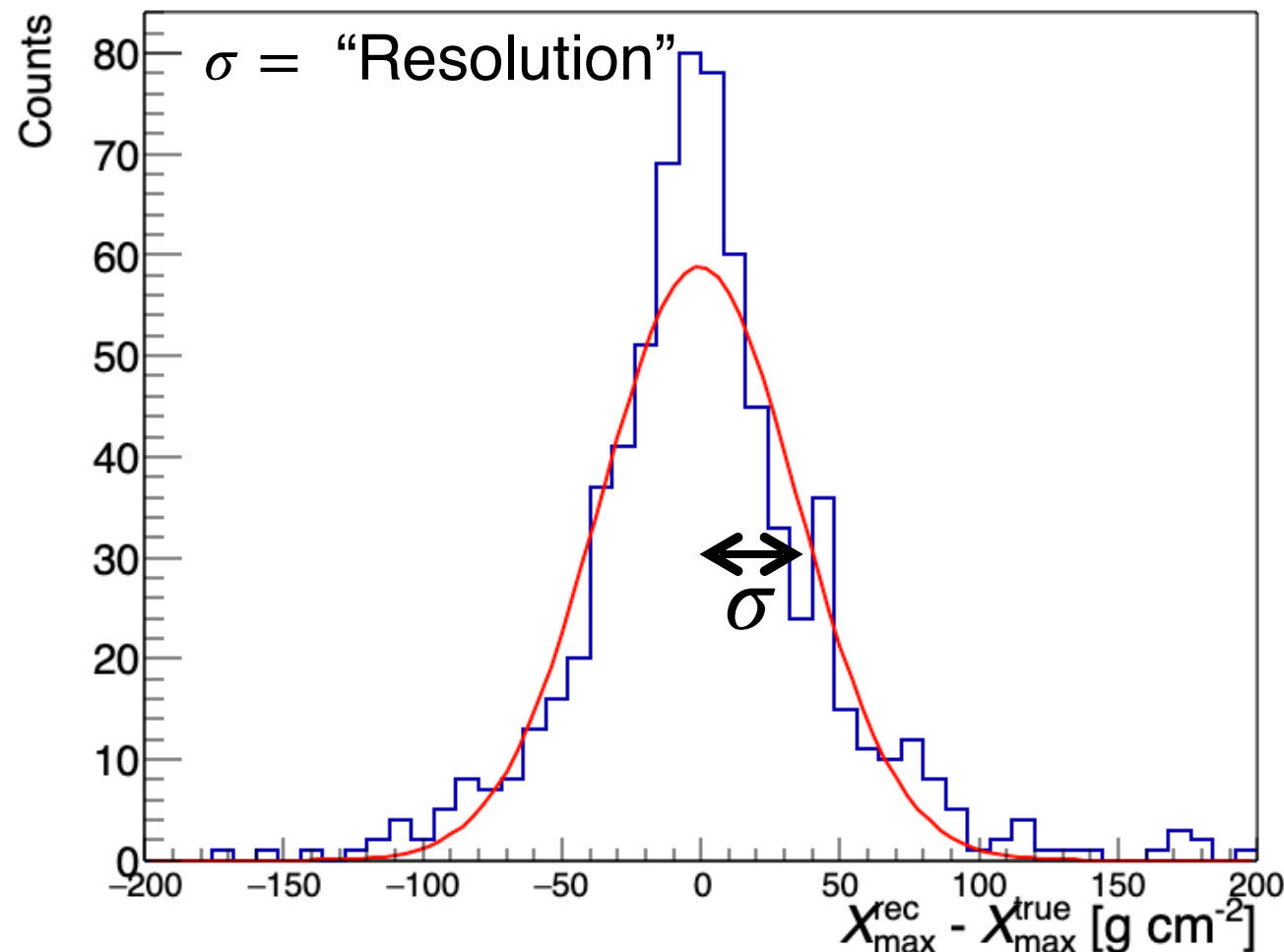
- Determine the **shower profile**
 - Amount of fluorescence light emitted by the shower as a function of atmospheric depth
 - $X_{\max}$ from profile maximum, Energy from integral



Current Largest Experiments

How do the Auger and TA fluorescence telescopes reconstruct $X_{\max}$ and energy?

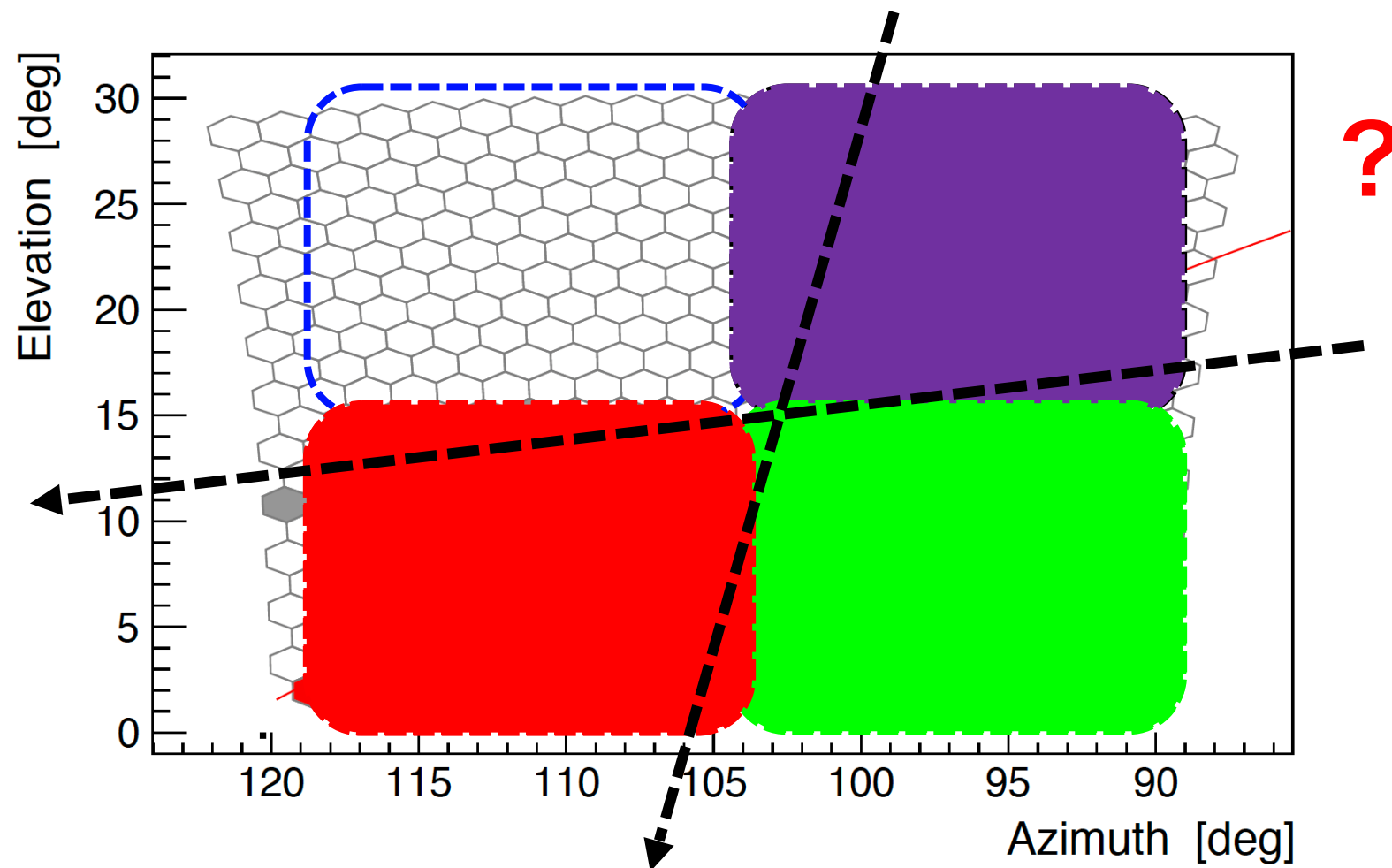
- Determine the **shower profile**
 - Amount of fluorescence light emitted by the shower as a function of atmospheric depth
 - $X_{\max}$ from profile maximum, Energy from integral
- **Resolutions** (reconstruction precision)
 - $X_{\max} \sim 25 \text{ g cm}^{-2}$,
 - Energy $\sim 10\%$



FAST: Top-Down Reconstruction (TDR)

**How does FAST
reconstruct the shower
parameters?**

**With just 4 pixels – can't
utilize same geometrical
reconstruction procedures
as Auger/TA!**



Discontinuity Sources

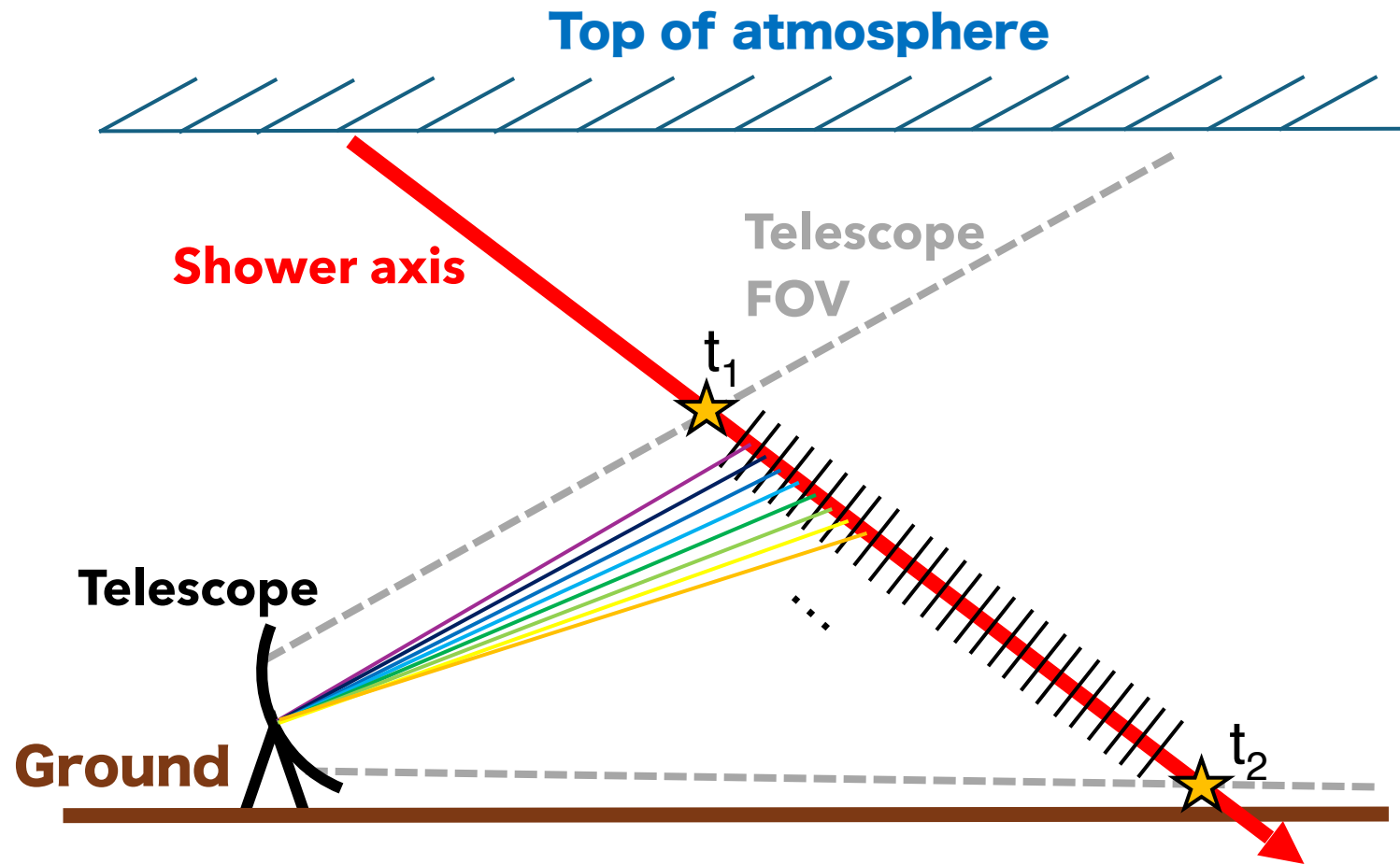
Three Main Issues:

Shower Axis Binning

- Signal in a trace bin comes from a single bin on shower axis
- This signal depends on bin size and position
- Old binning method caused discontinuities

Azimuthal Difference

Directional Efficiency

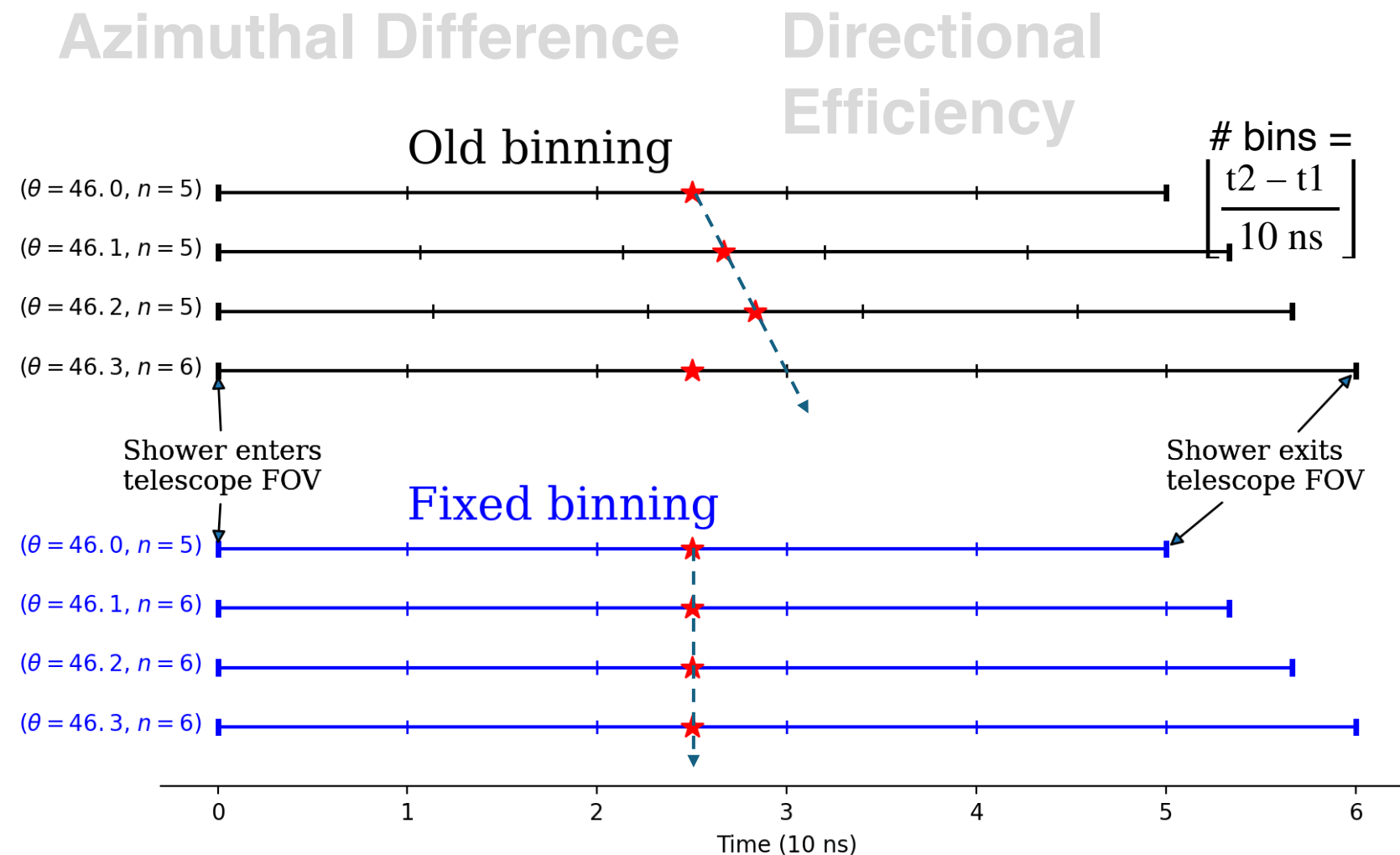


Discontinuity Sources

Three Main Issues:

Shower Axis Binning

- Signal in a trace bin comes from a single bin on shower axis
- This signal depends on bin size and position
- Old binning method caused discontinuities



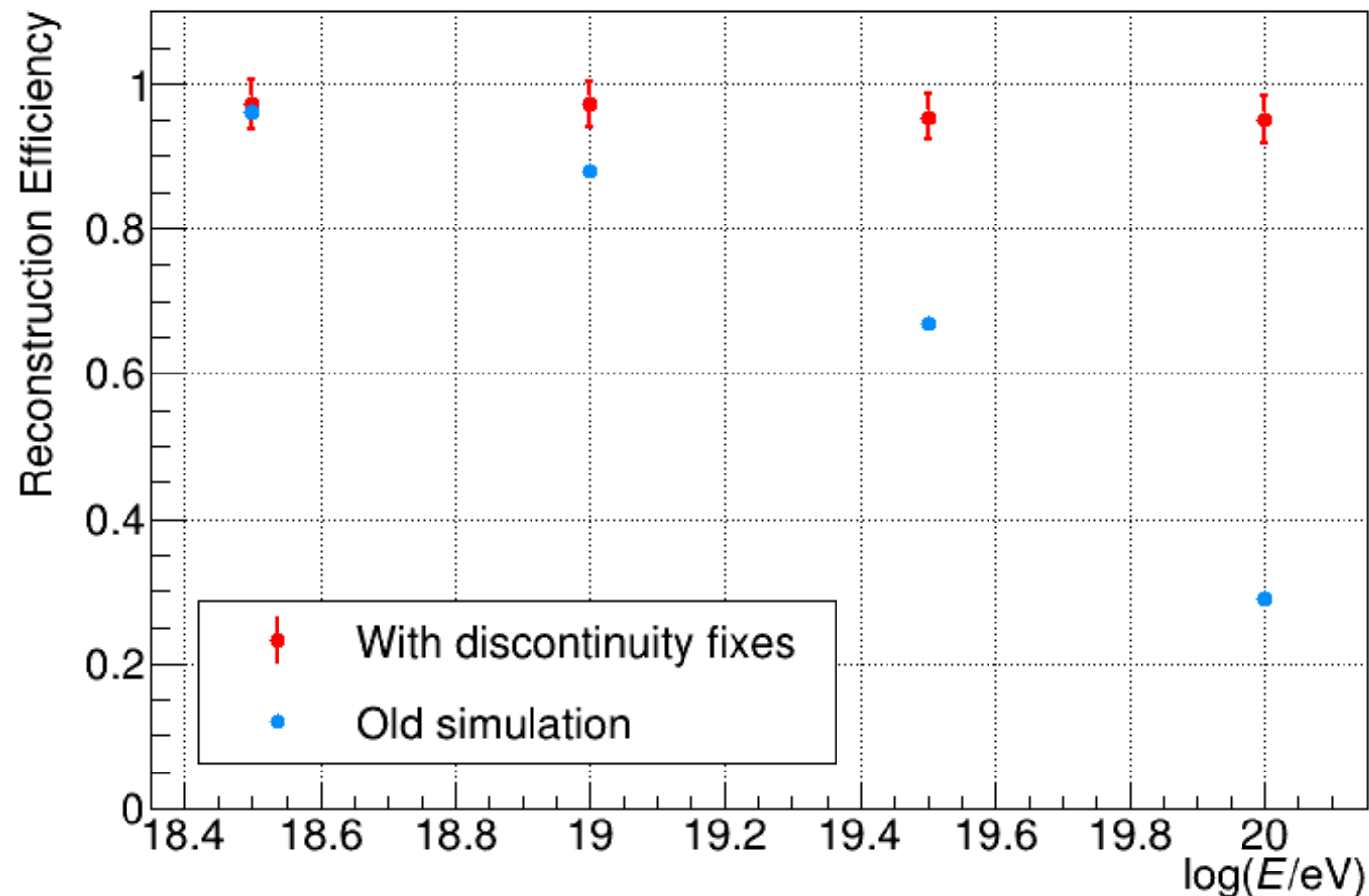
Shower Axis Binning

Directional Efficiency

-

Updated Efficiency

- Using updated simulation, perform “same” reconstruction as Albury – TDR with first guess of true values with smearing
 - **$X_{\max}$** : 40 g cm⁻²
 - **Energy**: 10%
 - **Core**: 300 m
 - **Arrival direction**: 3°
- Efficiency (i.e. minimisation success) now ~ 95% across whole energy range



Model Comparison

- Perform hyper-parameter optimization with Basic DNN and TSFEL DNN
 - Hidden layer node structure 512/256/128/64/32
 - Learning rate 0.0003
- No hyper-parameter search for LSTM (time constraints)
- Train on full dataset and compare validation losses (MSE)

	FAST-Single	FAST-TA	FAST-MiniV1	FAST-MiniV2
Basic	0.1144	0.0827	0.0765	0.0685
TSFEL	0.1006	0.0689	0.0649	0.0570
LSTM	0.0906	0.0672	0.0632	0.0565

Better than
Basic DNN

- **TSFEL DNN faster, more robust (?) when applied to data, so proceed with it**

Full Reconstruction: Setup

TSFEL DNN First guess + TDR

- Use first guess from TSFEL DNN as input to TDR
 - Only use first guesses with values inside the range of the training data
- TDR cuts
 - Successful minimization
 - $X_{\max}$ in FOV
 - Relative uncertainty in $X_{\max}$ and energy both < 0.5
 - Absolute uncertainty in core x and core y both < 1000 m

Neural Networks: Summary

Summary

- With only 1 or 2 pixels, can't predict parameters very well
- Need 3 or more pixels in one eye or stereo observation
- ML + TDR shows ideal theoretical resolutions in $X_{\max} \sim 30 \text{ g cm}^{-2}$ and $< 10\%$ for energy – promising! But need to include additional reality...

Shortcomings

- Need to train different models for different layouts
- No uncertainty estimate on first guess

First Guess II – Template Method

Overview

- Is there a way to produce a first guess regardless of the telescope layout?
- Can we get an uncertainty on the first guess?

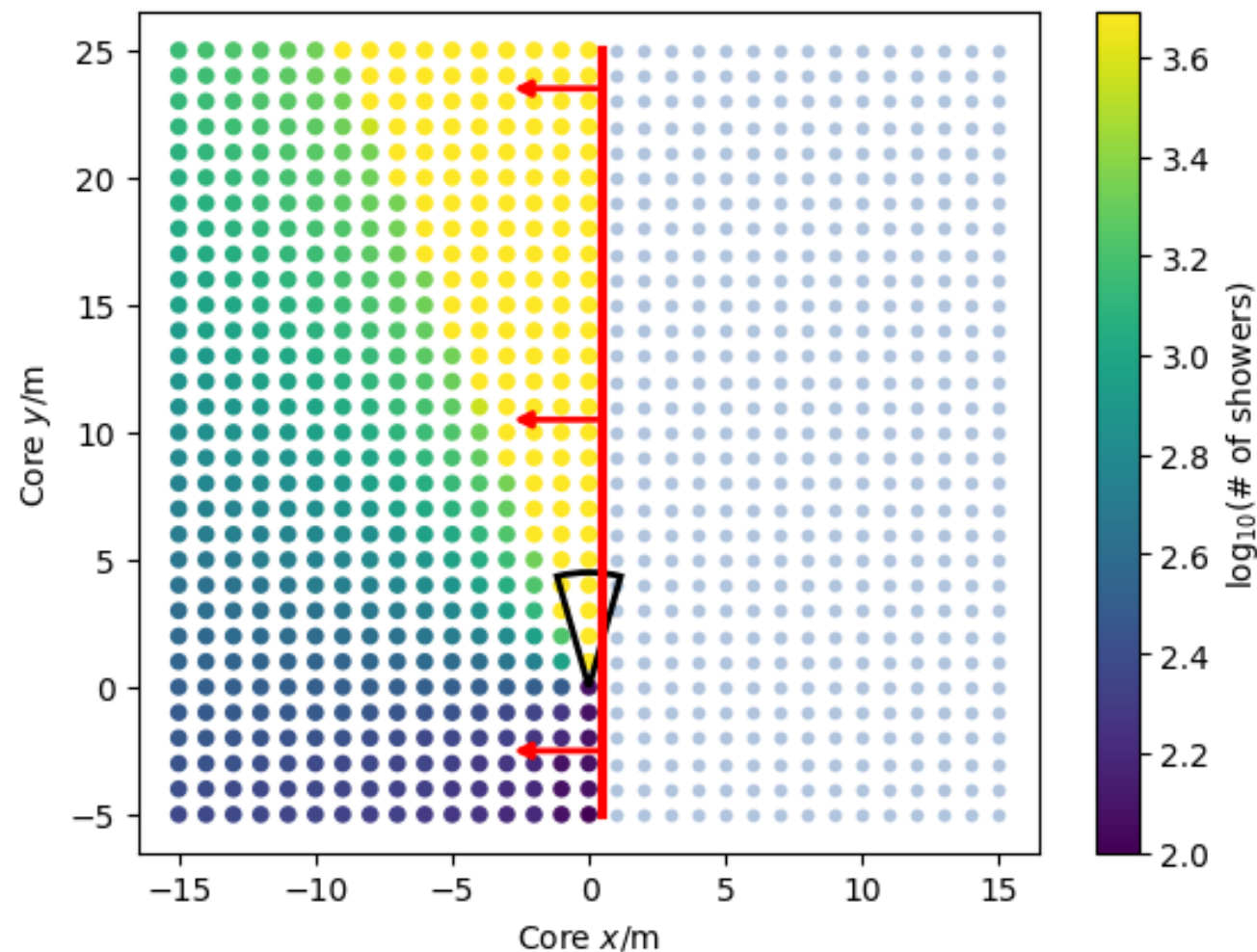
Template Method

- Generate a library of template traces for a single telescope
- For a set of data traces, compare traces to each template and **select the best match via the same maximum likelihood estimation as TDR.**
Can do for each telescope and combine results

Template Library

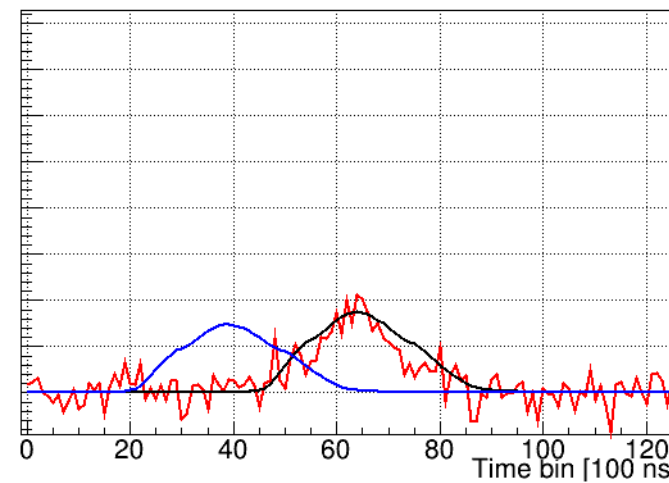
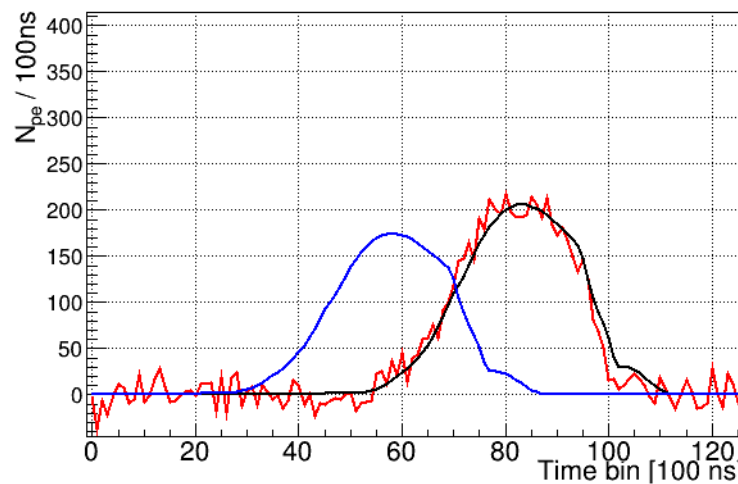
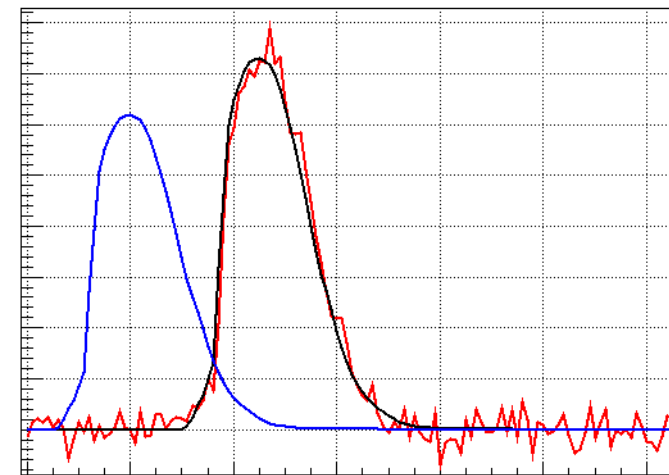
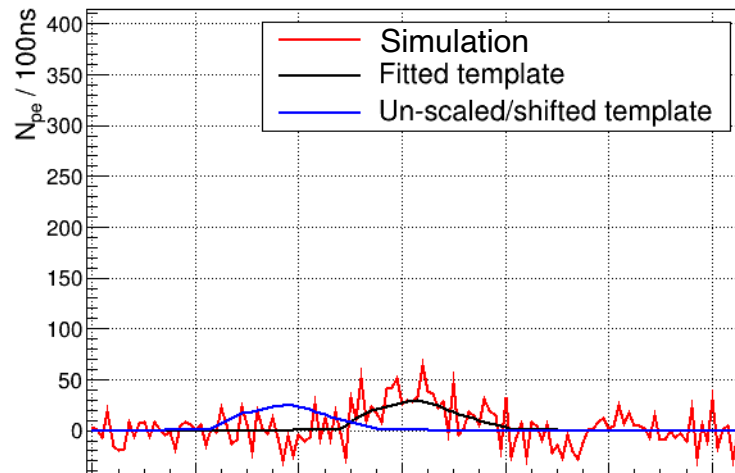
- Symmetric response of telescope in simulations → only simulate core $x \leq 0$
- Total of 980, 000 showers simulated (ignore showers outside FOV)
- Fixed energy – can scale template traces for different energies

Parameter	Values
$X_{\max}$	500, 600, 700, ..., 1200 g cm ⁻²
Energy	Fixed (10 ¹⁹ eV)
θ	0, 5, 10, ..., 80°
ϕ	-170, -160, -150, ..., 180°
Core x	-15, -14, -13, ..., 15 km
Core y	-5, -4, -3, ..., 25 km



Template Comparison

- For some data (red) and some template (blue), need to find
 - Best fit time offset t_{off}
 - Scale factor A
- Use same likelihood function as TDR for quantitative comparison



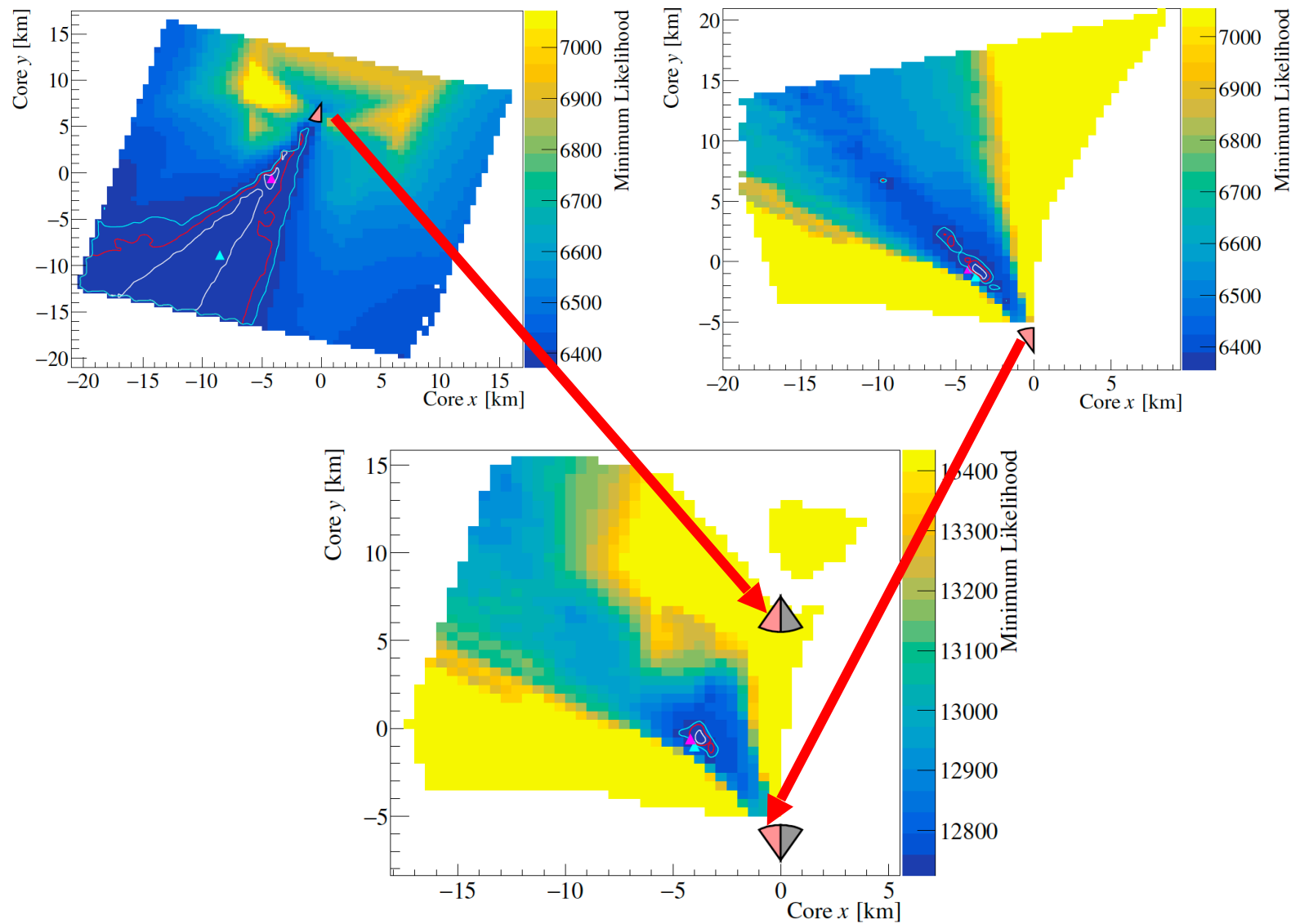
Parameter Estimation

Single Telescope

Just take the best fitting template

Multiple Telescopes

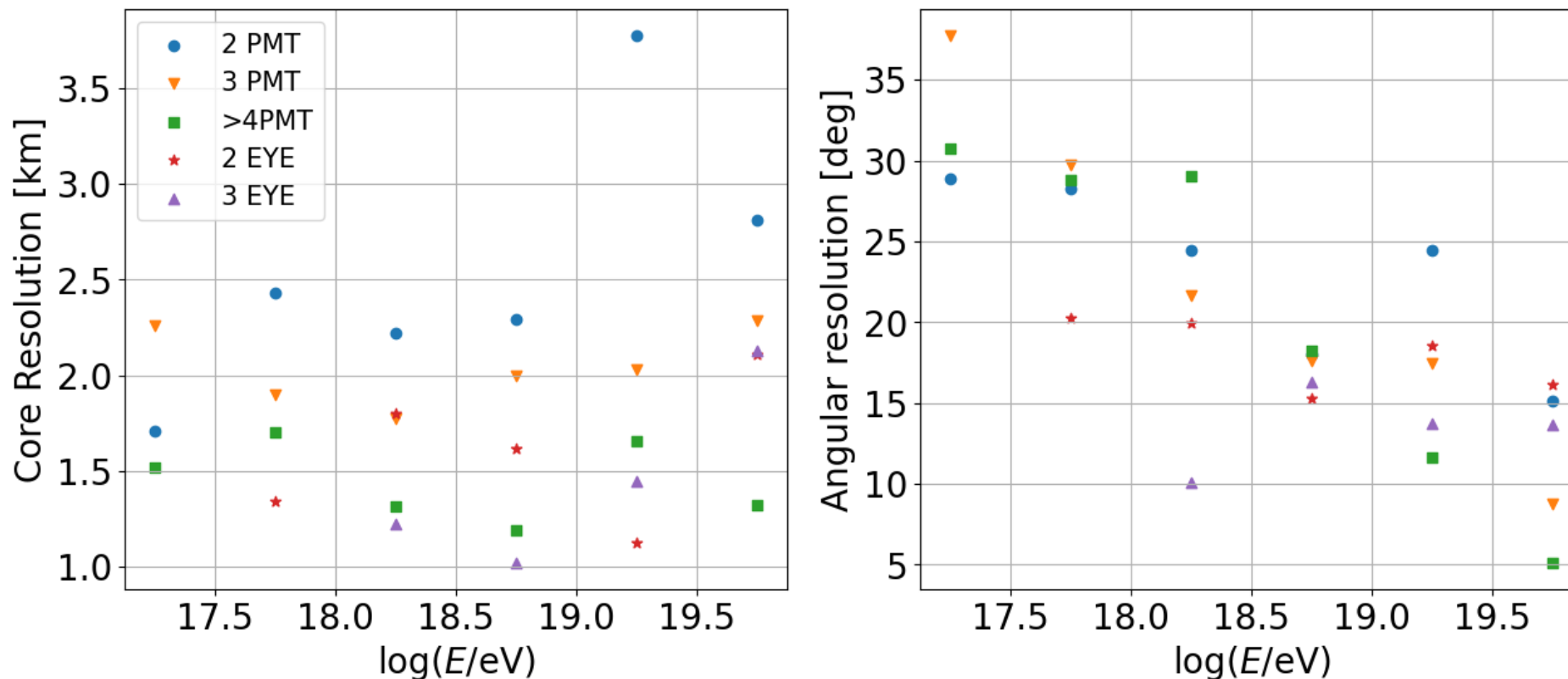
- Calculate minimum likelihood maps in slices of core position, arrival direction and X_{max} /energy
- Interpolate each map on set grid points and sum $\rightarrow$ first guess given by minimum locations from each summed map



Template Method Performance

Geometry:

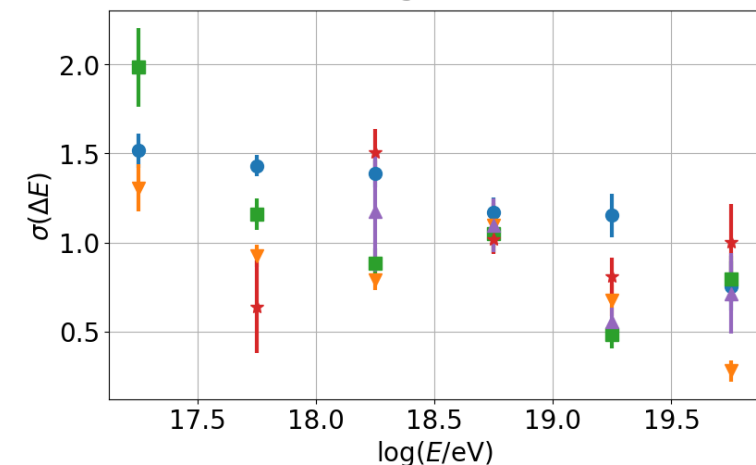
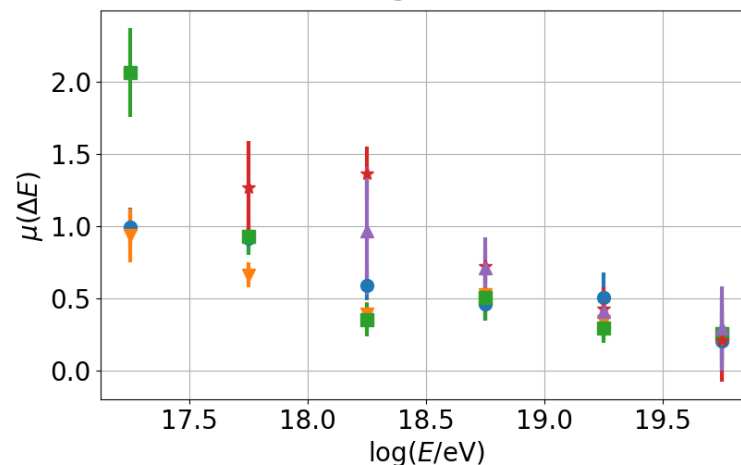
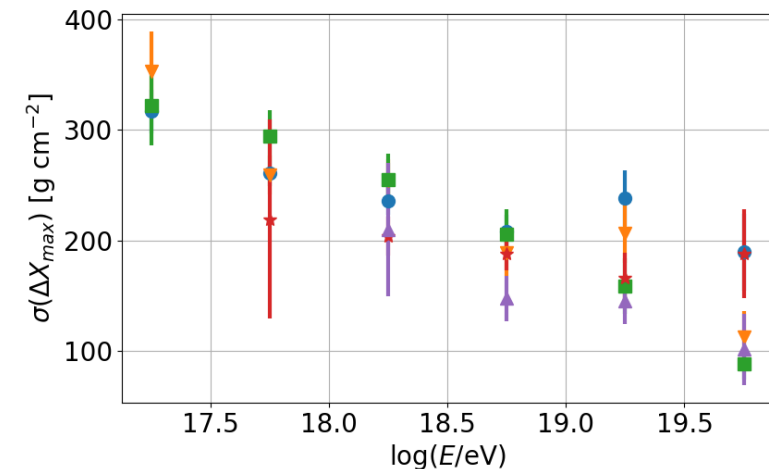
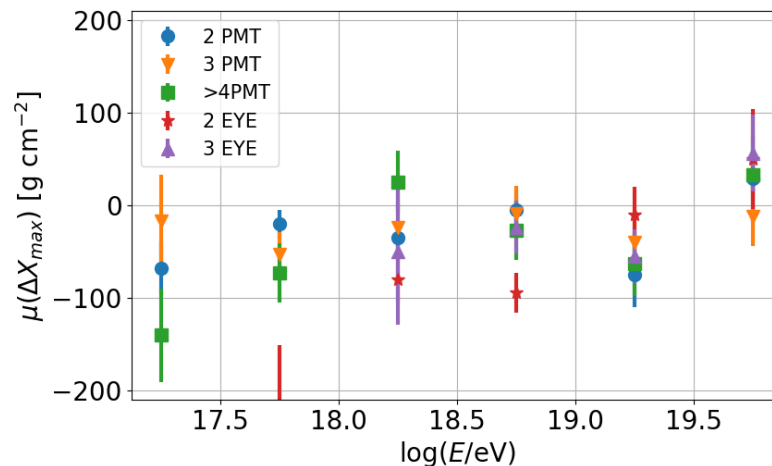
- Core and angular resolutions ok for many PMTs/eyes and higher energies
- Getting geometry right is most important for a first guess



Template Method Performance

$X_{\max}$ & Energy:

- Best case resolutions:
 - $X_{\max} \sim 100 \text{ g cm}^{-2}$
 - Energy $\sim 50\%$ (also non-zero bias)
- Why?
 - Coarse template spacing
 - Method of combining results from multiple telescopes and energy estimation not ideal



Template Method: Summary

Summary

- Flexible method allowing for any telescope configuration to get a first guess
- Core angular resolutions at highest energies reasonable for first guess $\sim 10^\circ$ and 1 km in best case

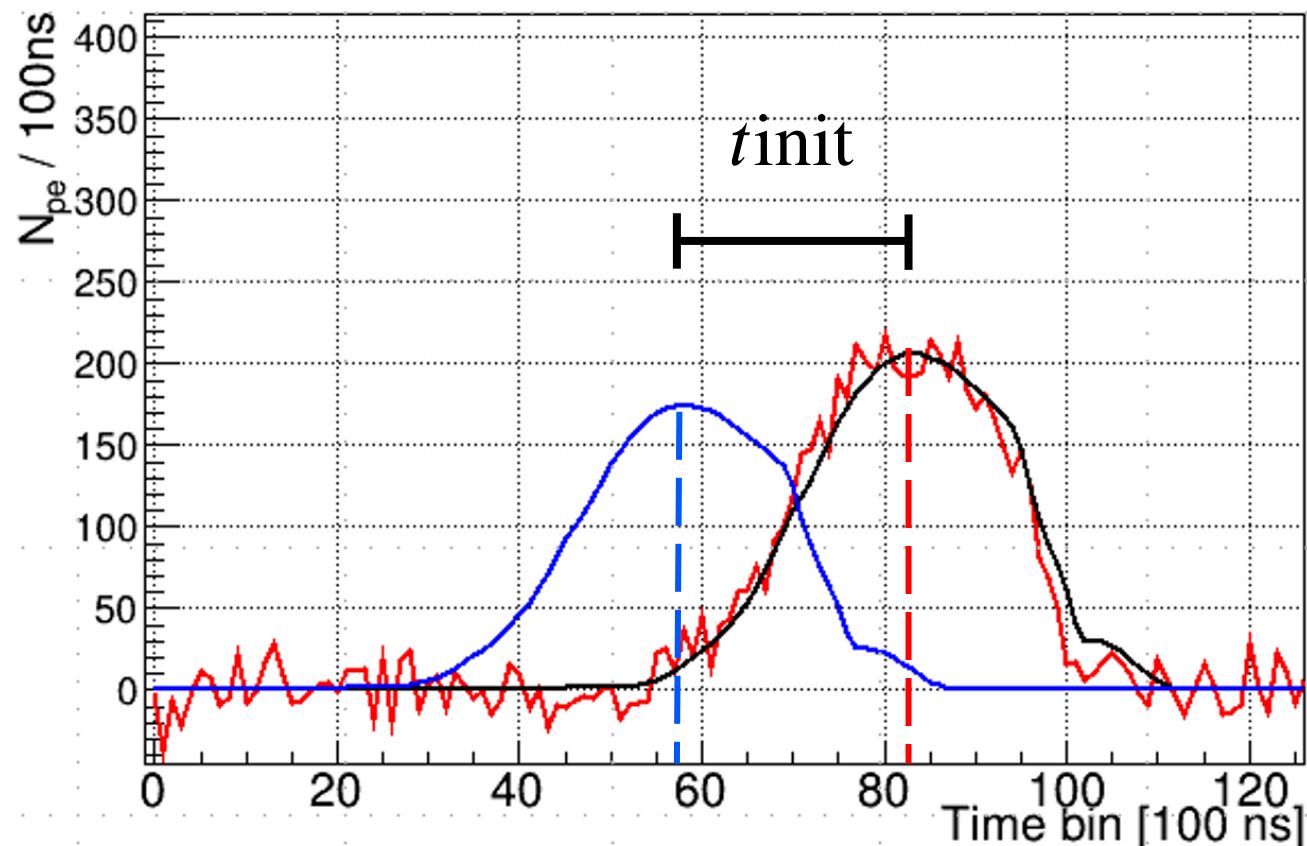
Shortcomings

- Density of templates insufficient for quadratic interpolation around the minimum - poorer resolutions in $X_{\max}$ and energy
- Time intensive (up to ~ 4 minutes for single telescope)

Template Comparison

Finding the best t_{off} and A :

- Let t_{init} = difference (in bins) between centroid time of first triggered PMT in **data** and corresponding PMT in **template**
- Try time offsets
 $t_{\text{off}} \in (t_{\text{init}} - 5, t_{\text{init}} + 5)$
- At each t_{off} estimate best scale factor A from ratio of signals.
- Maximum likelihood over all t_{off} values is assigned to the template

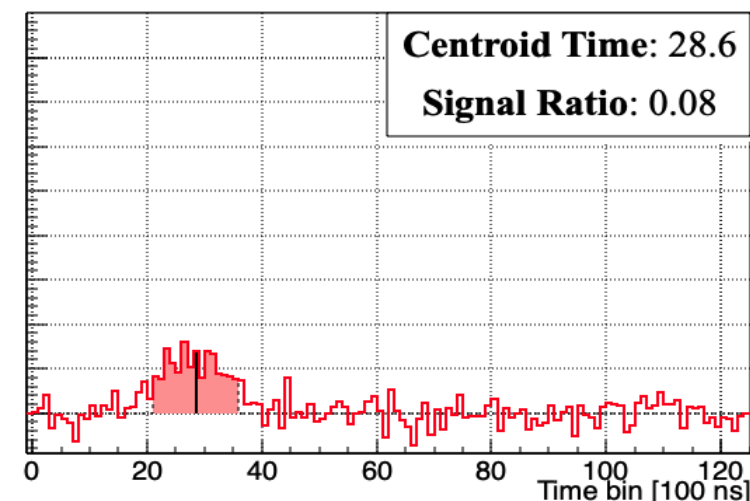
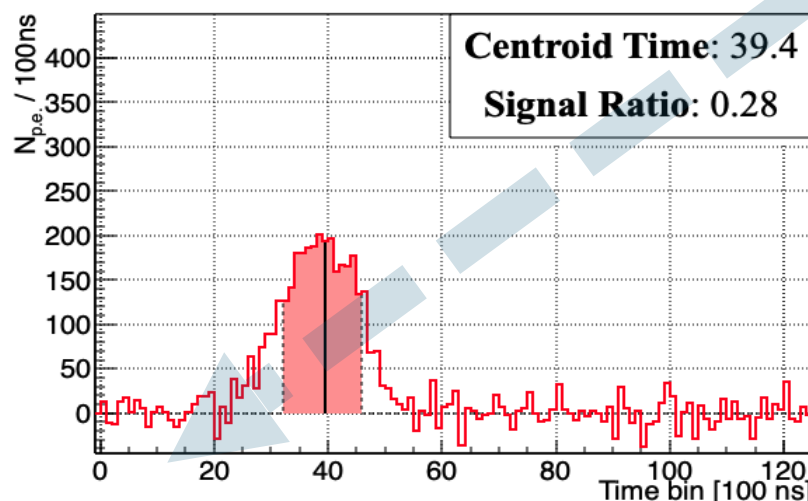
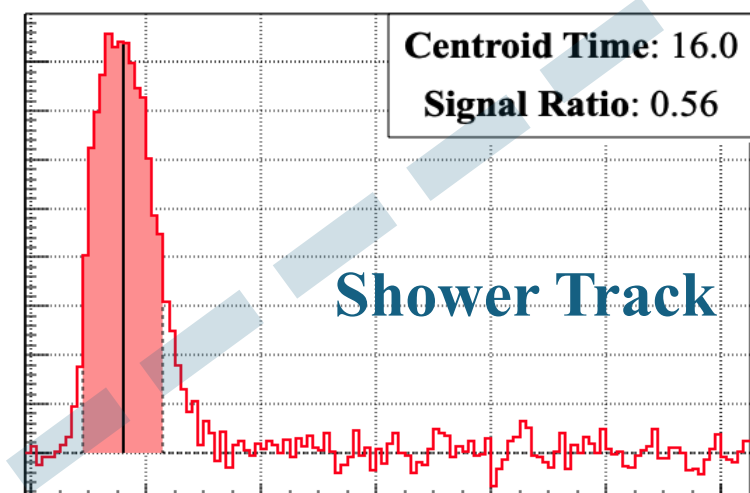
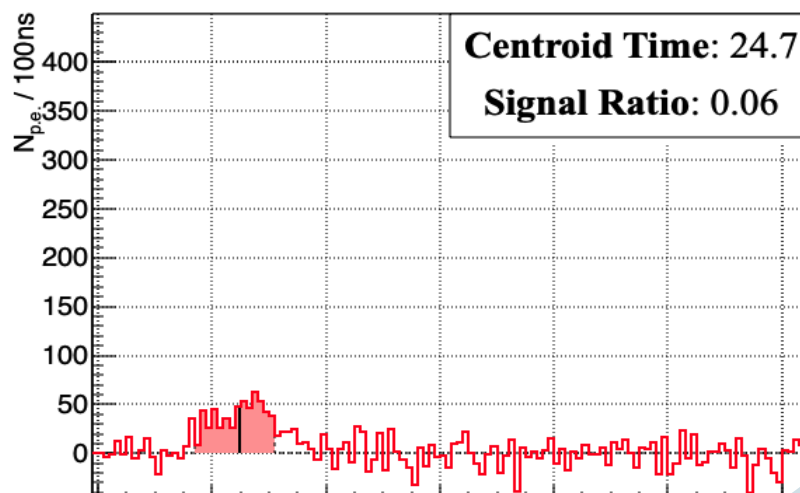


Template Selection

- Comparing every template would take > 1 hr.
 - Too long for a first guess, want $<$ few minutes
- **Solution:** Narrow down search space by only evaluating the likelihood for templates with **similar features to data**

Template Selection

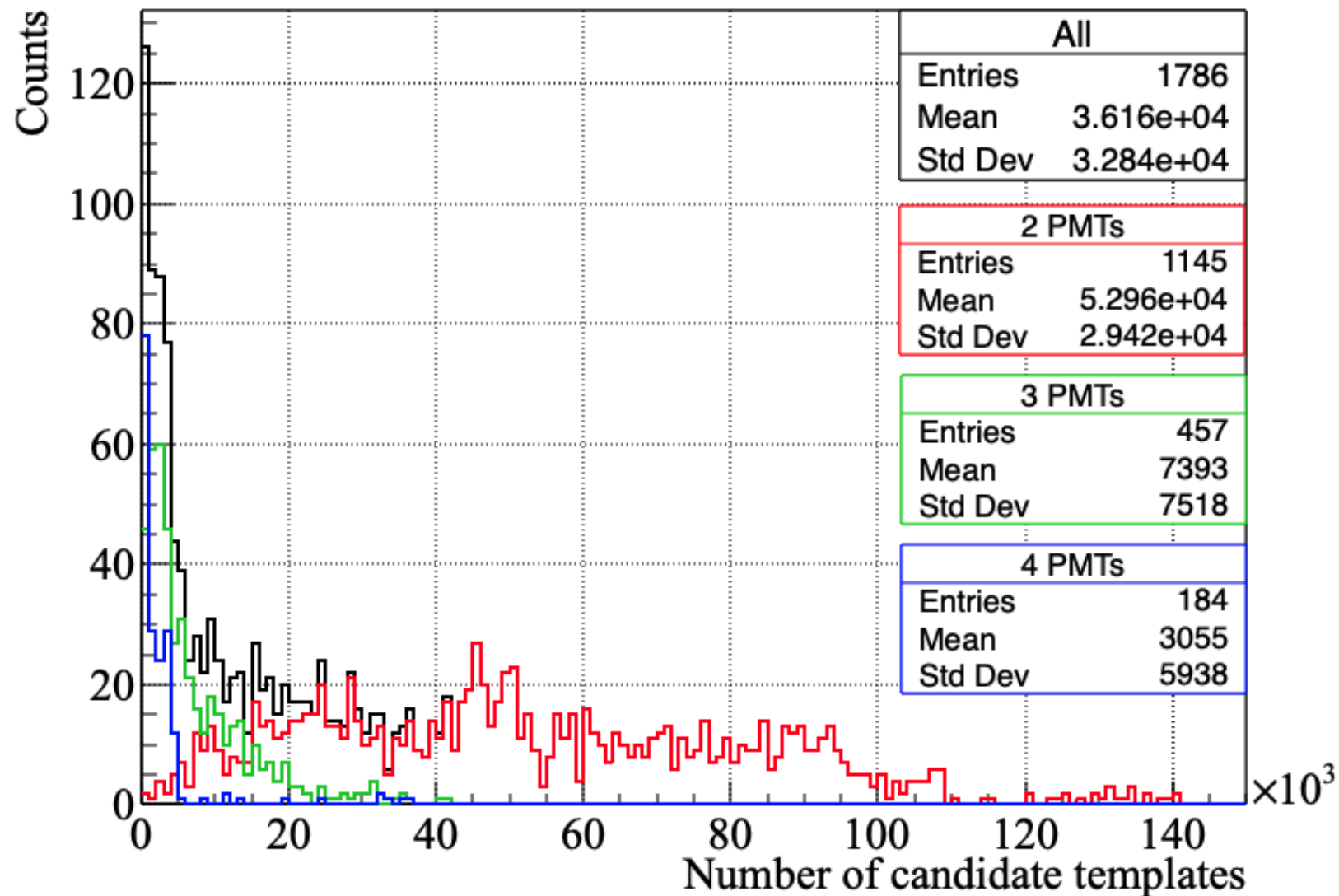
- Calculate **centroid times** and **signal ratios** from each triggered PMT in data
- Compare the *differences* in these values to those from each template
- Only calculate likelihood for templates which match the data within a certain threshold



Template Selection

On average, how many candidates for different numbers of PMTs?

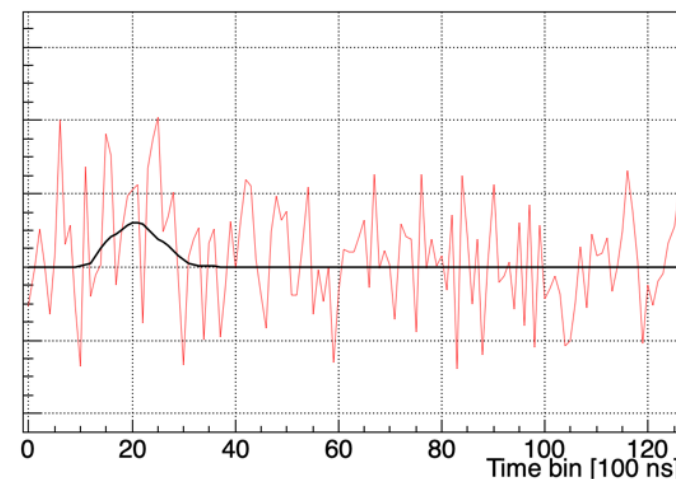
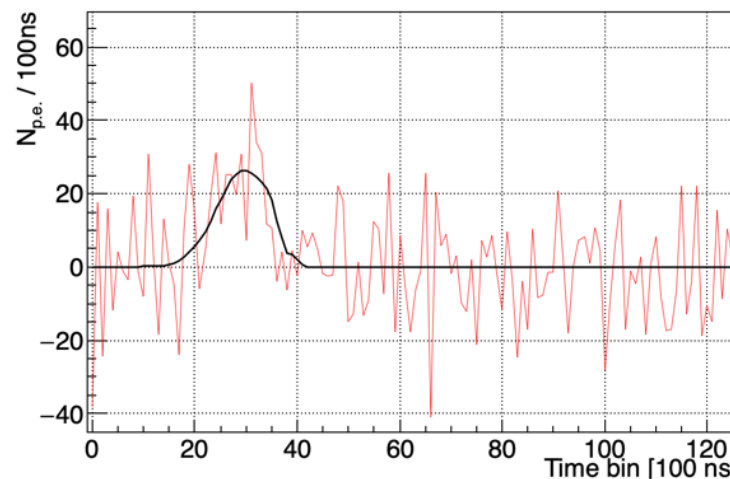
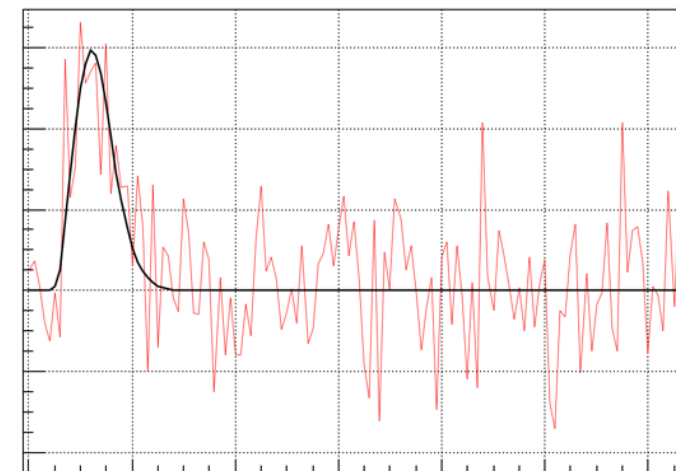
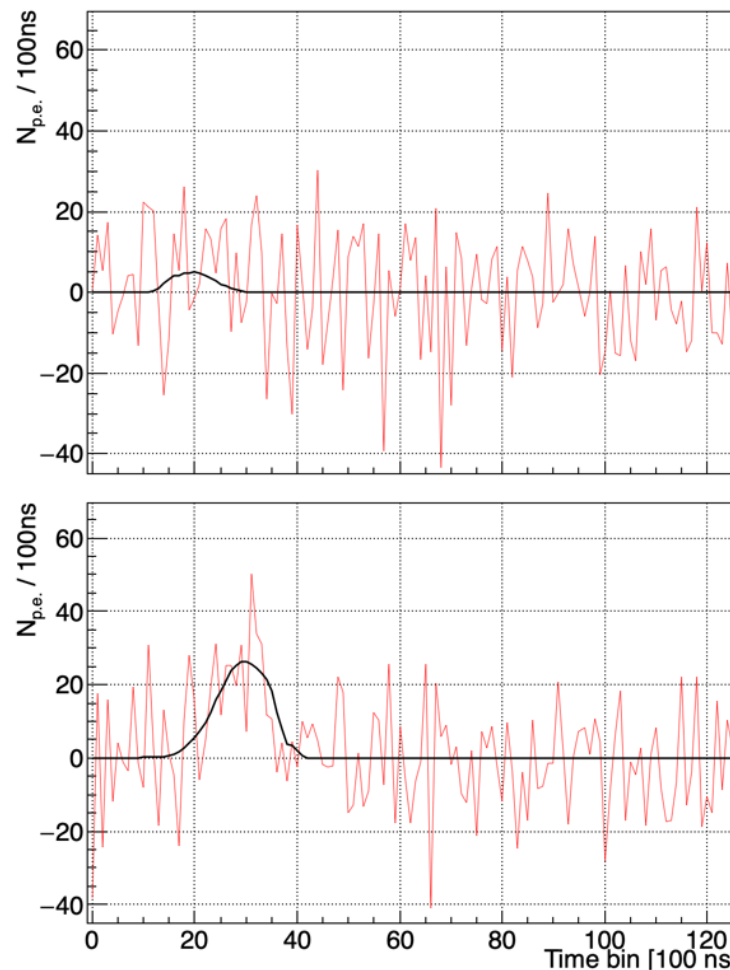
- 2 PMTs ~ 50,000
 - 2.5 minutes
- 3 PMTs ~ 7,000
 - 20 seconds
- 4 PMTs ~ 3,000
 - 10 seconds



Parameter Estimation

Single Telescope

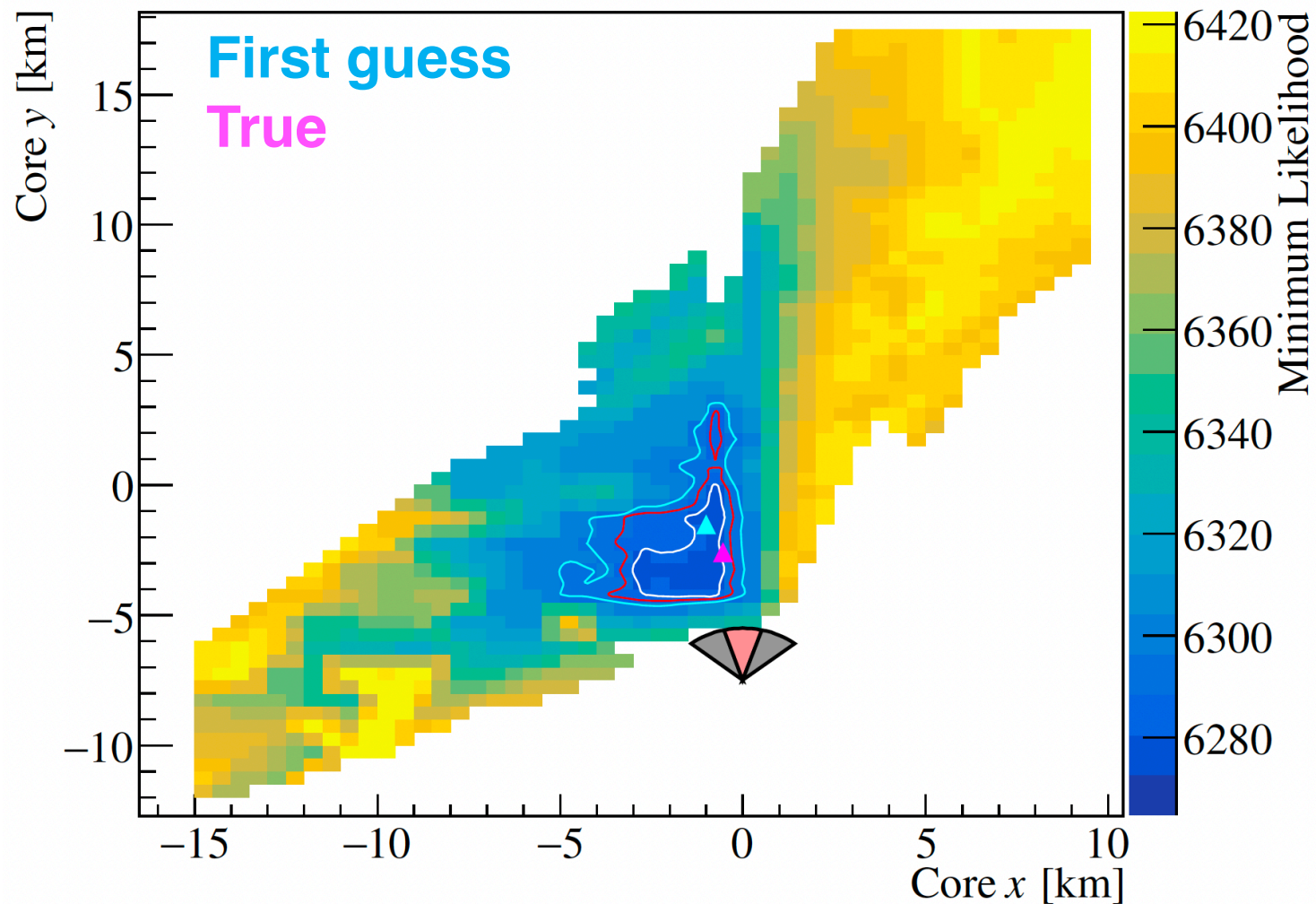
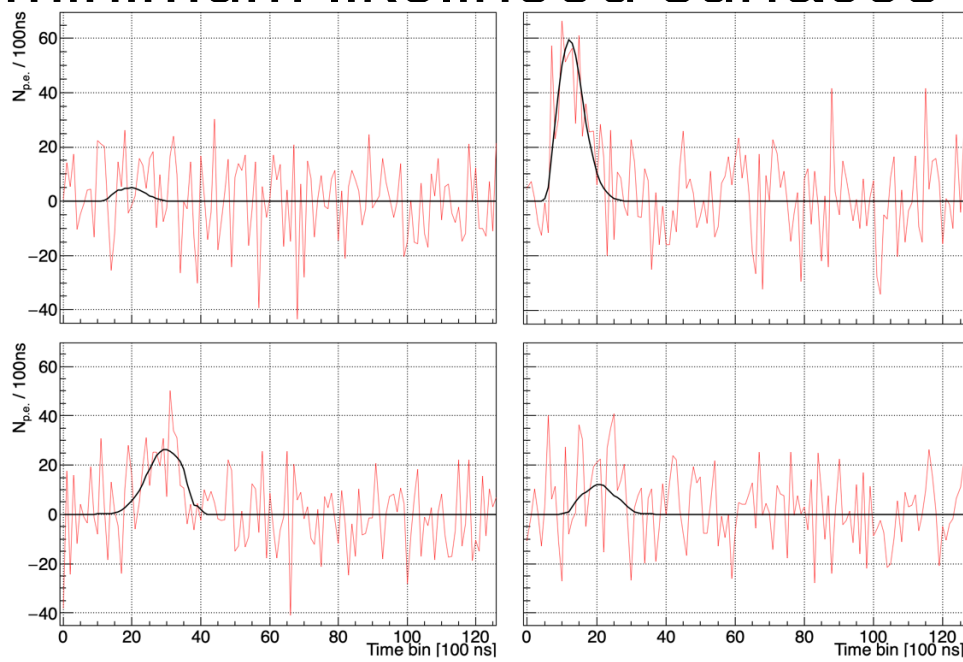
- Just take the best fitting template
- Uncertainty can be visualised by plotting $1/2/3\sigma$ contours on “minimum likelihood surfaces”



Parameter Estimation

Single Telescope

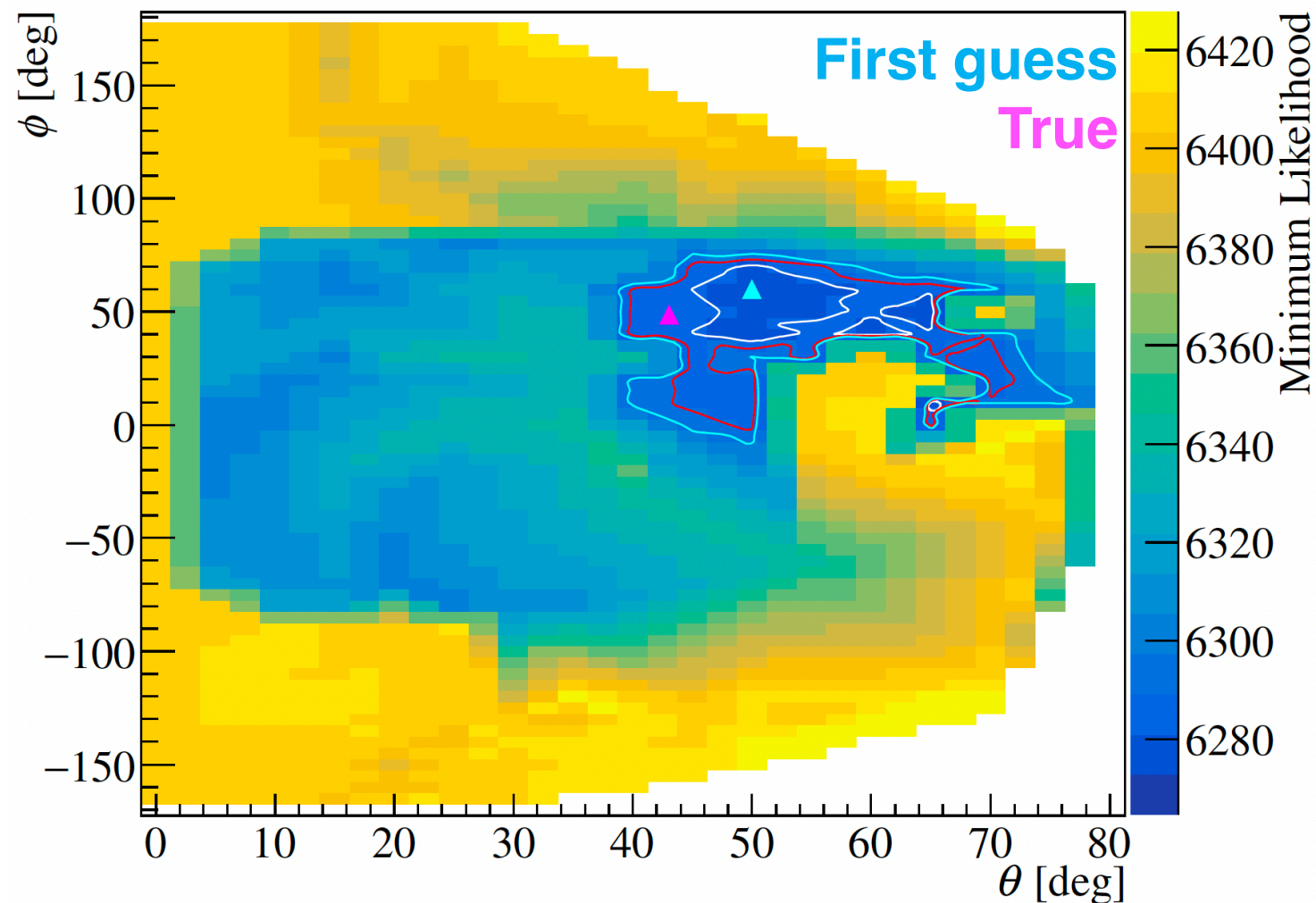
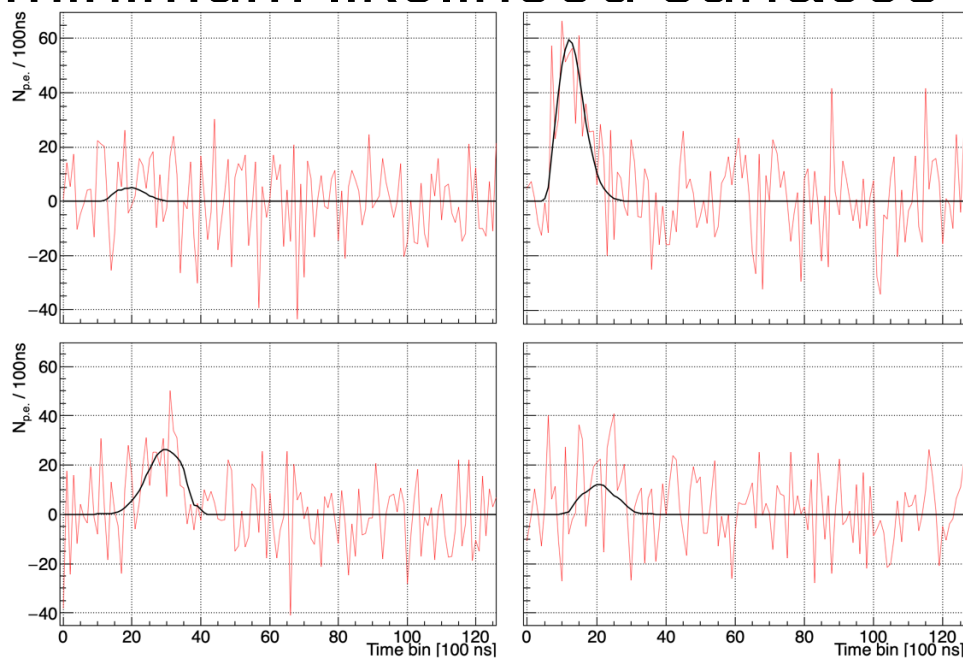
- Just take the best fitting template
- Uncertainty can be visualised by plotting $1/2/3\sigma$ contours on “minimum likelihood surfaces”



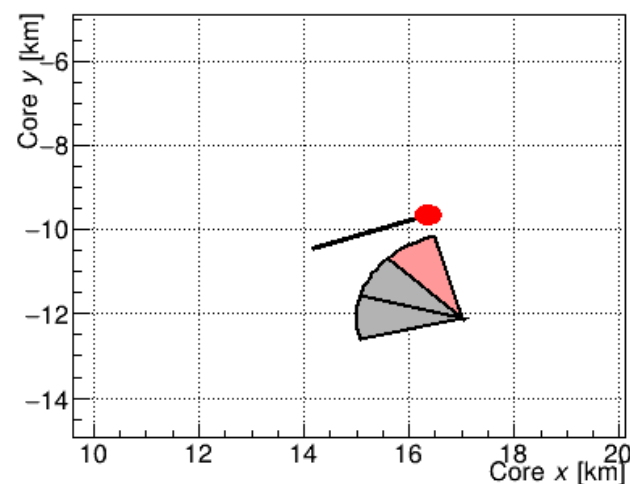
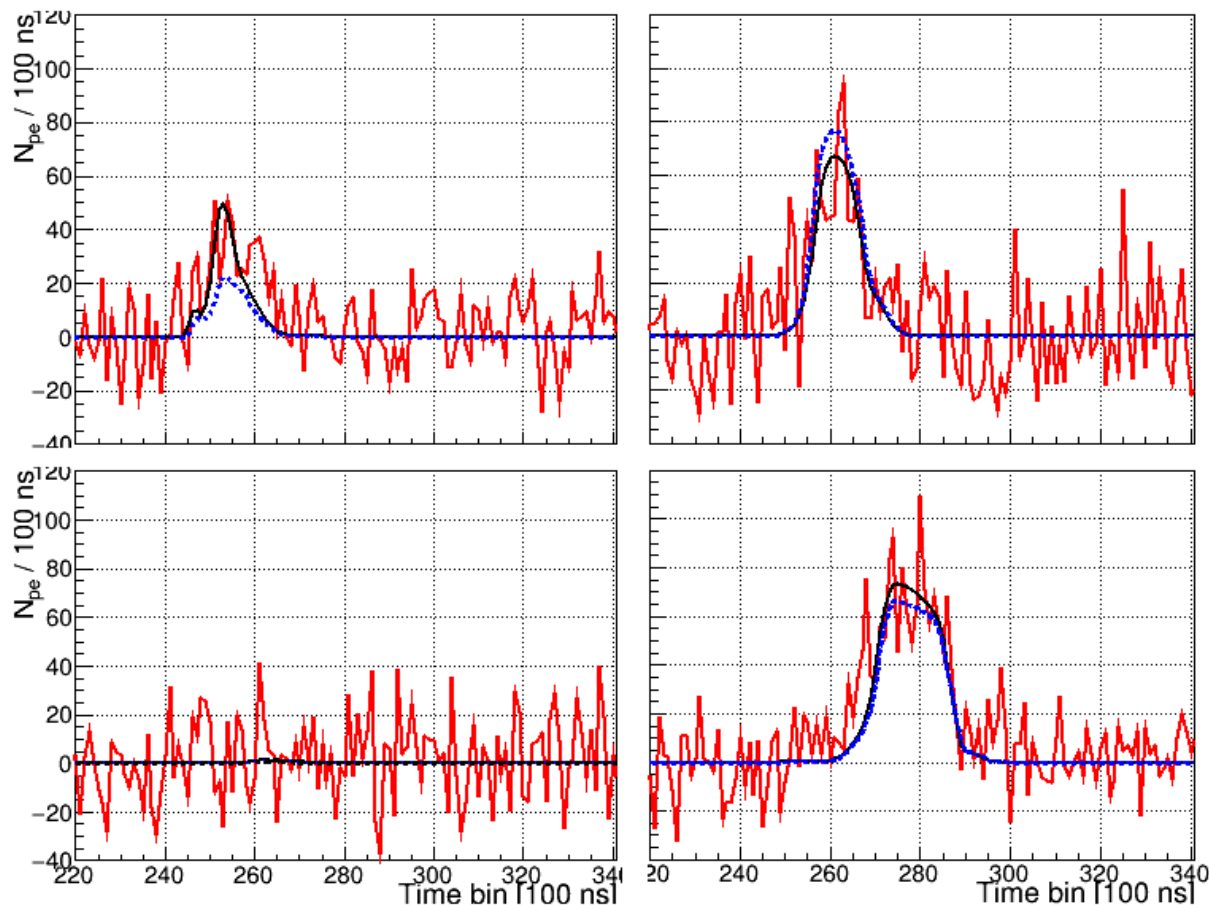
Parameter Estimation

Single Telescope

- Just take the best fitting template
- Uncertainty can be visualised by plotting $1/2/3\sigma$ contours on “minimum likelihood surfaces”



Example Reconstruction



RECONSTRUCTION RESULTS

$X_{\max}$: 802 ± 71 (796) [g cm^{-2}]

E : 17.9 ± 17 (17.92) [$\log(E/\text{eV})$]

θ : 29.5 ± 1.4 (28.2) [$^\circ$]

ϕ : 199.1 ± 4.6 (-160.2) [$^\circ$]

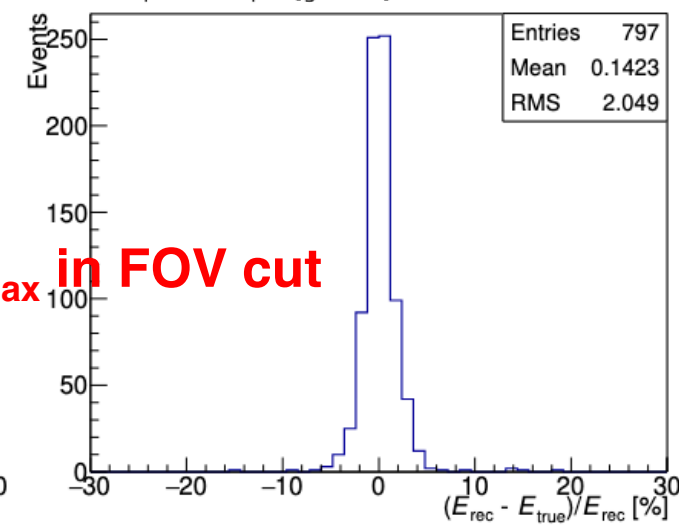
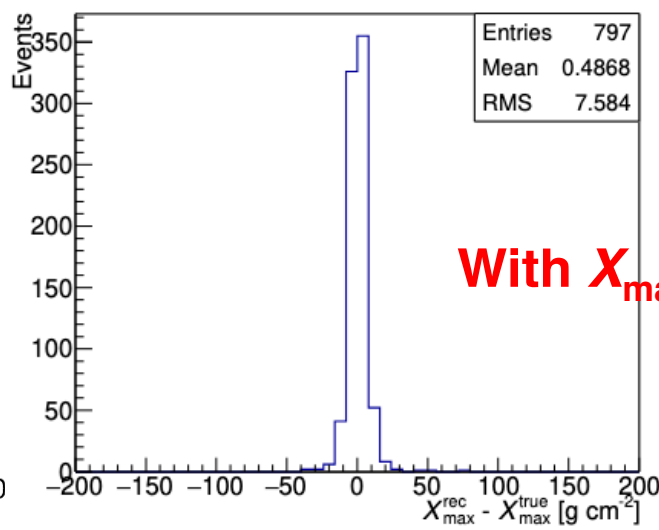
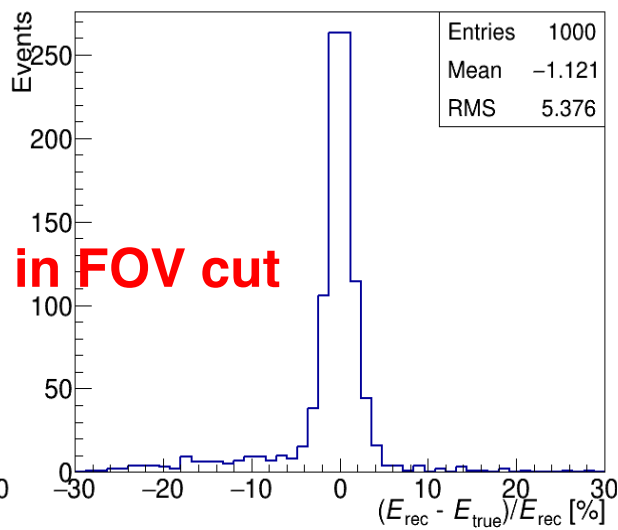
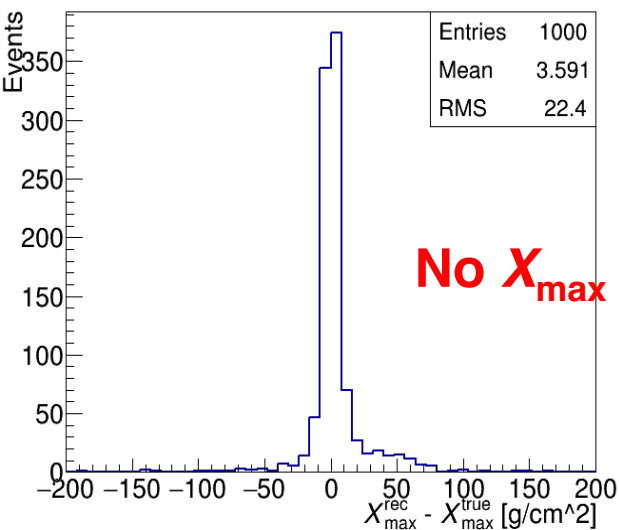
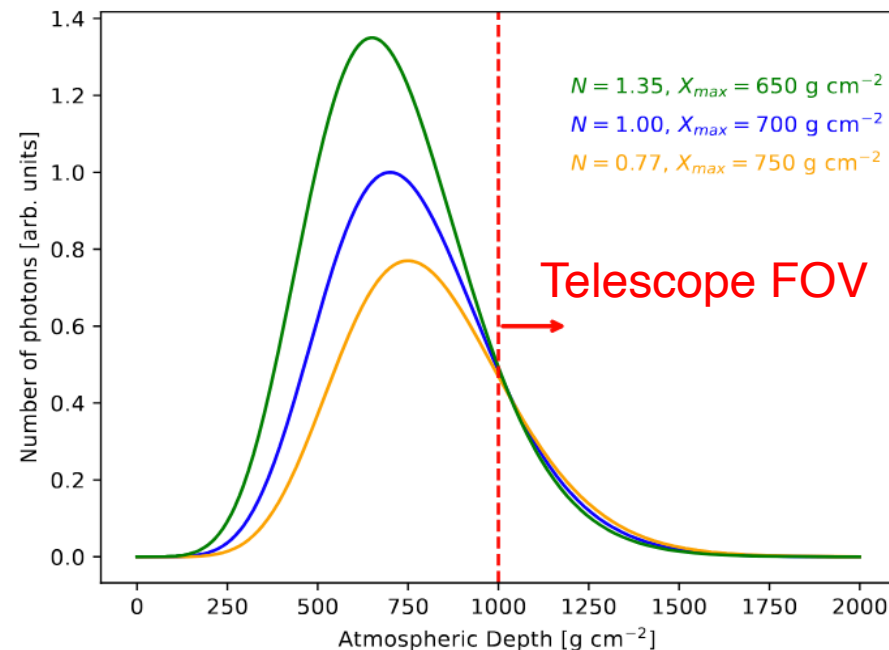
x : 16352 ± 33 (16360) [m]

y : -9675 ± 57 (-9650) [m]

- Reconstruct all six shower parameters simultaneously with TDR
- Use TA / Auger reconstructed values as first guess

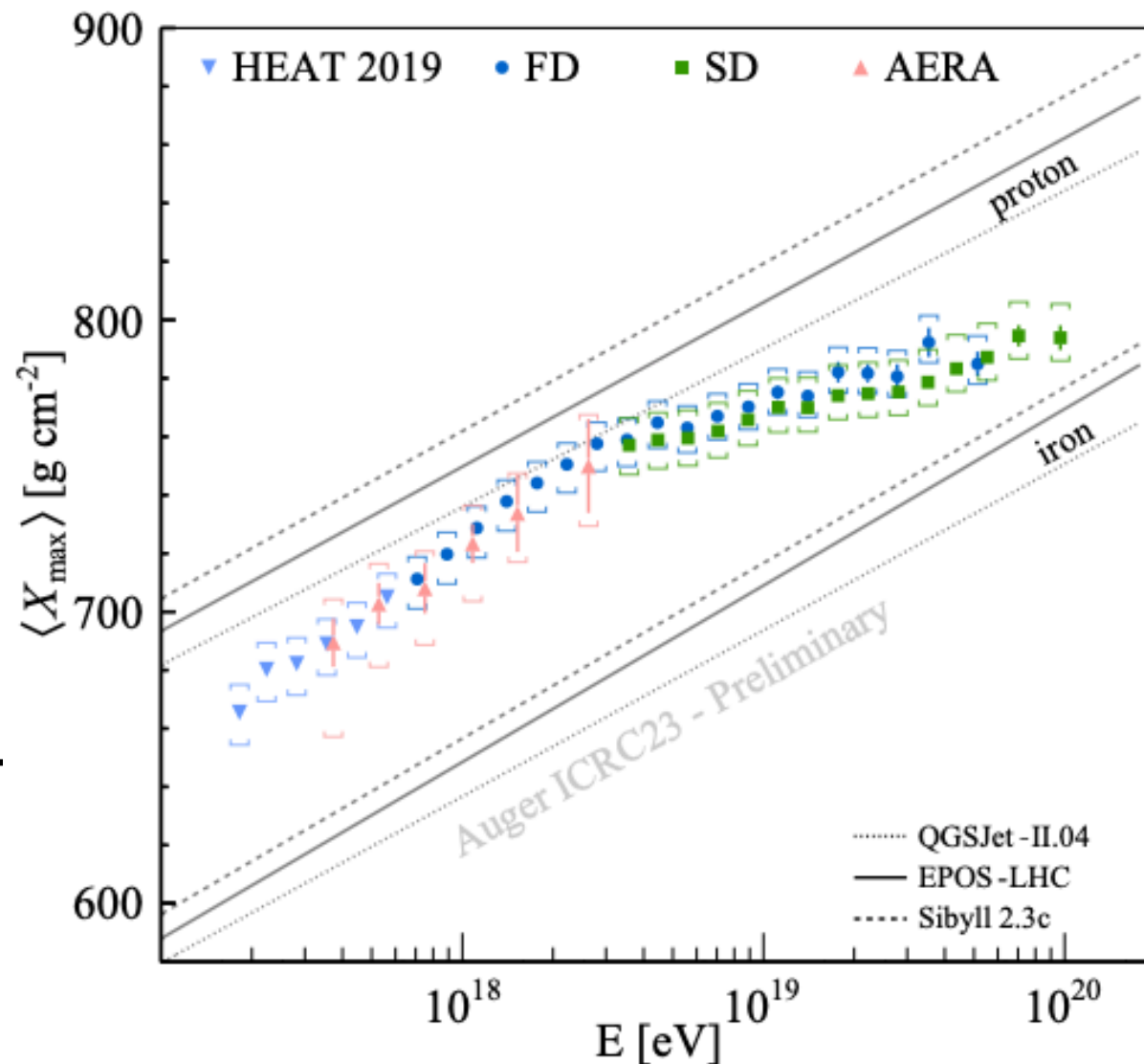
$X_{\max}$ & Energy Correlation

- Depending on shower geometry different profiles can appear similar to the telescope
- Causes degeneracy in reconstructed $X_{\max}$ and energy
- Solution:** Apply $X_{\max}$ in FOV cut

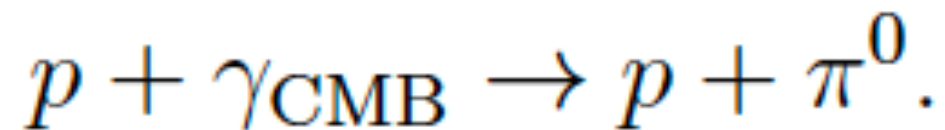
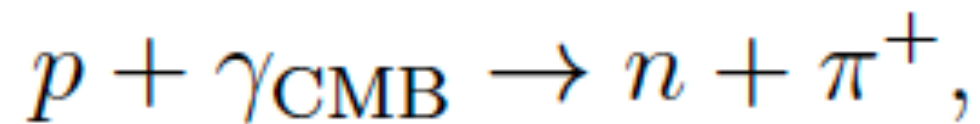
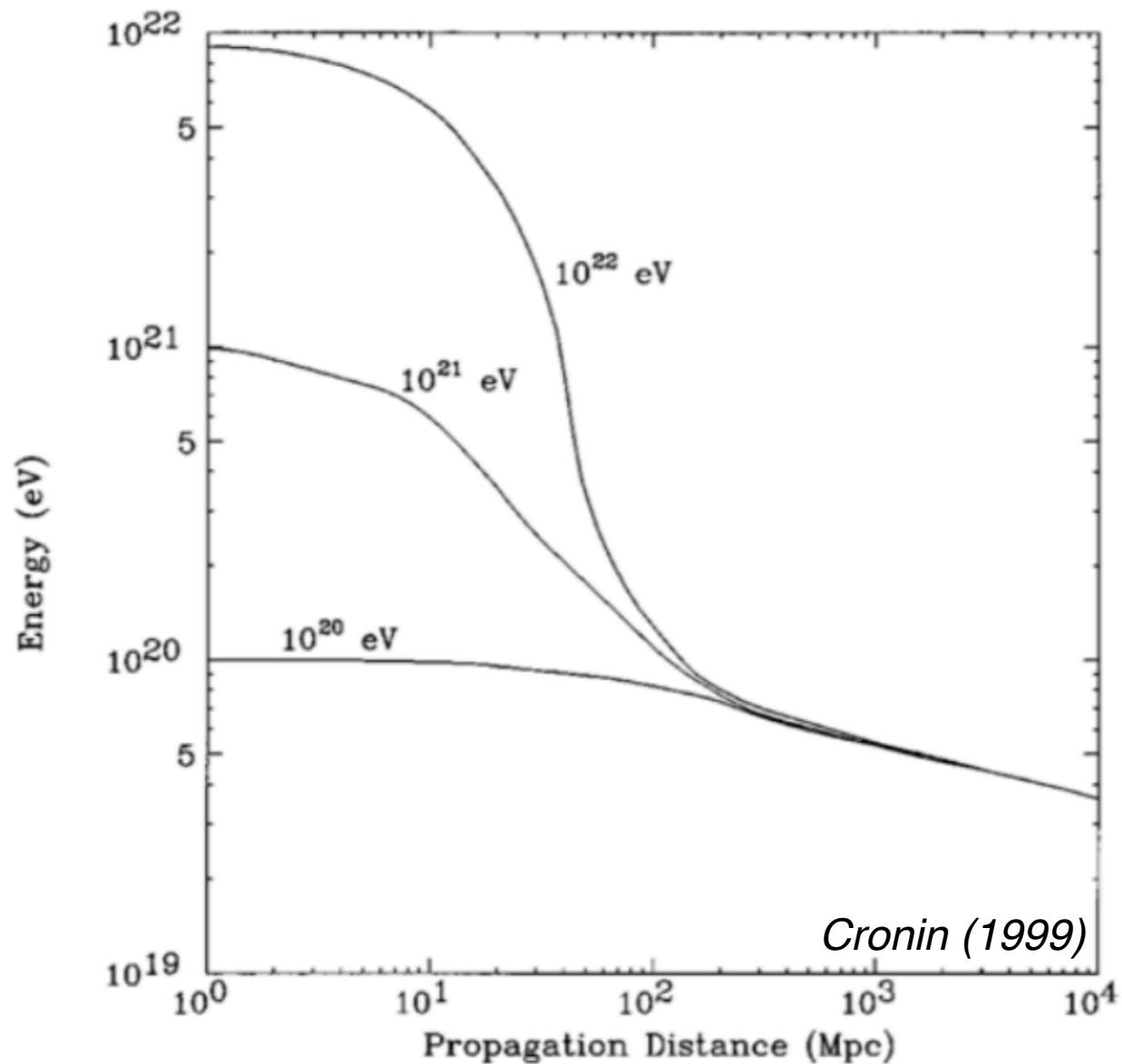


$X_{\max}$ & Mass Composition

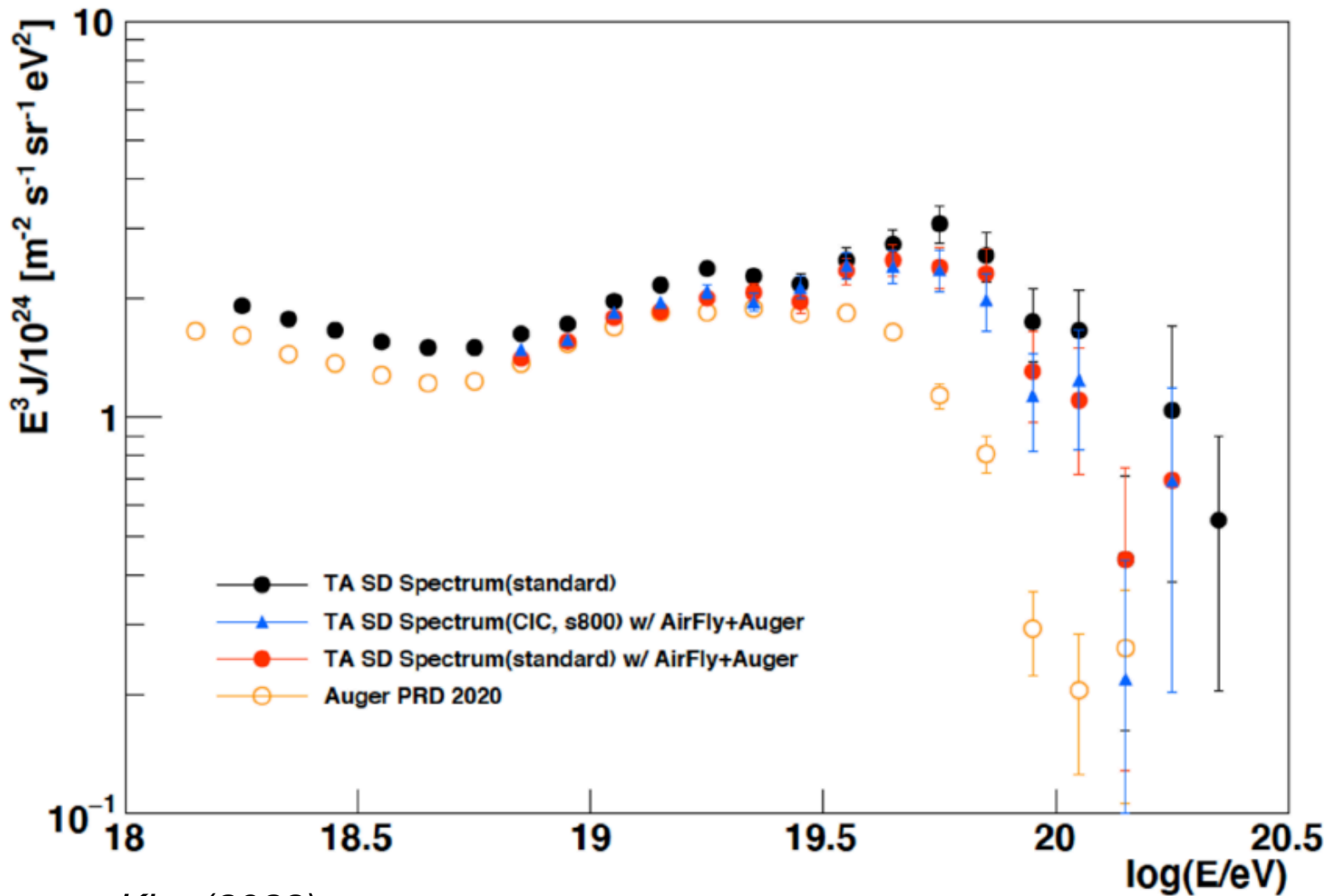
- “Mass composition” = what nuclei make up observed cosmic rays
- $\langle X_{\max} \rangle$ is a proxy for mass composition
- $\langle X_{\max} \rangle$ is energy dependent
 - Lighter nuclei around $10^{18.5} - 10^{19}$ eV
 - Heavier nuclei at the highest energies..
- Cosmic ray trajectories are bent by magnetic fields $\propto Z$ –
 - **Important to know mass**



GZK Cutoff



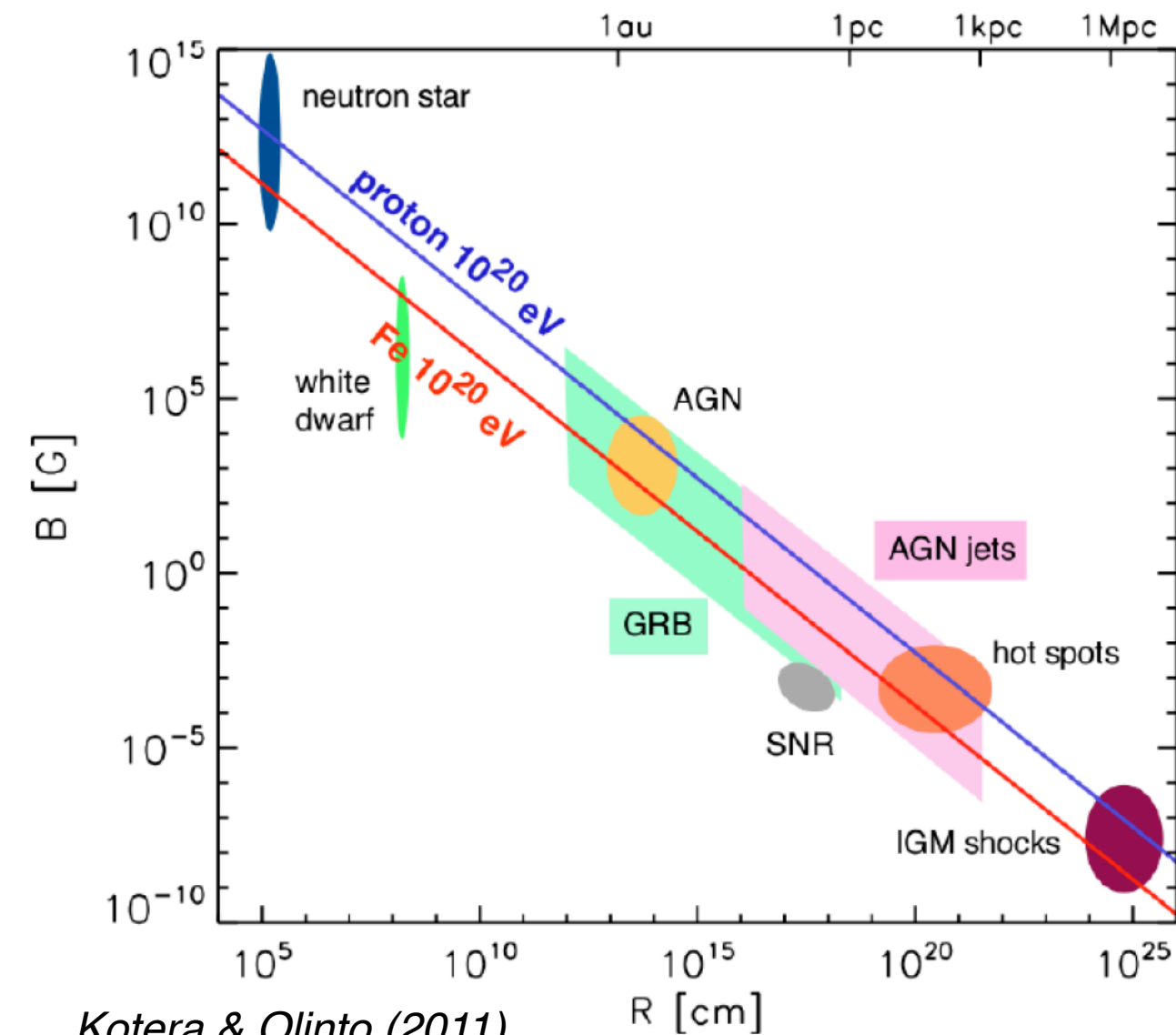
UHECR energy spectra



Kim (2023)

Agreement between Auger and TA below $10^{19.5} \text{eV}$ when using same fluorescence yield model and event selection procedure

Possible sources

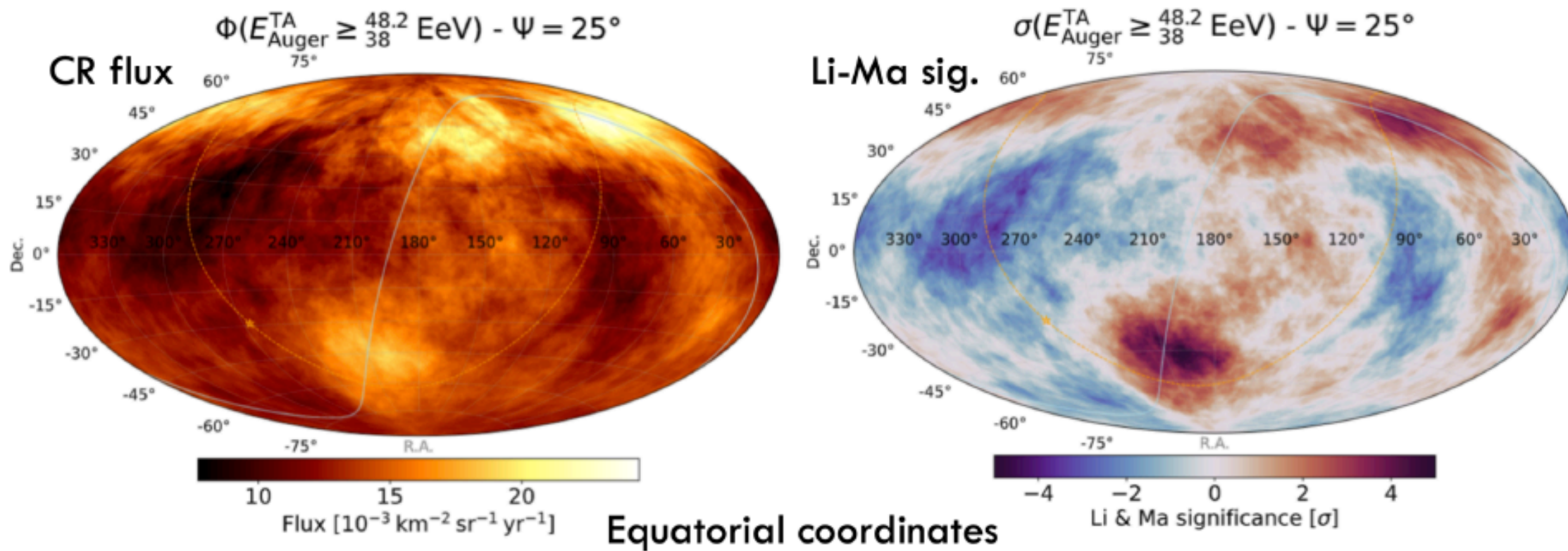


Kotera & Olinto (2011)

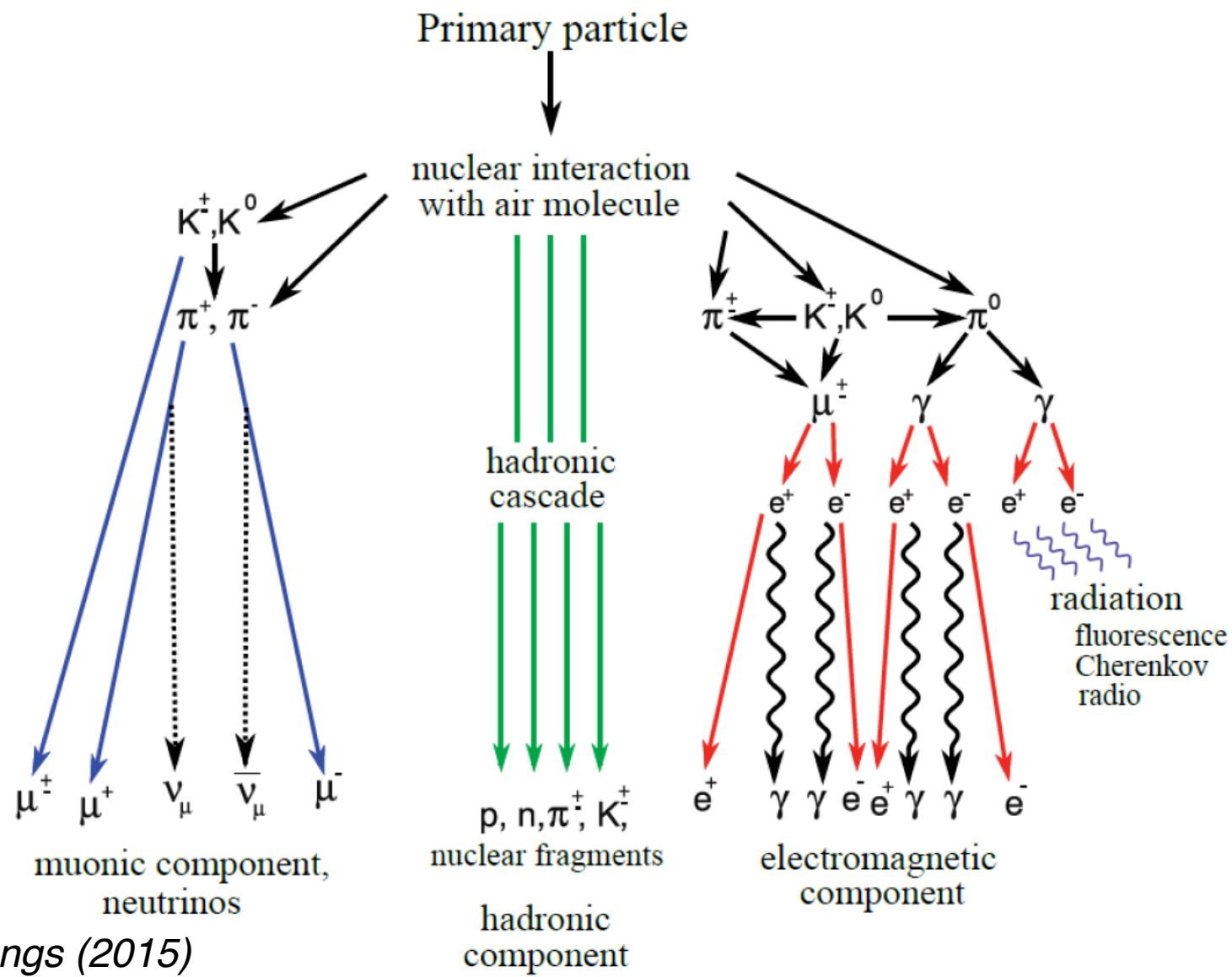
Hillas Plot

- Size and magnetic field strength of different astrophysical objects

Anisotropy

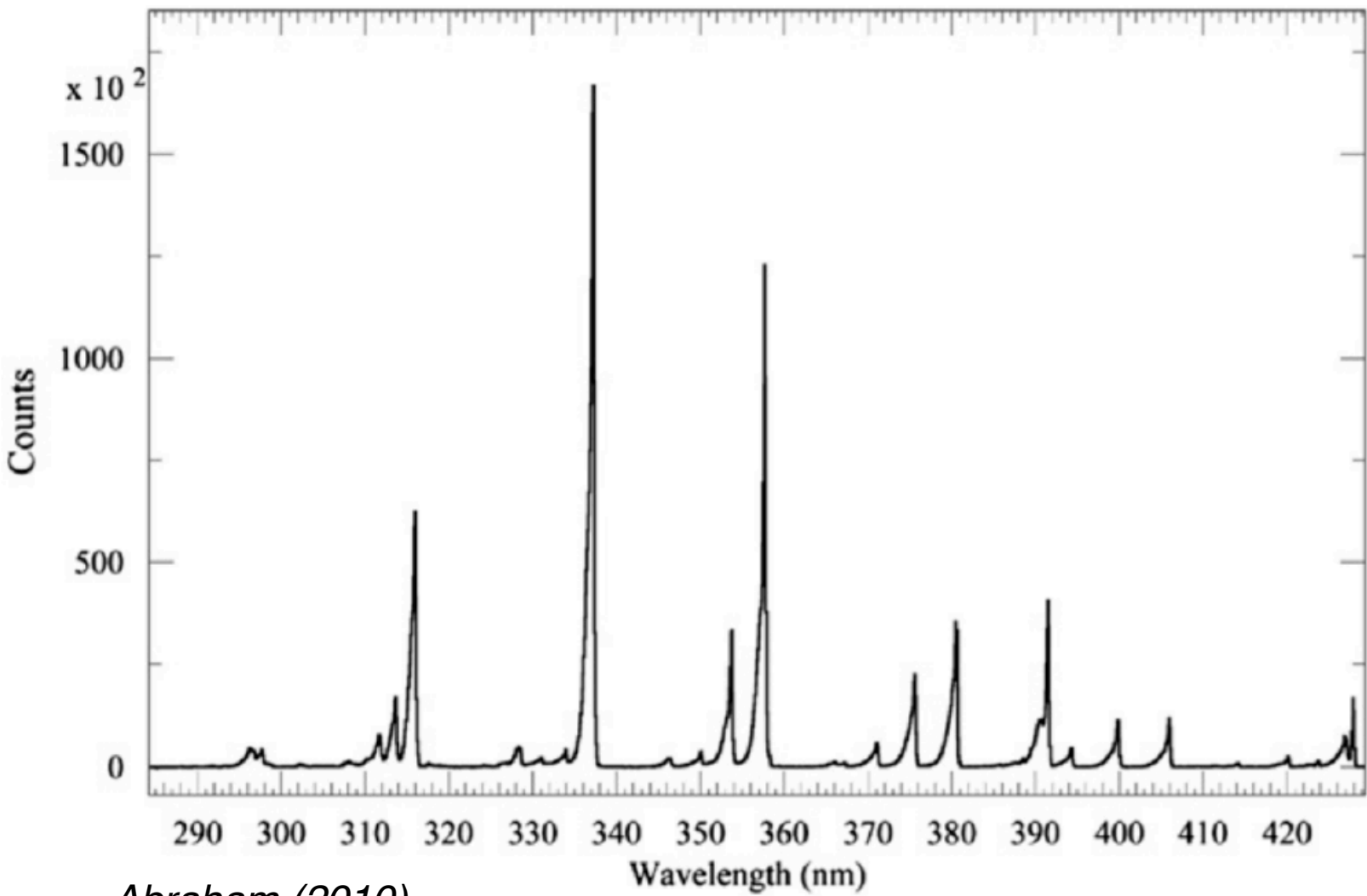


EAS components



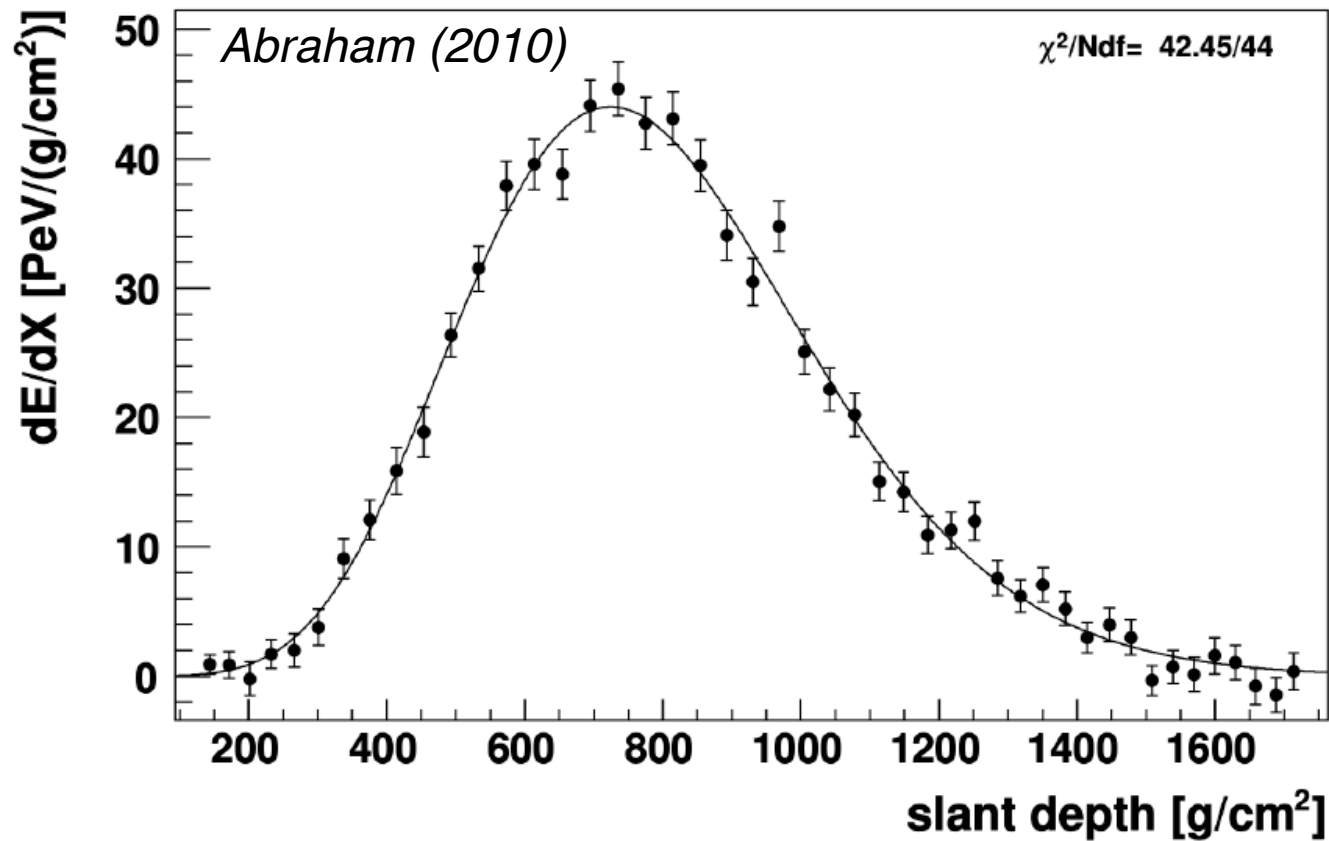
Haungs (2015)

Fluorescence spectrum



Nitrogen in dry air

Reconstructing shower profile

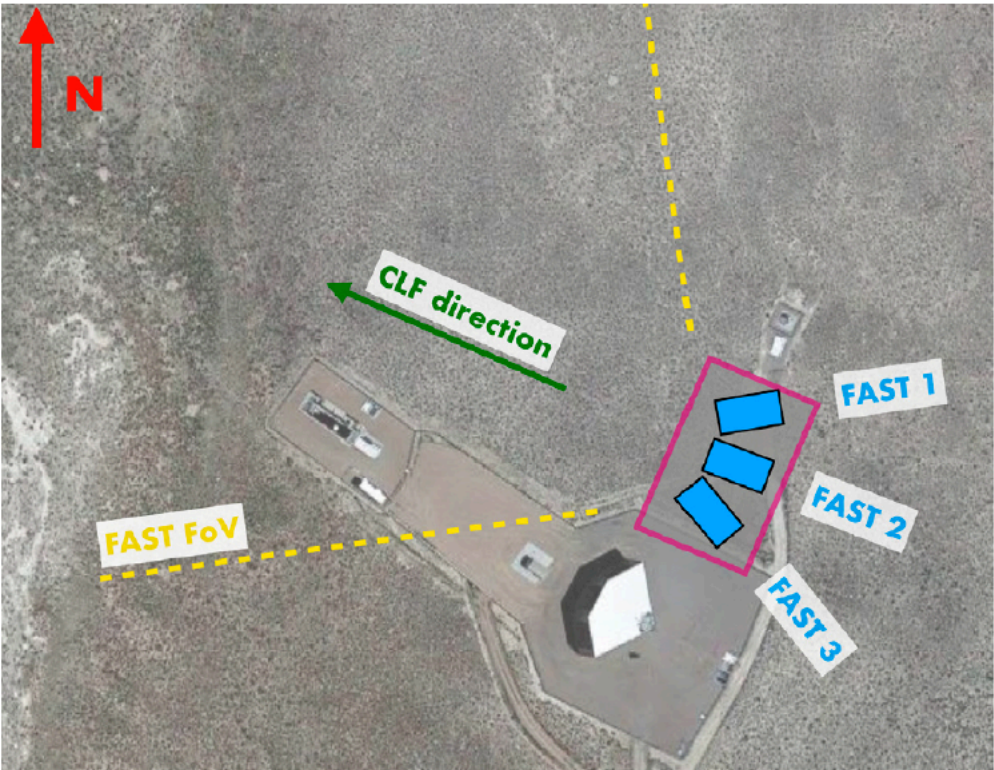


Fit Gaisser Hillas function to energy deposit as function of depth

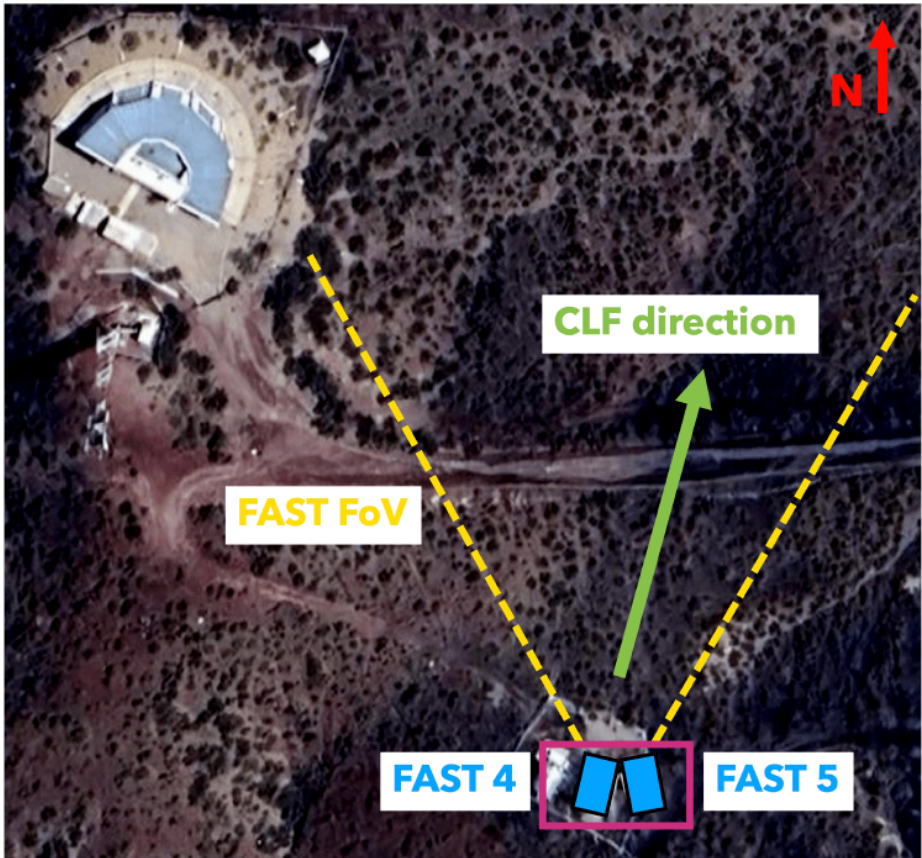
$$\frac{dE}{dX}(X) = \left(\frac{dE}{dX} \right)_{\max} \left(\frac{X - X_0}{X_{\max} - X_0} \right)^{\frac{X_{\max} - X}{\lambda}} \exp \left(\frac{X_{\max} - X}{\lambda} \right)$$

Birds eye view of FAST

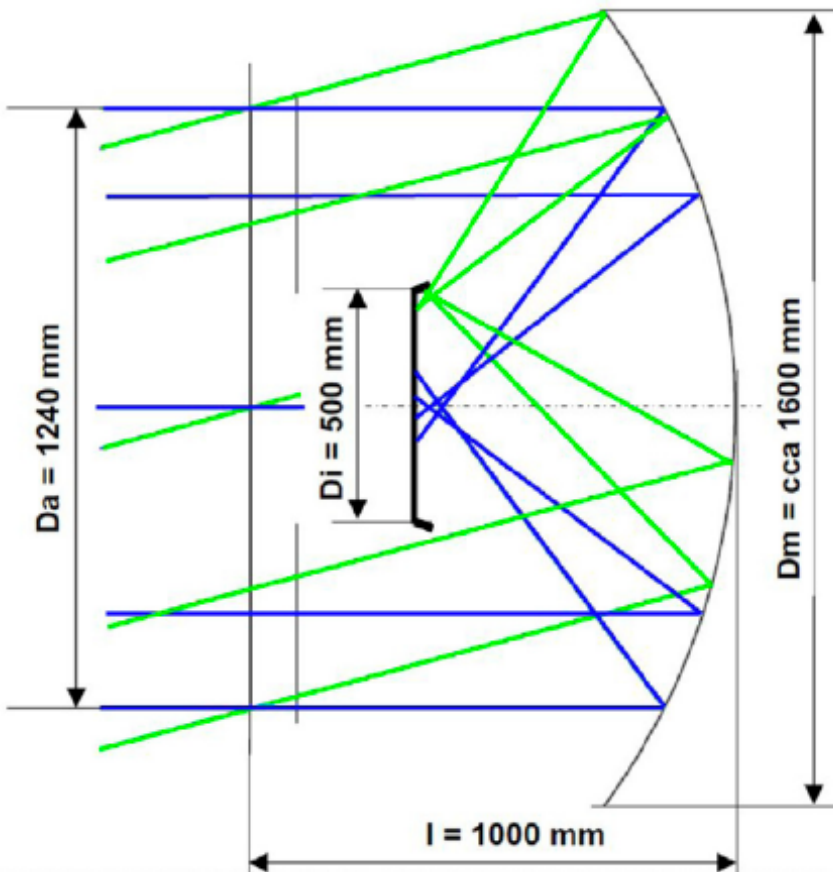
FAST@TA



FAST@Auger

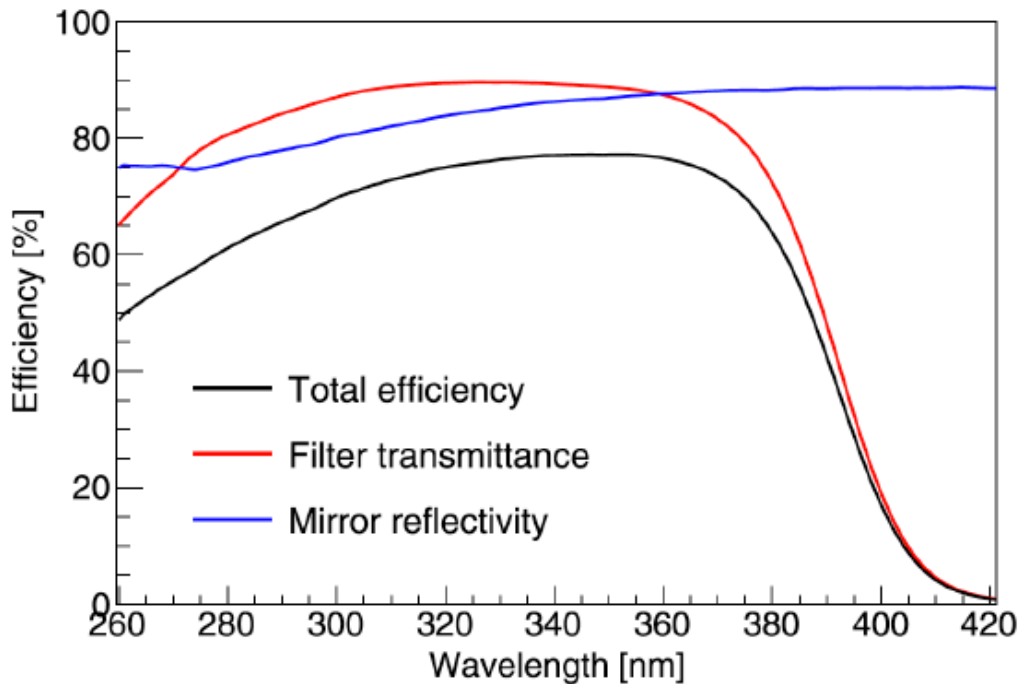


FAST Optics

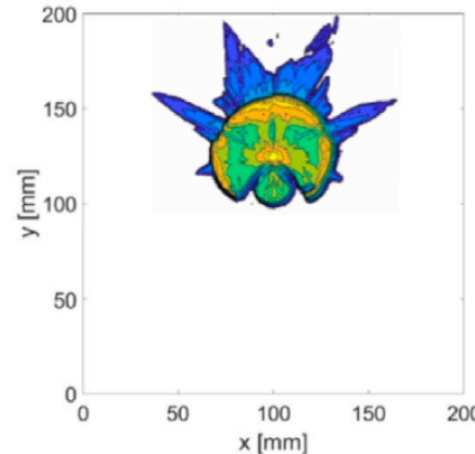
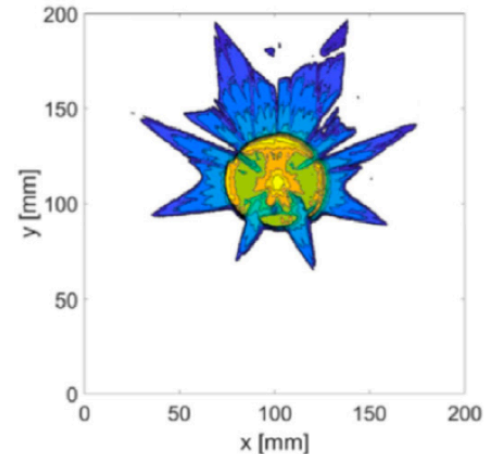
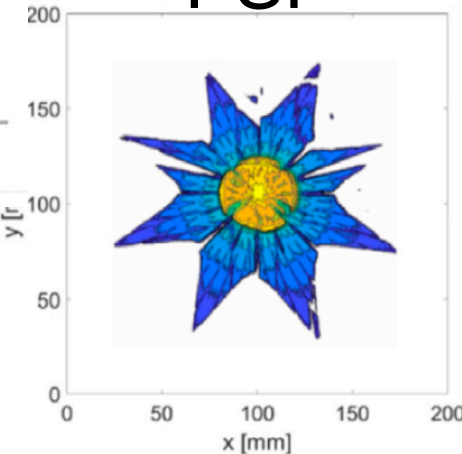


Mirror / camera dimensions

Spectral Efficiency



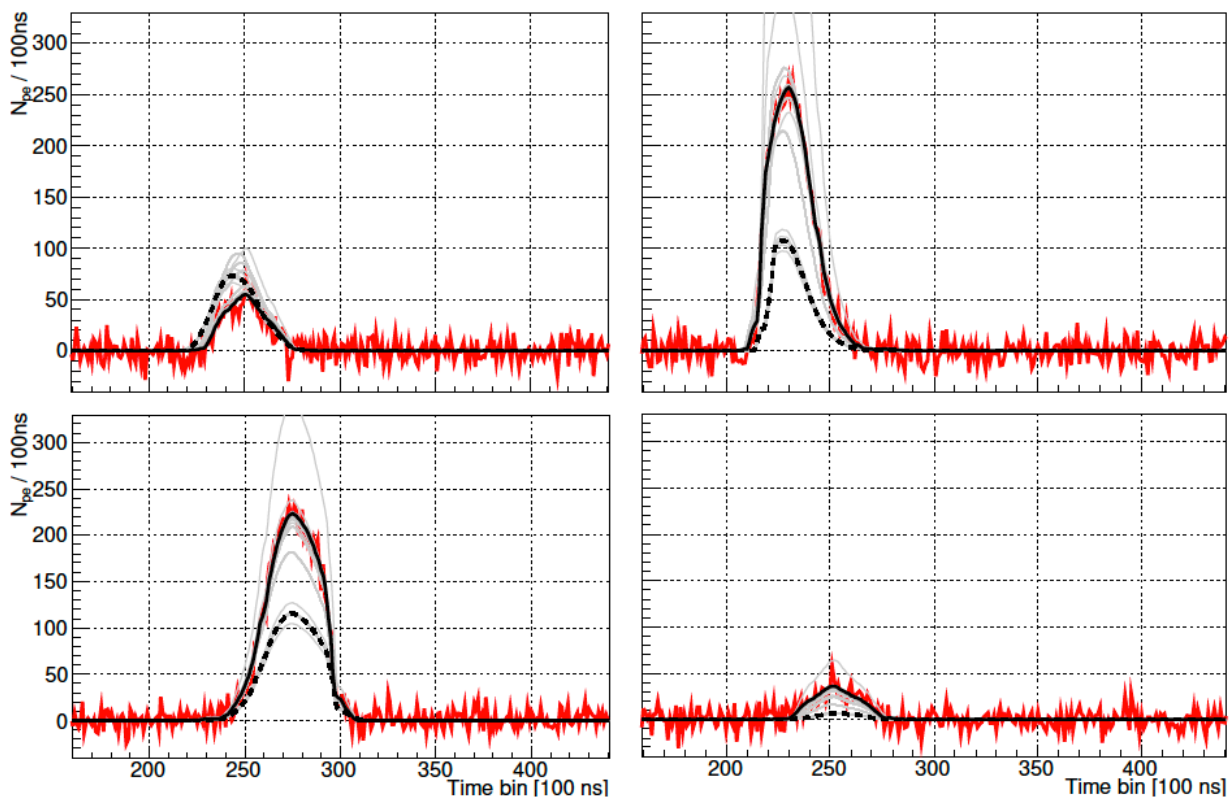
PSF



Top-down reconstruction

n		1	2	3	4	5	6
x_c		1	2.28	3.53	4.72	5.89	7.04

Table 3.1: Values x_c by which the negative log-likelihood must be increased by to give the uncertainties on the n reconstructed parameters.



$$P(x_i|\mu_i, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[-\frac{(x_i - \mu_i)^2}{2\sigma^2} \right]$$

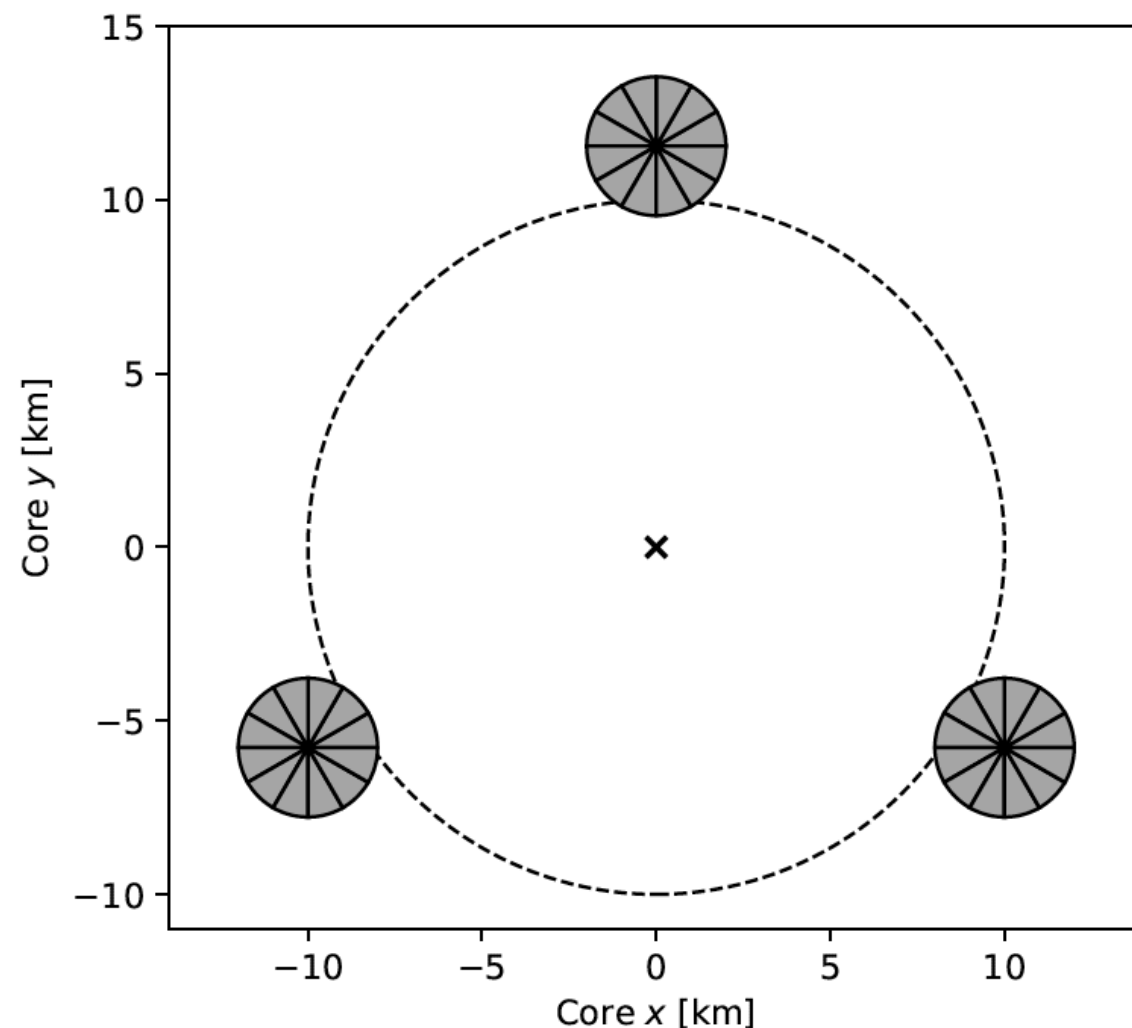
$$\sigma^2 = \sigma_{\text{bckg}}^2 + \mu_i(1 + V_g)$$

Reconstruction Efficiency

- Investigate using simulated dataset of 1000 showers

$X_{\max}$	EPOS-LHC (500 – 1200 g cm ⁻²)
Energy	Fixed (30 EeV)
θ	$\sin \theta \cos \theta$ (0 – 80°)
ϕ	Uniform (0 – 360°)
Core	Uniform in circle centred at (0,0), $r = 10$ km

- Use reconstruction of zenith angle as measure of performance. Other parameters fixed to true values.
- Initial result:**
 - 274 events with invalid minimum
 - 232 events with $\theta_{\text{err}} > 100^\circ$



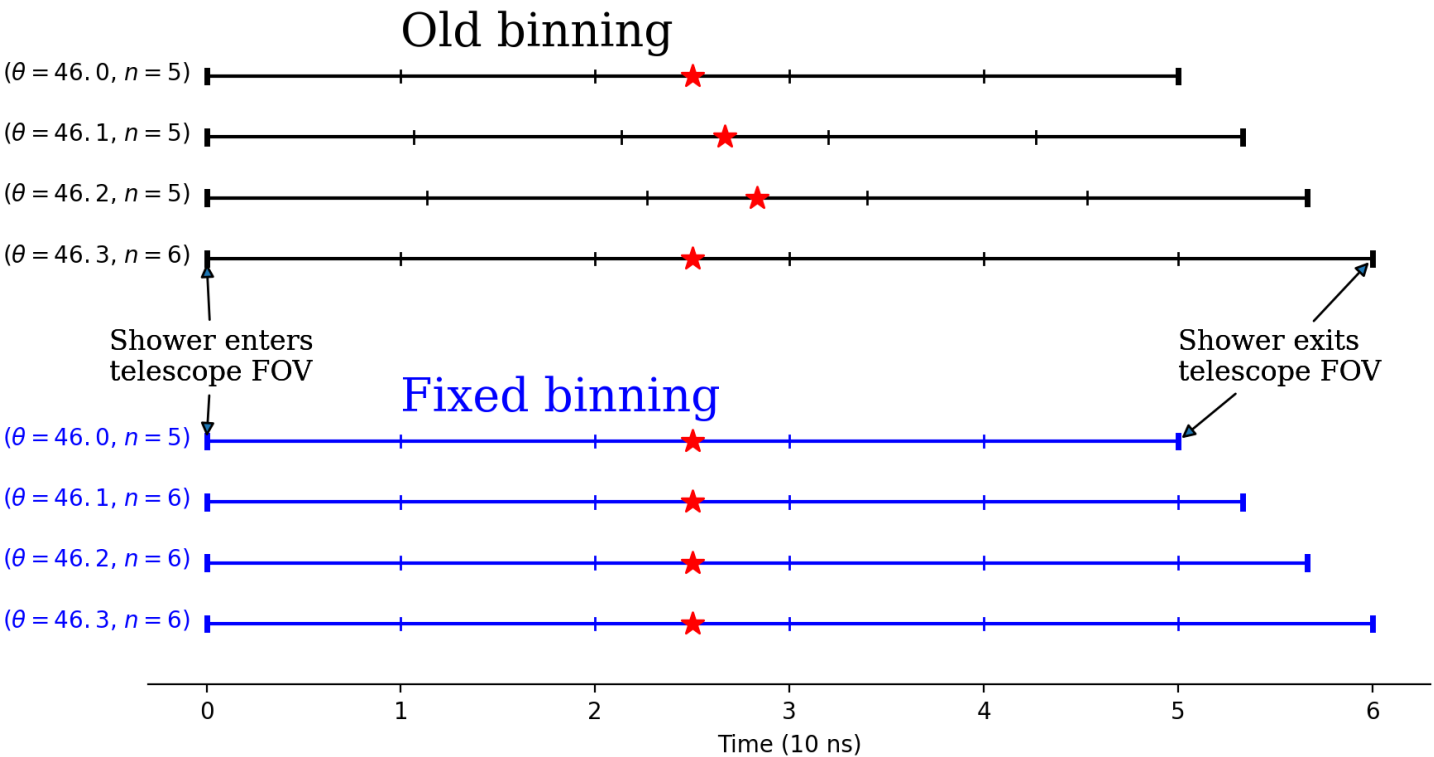
Reconstruction Efficiency

Problem 1a: Binning

- In original simulation, the method of binning the shower axis generated jumps in the locations and sizes of bins
- Why?
 - Number of bins set via $n = \Delta t / 10\text{ns}$
 - *Then*, length of one bin set via $t_{\text{bin}} = \Delta t / n$
 - In above, both n and Δt integers

Old binning using # bins

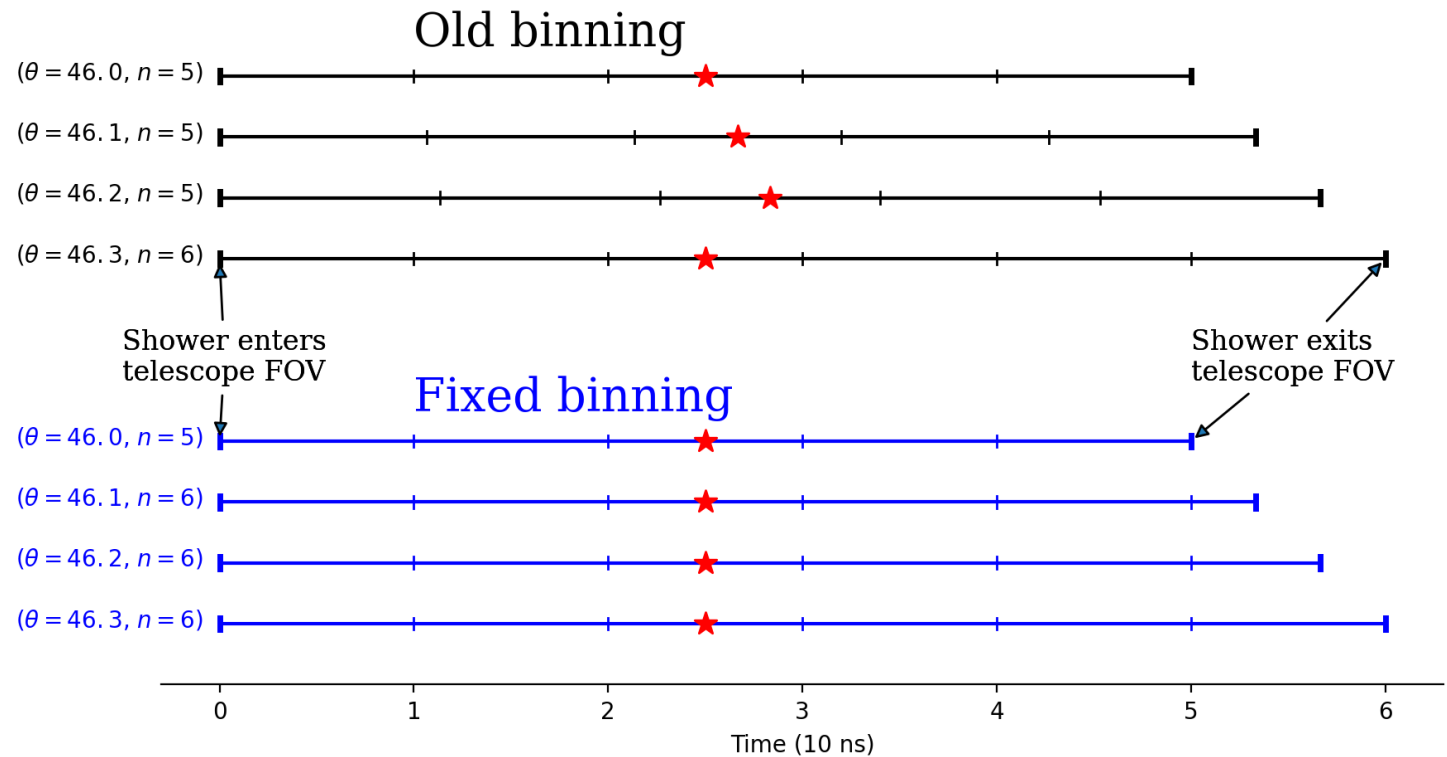
$$= \left\lfloor \frac{t_2 - t_1}{10 \text{ ns}} \right\rfloor$$



Reconstruction Efficiency

Problem 1a: Shower axis binning

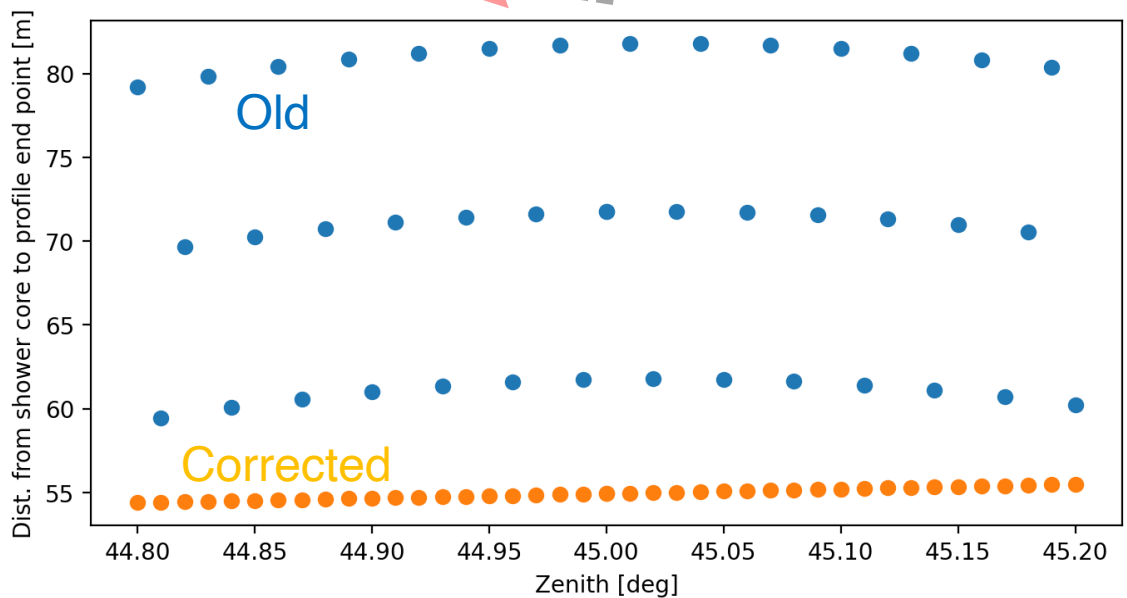
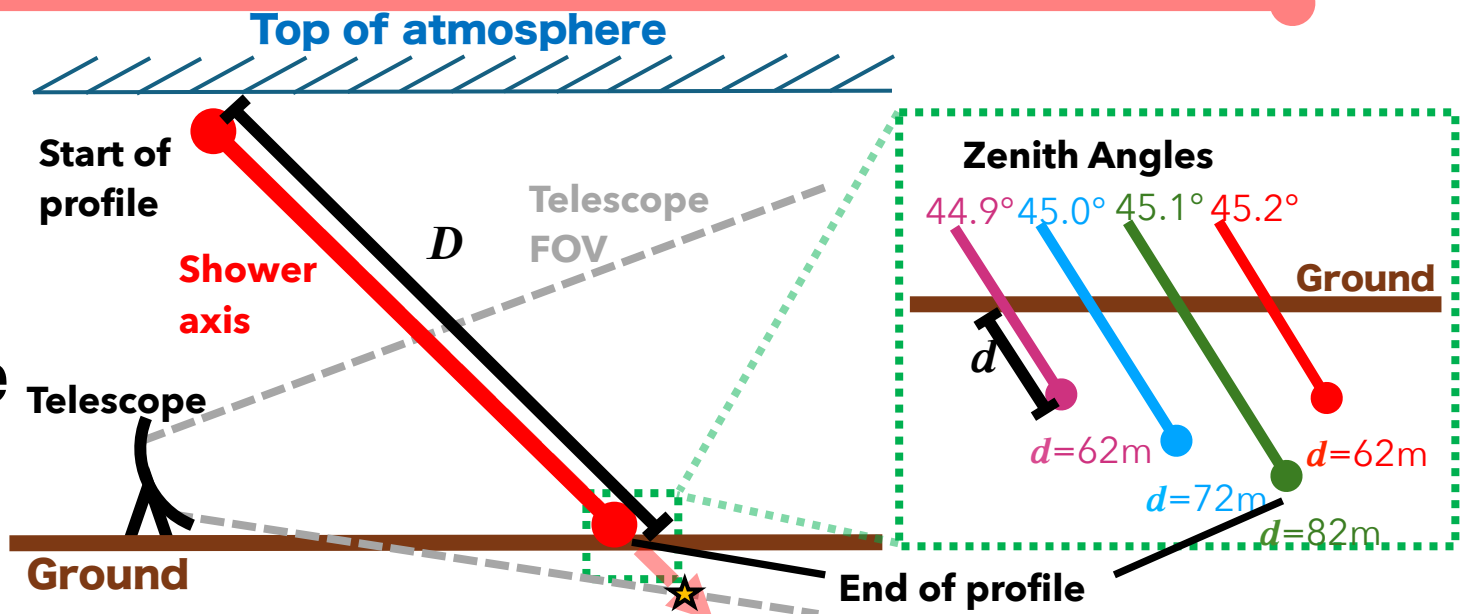
- Correct by using fixed $t_{bin}=10ns$ and double precision for Δt
- Store fraction of last bin included and scale signal accordingly
- Bin centers and sizes now stay the same



Reconstruction Efficiency

Problem 1b: Shower axis binning

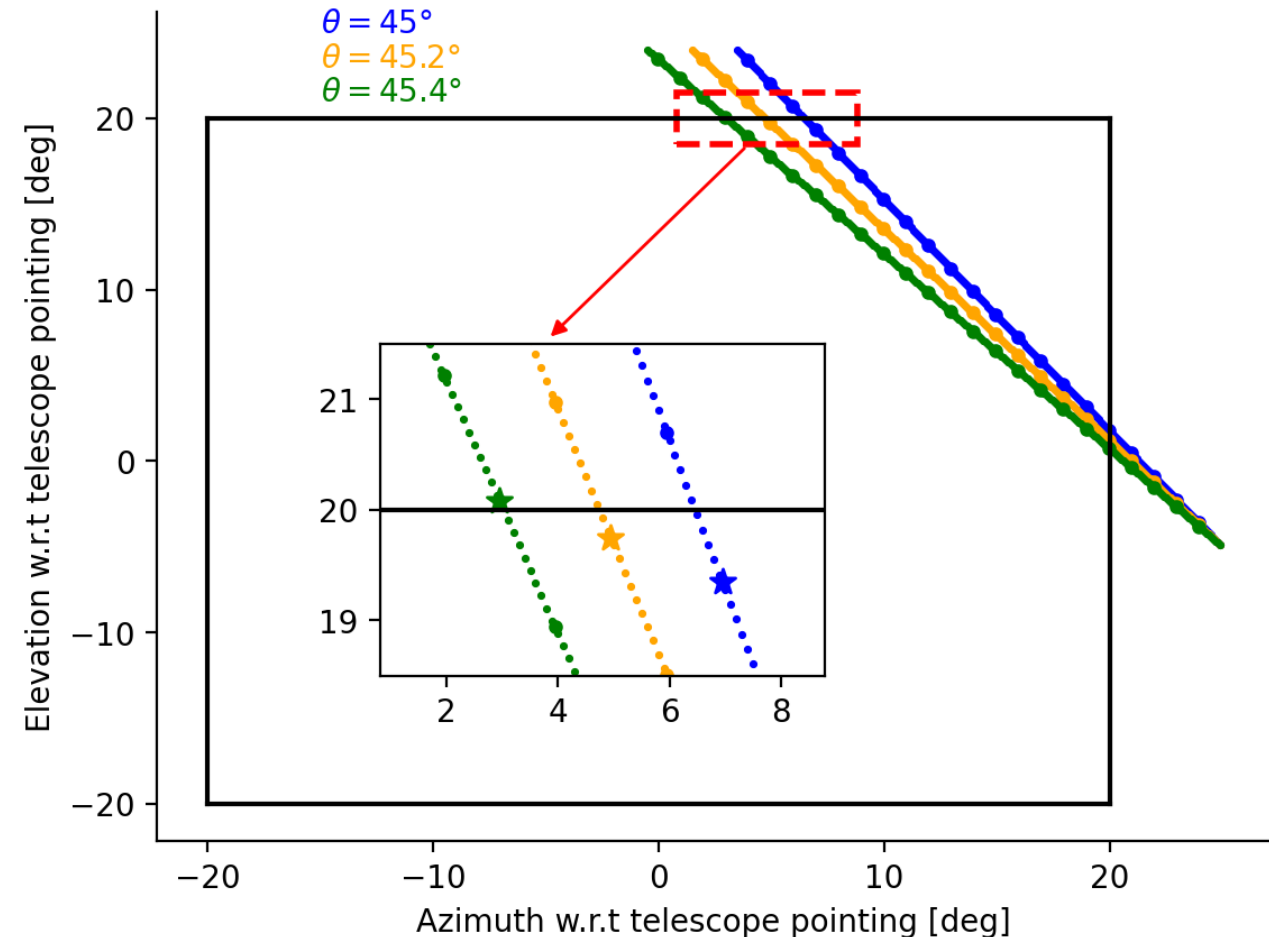
- Location of the end of the shower profile, which may be used to determine binning, found to change erratically
- Calculated in Auger module(?!)
 - Using integers to determine profile binning
- Fix by copying Offline code and correcting with doubles



Reconstruction Efficiency

Problem 3: Signal re-binning

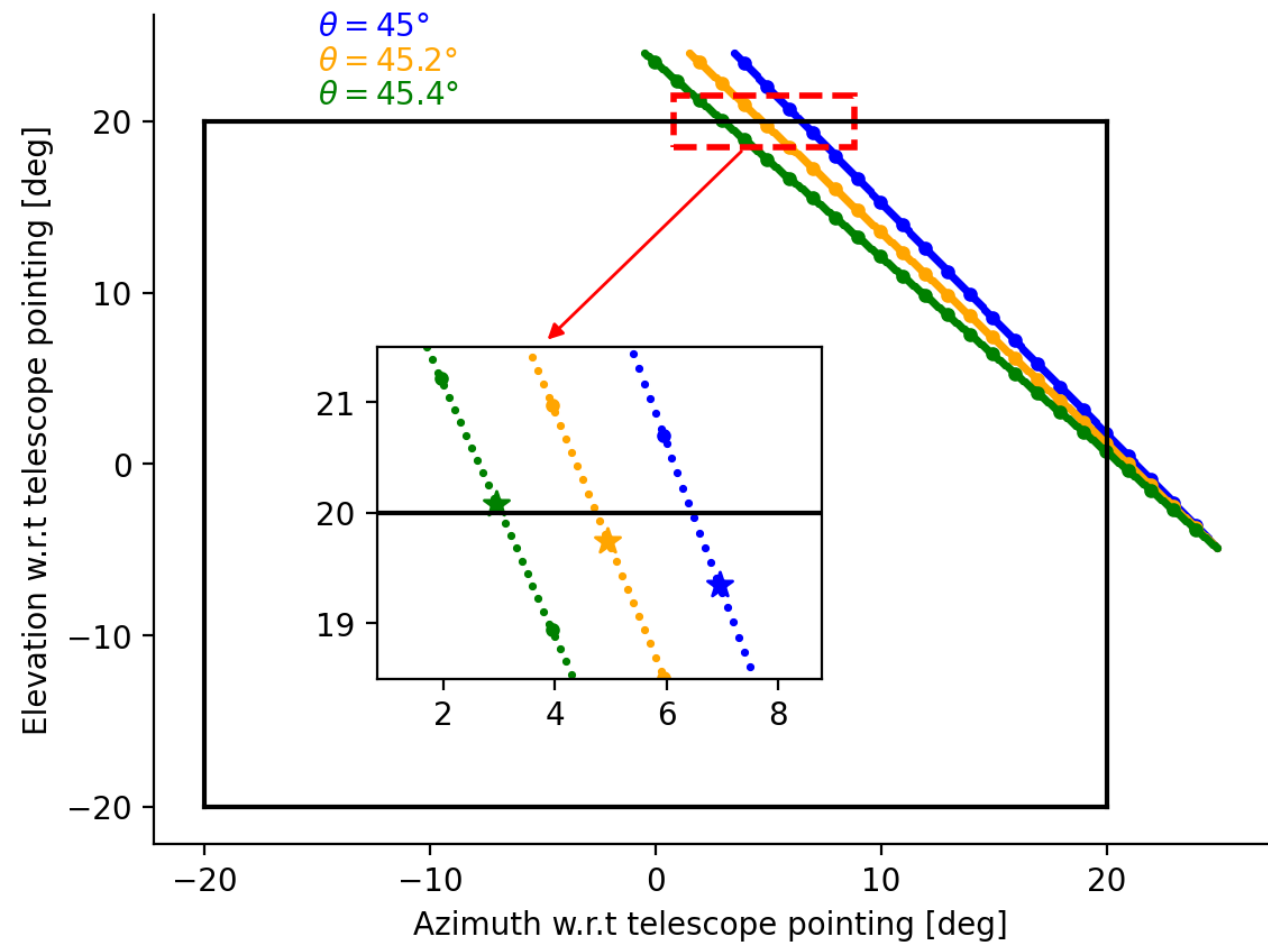
- After light at aperture calculation and filter response, multiply by directional efficiency
- Right plot
 - Small circles - 10 ns bin centers
 - Big circles - 100 ns bin centers
 - Stars - centers of 4th 100 ns bin



Reconstruction Efficiency

Problem 3: Signal re-binning

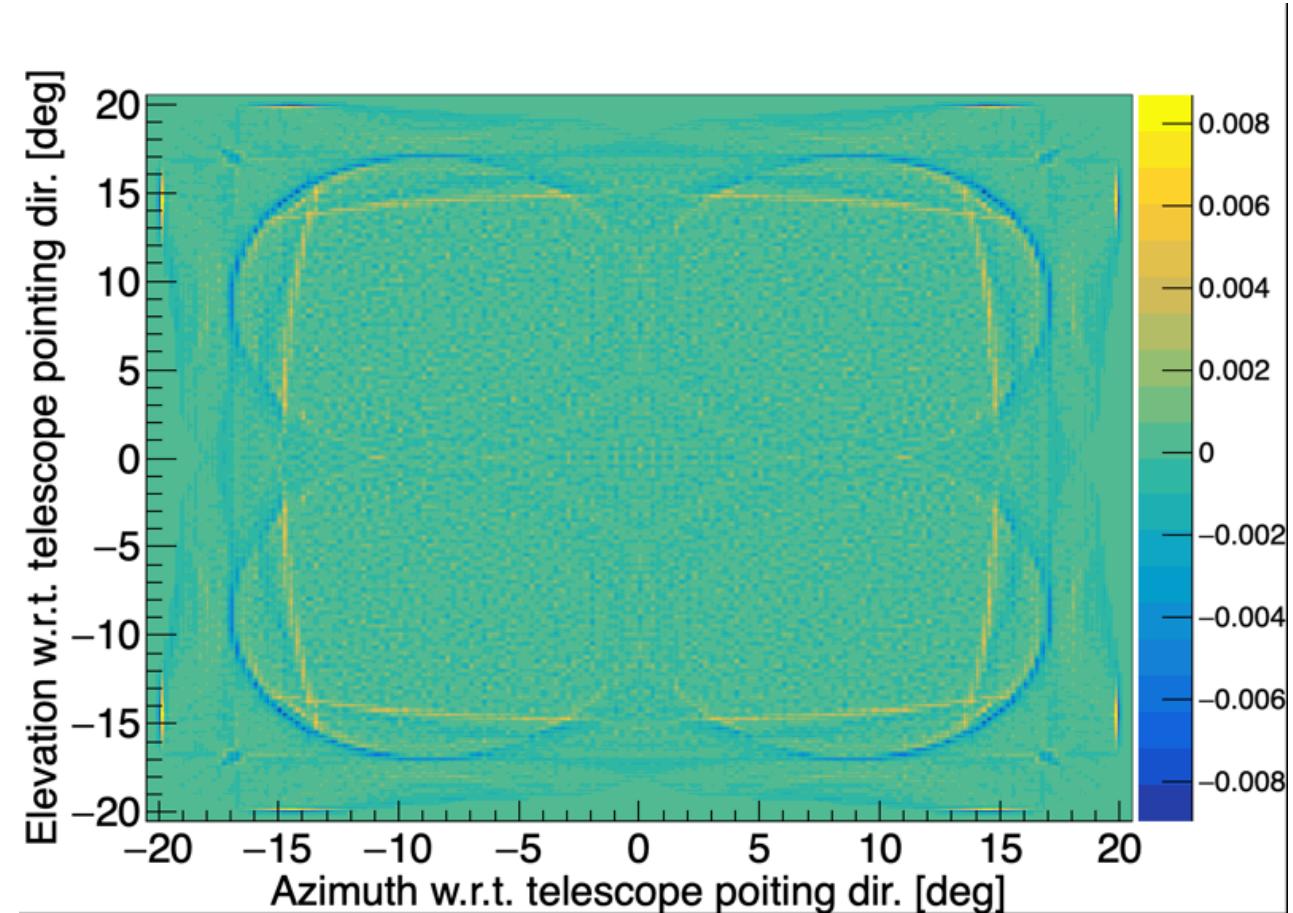
- Efficiency factor for 100 ns bin
 - For elevation/azimuth outside $(-20^{\circ}, 20^{\circ})$, efficiency = 0
 - Inside range non-zero. Boundary has areas where efficiency = 0.02~0.03 – doesn't tend to zero sufficiently smoothly
- Small changes in geometry can cause bin inside range to fall outside range – get discontinuity



Reconstruction Efficiency

Problem 3: Signal re-binning

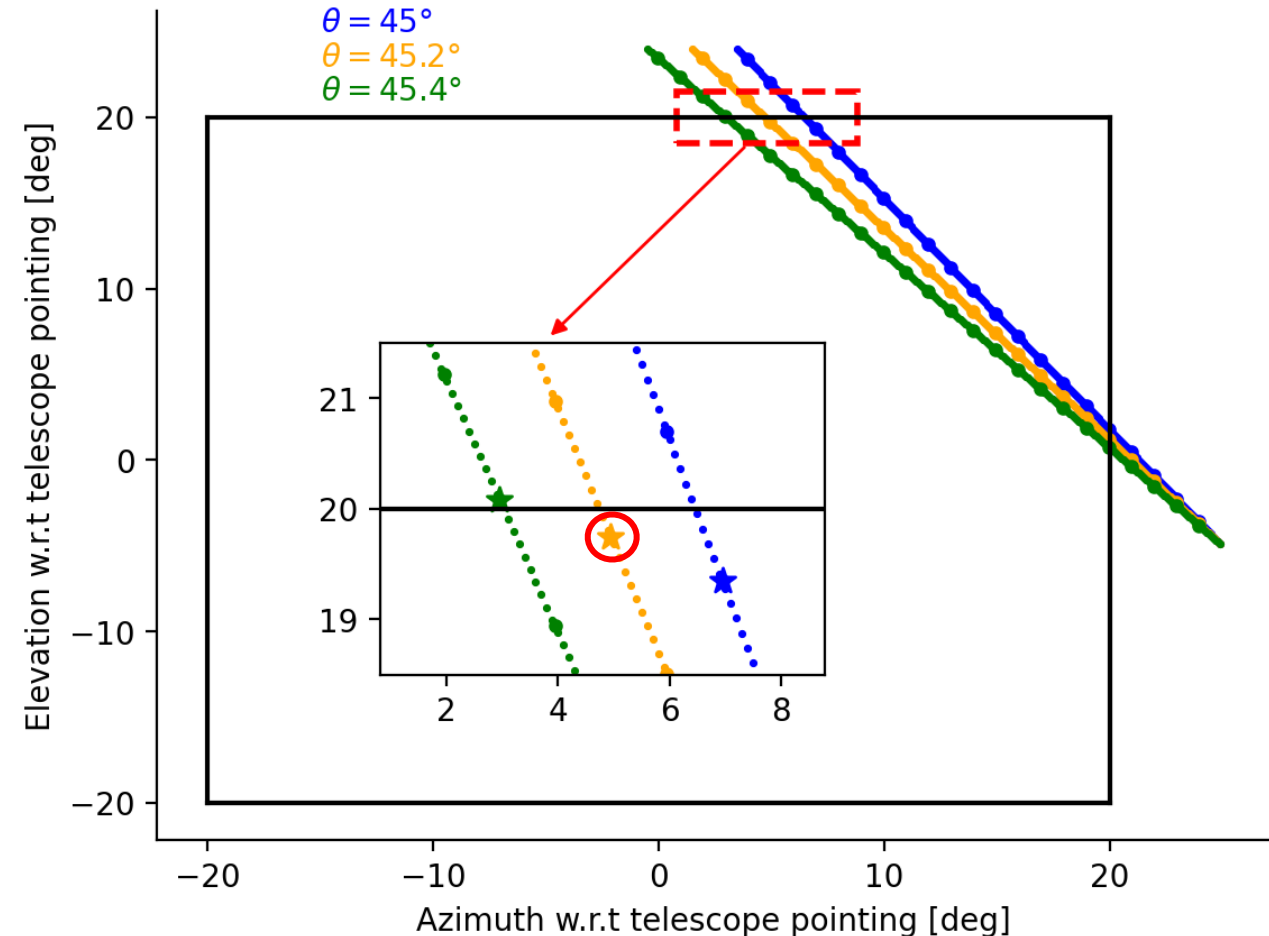
- Solution Part 1:
 - Ideally would perform real measurements until values at boundary “sufficiently close to 0”
 - Emulate by smoothing map. Extend to $(-20.5^\circ, 20.5^\circ)$ in elevation/azimuth with zeros, then use ROOT TH2D.Smooth()
 - Right plot - difference between original and smoothed map



Reconstruction Efficiency

Problem 3: Signal re-binning

- Solution Part 2:
 - With smoothed map, calculate signal in 100 ns bins next to edges using surrounding 10 ns bins directly
 - Each 10 ns bin contains less signal so jumps to 0 are less impactful on final likelihood
- **After fix:**
 - 82→79 events with invalid minimum
 - 8→2 events with $\theta_{\text{err}} > 100^\circ$



Reconstruction Efficiency

Reconstructing the test data set of 1000 showers with binning corrections

Problem 1: Shower axis binning

- 274→109 events with invalid minimum
- 232→20 events with $\theta_{err}>100^\circ$

Problem 2: Azimuthal difference

- 109→82 events with invalid minimum
- 20→8 events with $\theta_{err}>100^\circ$

Problem 3: Directional efficiency

- 82→79 events with invalid minimum
- 8→2 events with $\theta_{err}>100^\circ$

Reconstruction state	Failed reconstructions	$\theta_{err} > 100^\circ$
Original simulation	274	232
+ Shower axis binning	109	20
+ Azimuthal difference	82	8
+ Signal re-binning	79	2
+ Strategy Improvement	1	1

Reconstruction Efficiency

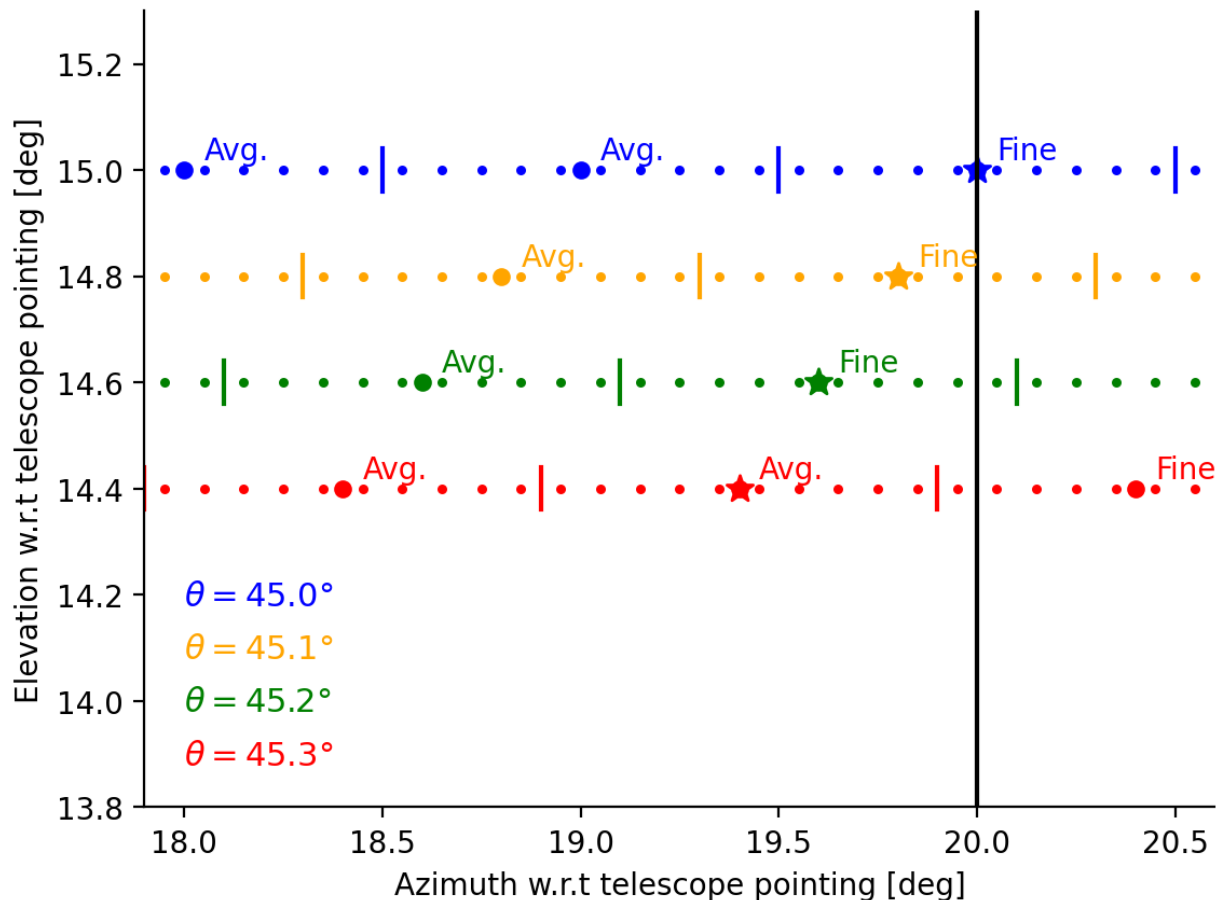
Remaining reconstruction failures

- Remaining events with invalid minimum caused by artefact of reconstruction setup
 - Starting with true values, occasionally fitter does not search space thoroughly enough to determine minimum location within specified tolerance
 - Rectify by changing minimizer strategy setting in Minuit2 (from 0 to 1) in these cases
 - Should (almost certainly) not be an issue for real data reconstruction
- **After fix:**
 - 79→1 events with invalid minimum
 - 2→1 events with $\theta_{\text{err}} > 100^\circ$

Reconstruction Efficiency

Remaining reconstruction failures

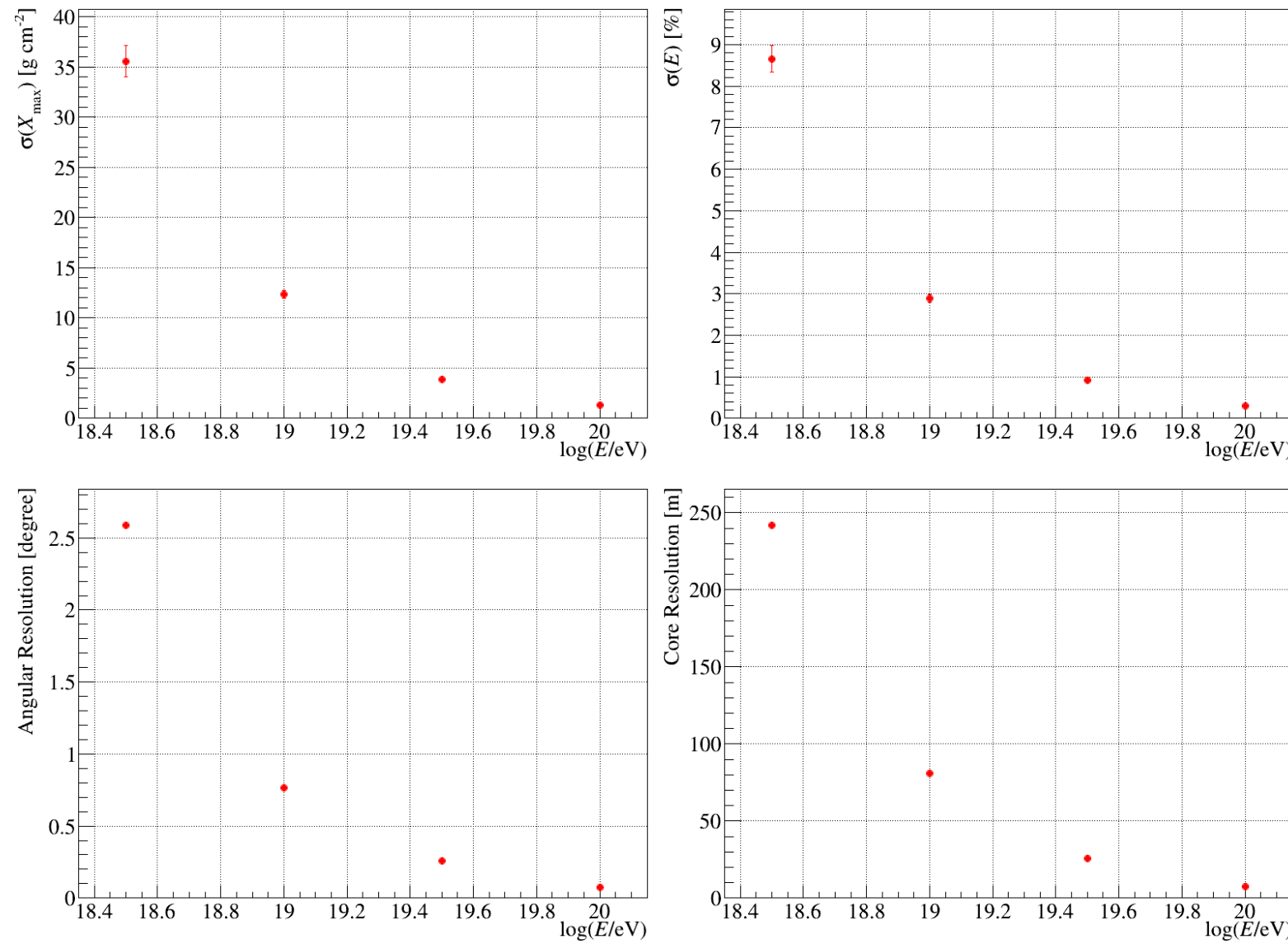
- Last event fails because signal re-binning solution not perfect – see thesis for details



Resolutions after improvements

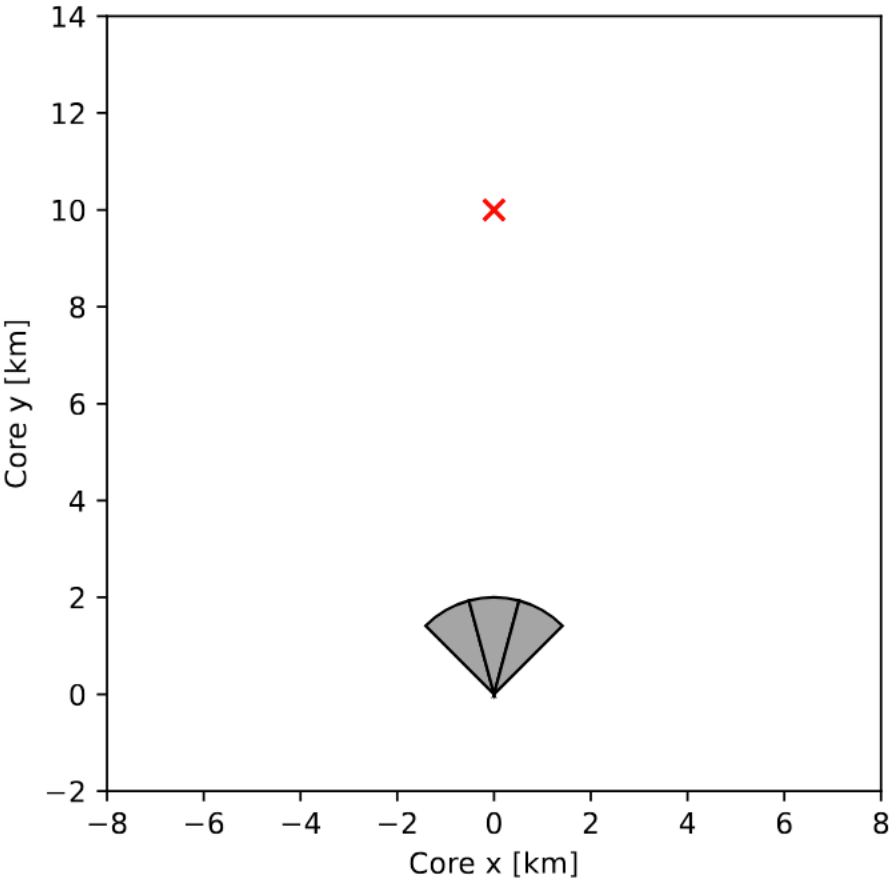
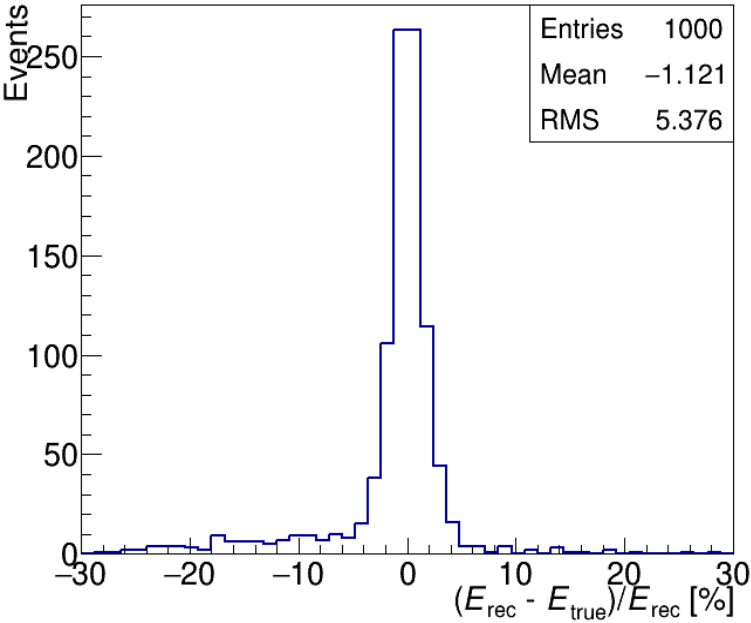
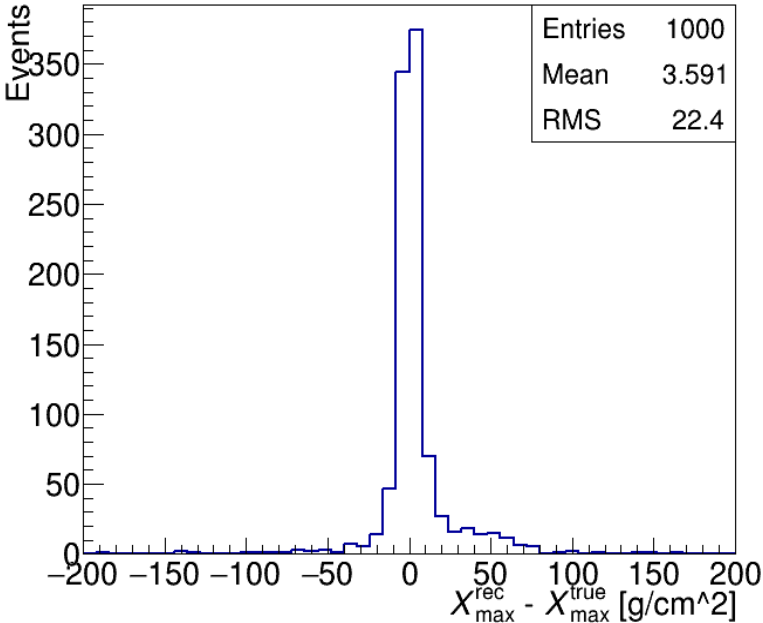
Ideal resolutions

- Resolutions at 30 EeV
 - Xmax: < 5 g cm⁻²
 - Energy: 1%
 - Core: < 50m
 - Arrival direction: < 0.5°
- Not representative of true resolutions
 - First guess already very good, data-independent
 - Ideal simulations: Gaussian background, no shower-to-shower fluctuations, no atmospheric/calibration uncertainties, no simulation of electronic response (e.g. saturation)



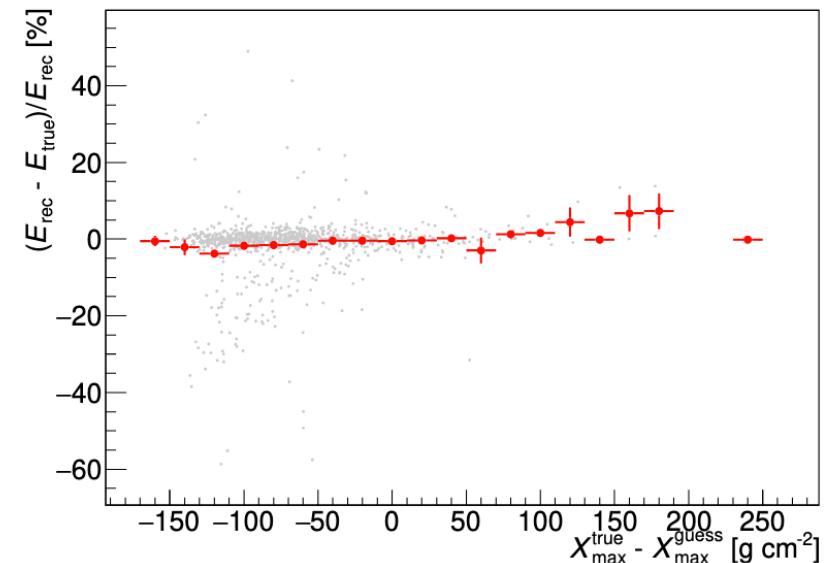
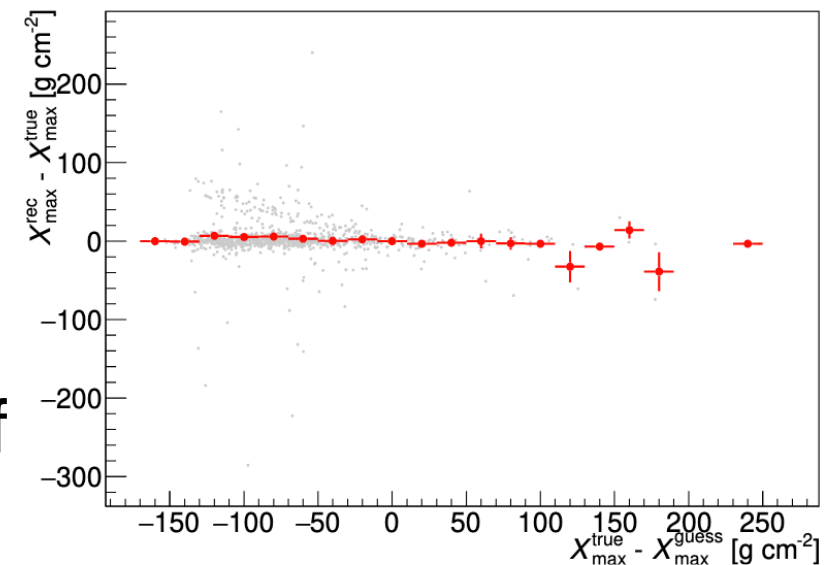
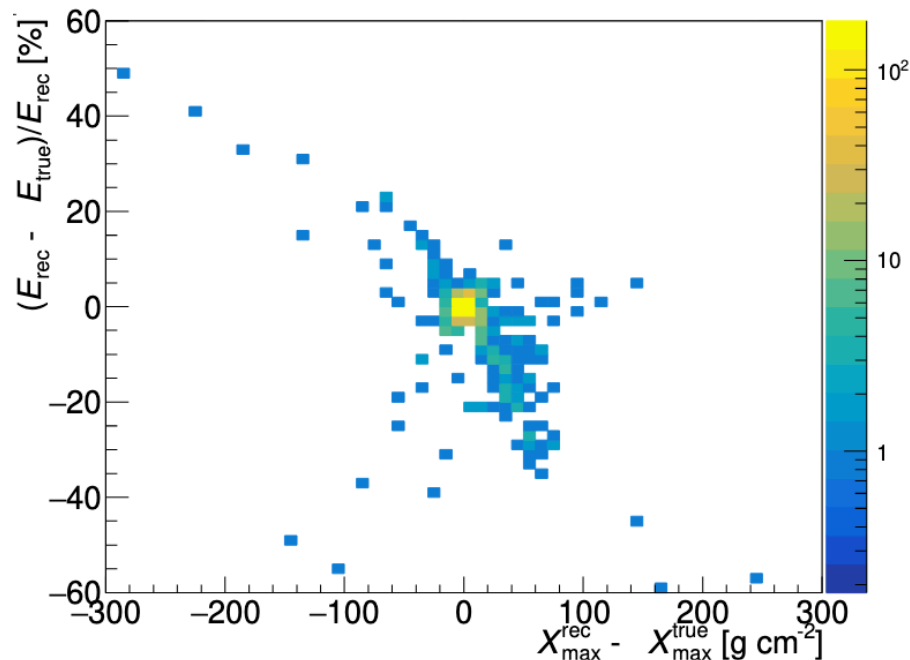
X_{max} & Energy Correlation

- When reconstructing X_{max} and energy simultaneously, get long tails, even when using true geometry



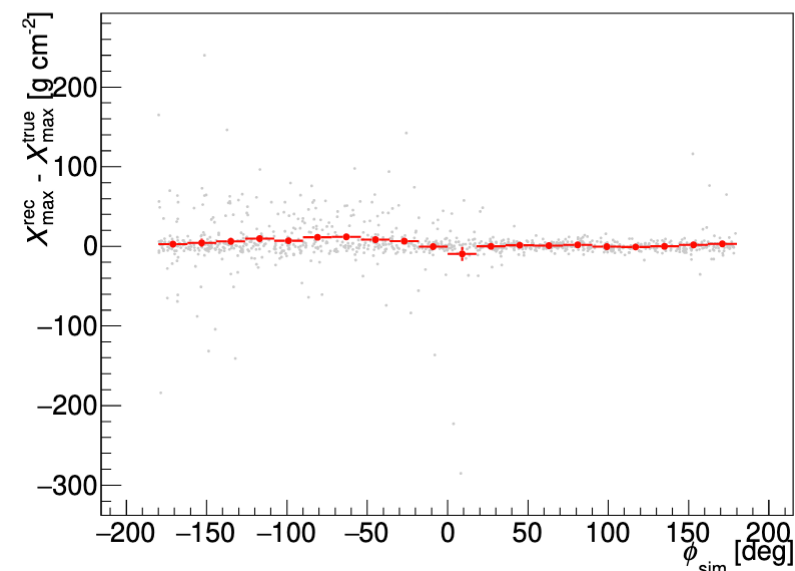
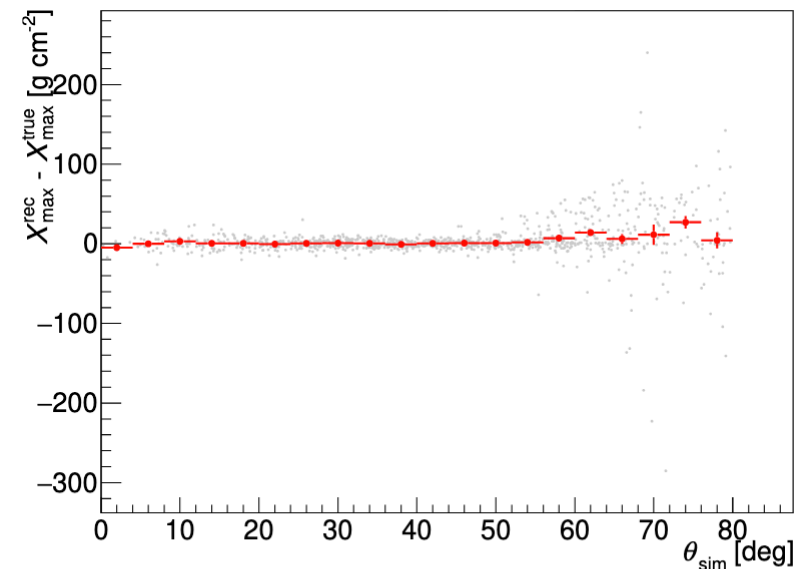
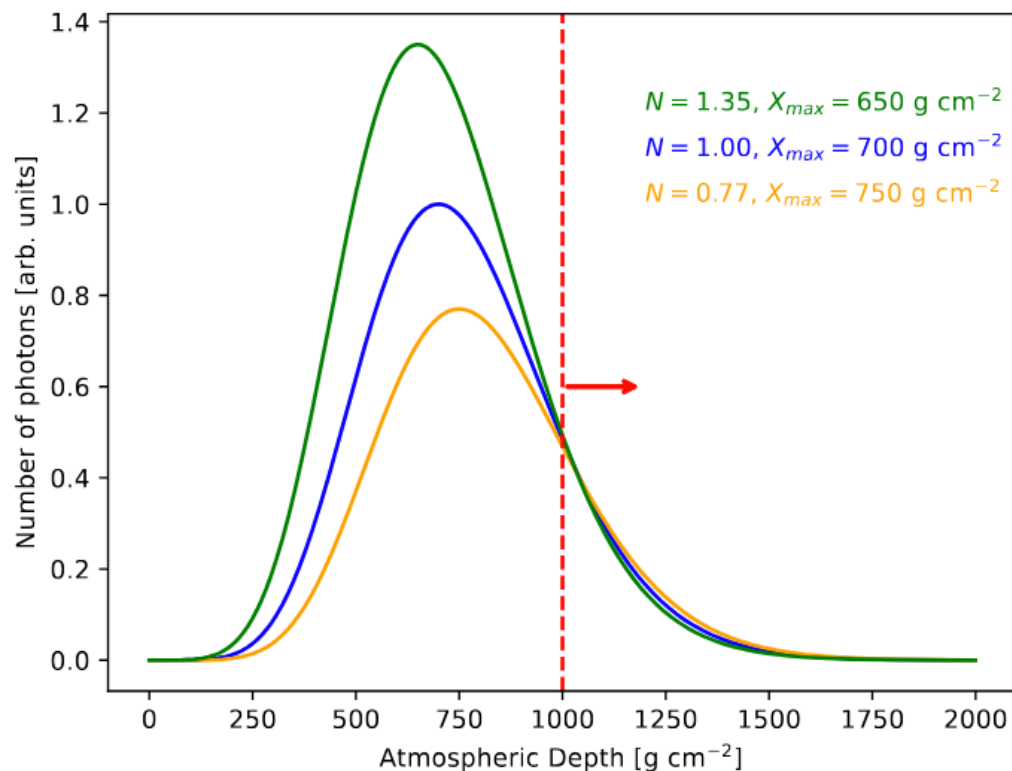
$X_{\max}$ & Energy Correlation

- Look at difference in reconstruction and true values as function of difference between true and first guess values
- Correlation between energy diff and $X_{\max}$ diff



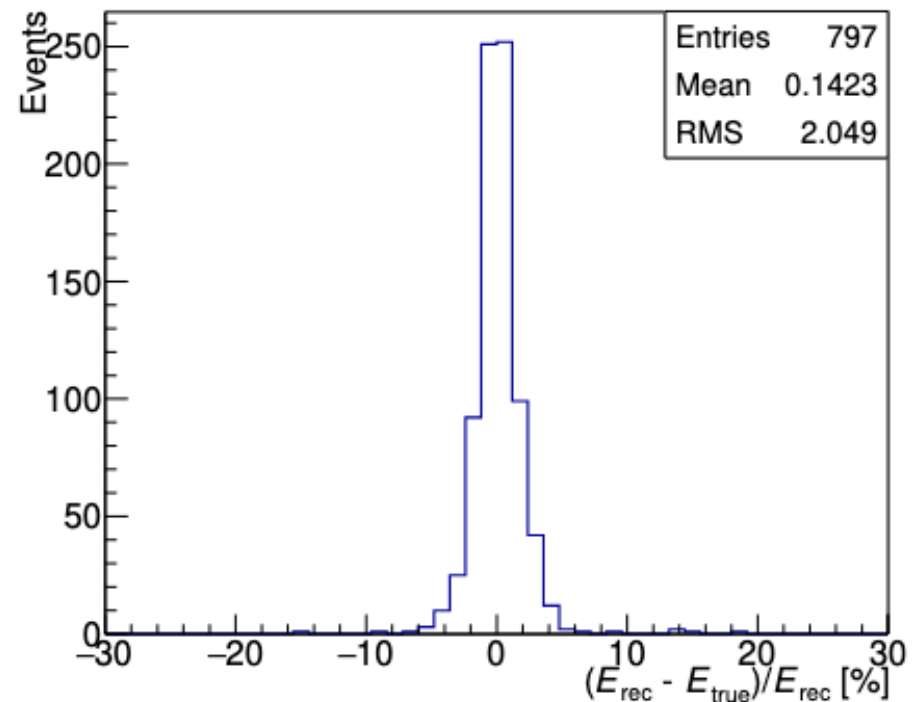
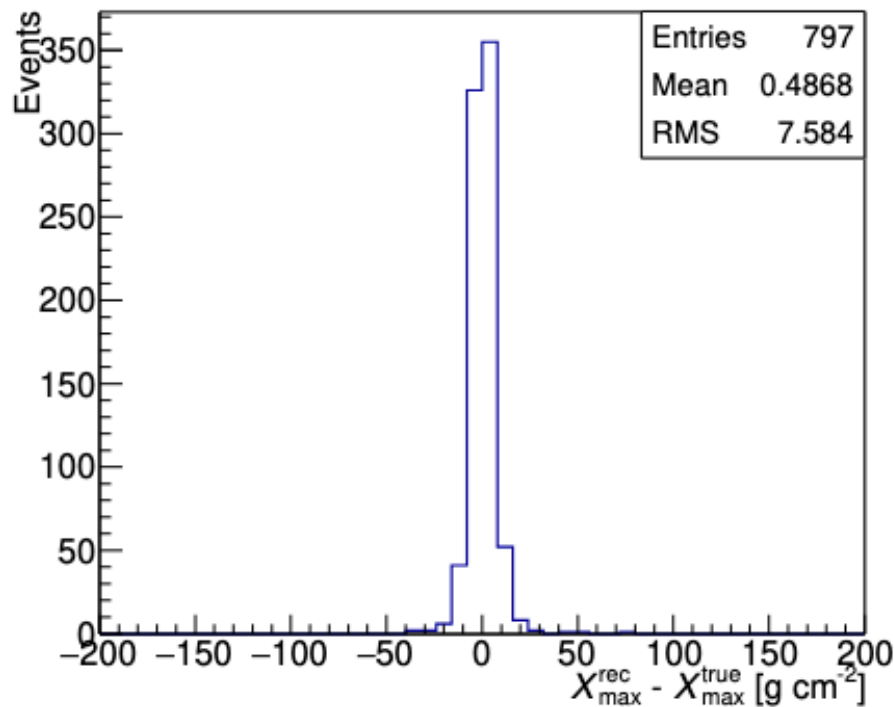
$X_{\max}$ & Energy Correlation

- Only applies for geometries with high zenith angles coming from behind telescope i.e. seeing only last part of profile!



$X_{\max}$ & Energy Correlation

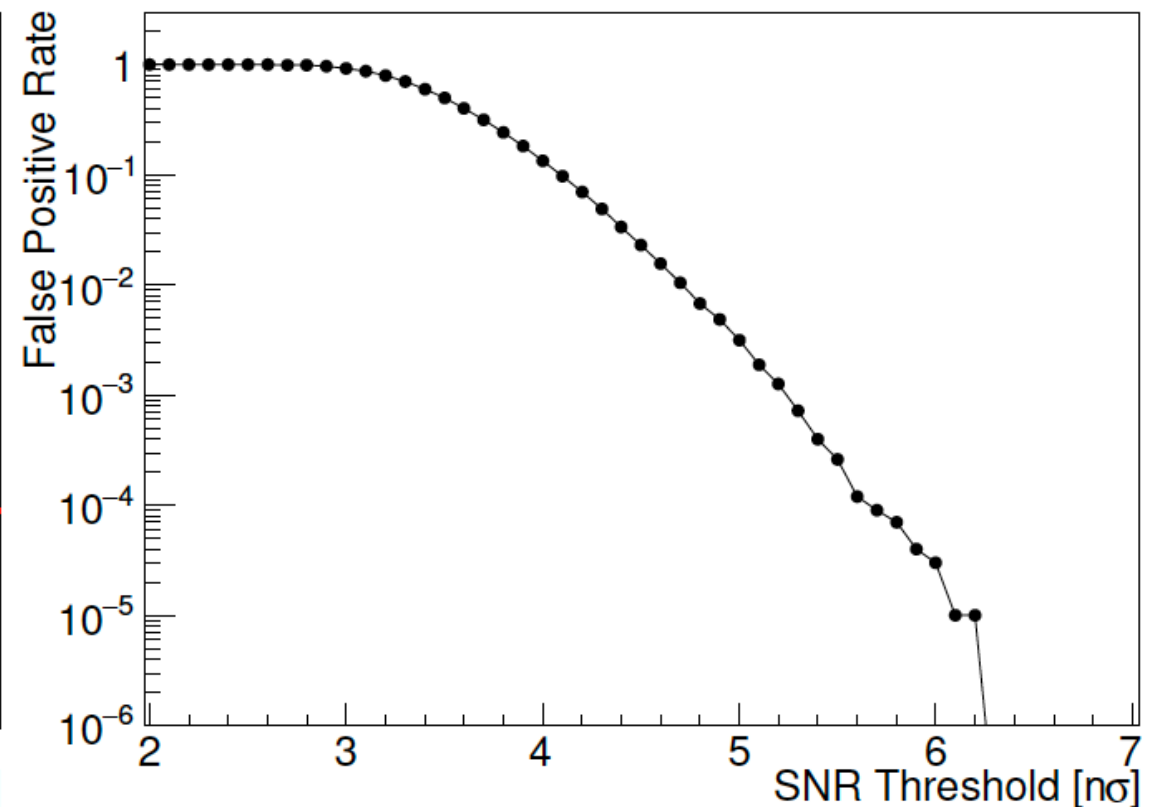
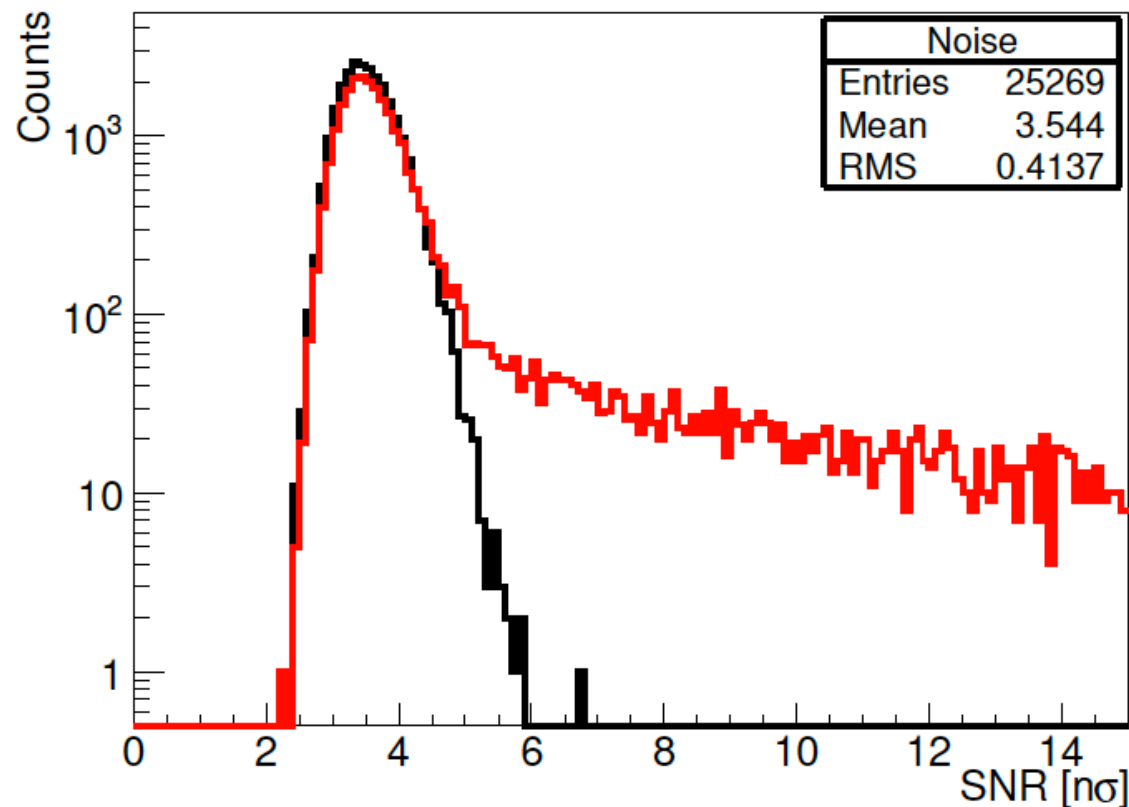
- Cut out events with $X_{\max}$ outside of FOV of triggered telescopes
 - Use telescope location, pointing direction and reconstructed parameters
- Tails in distribution disappear - **important cut for FAST**



First guess I – Neural Networks

$$\text{SNR} = \frac{S}{N} = \frac{\sum_{i=k_{\text{start}}}^{k_{\text{stop}}-1} S_i}{\sigma_{\text{nsb}} \sqrt{k_{\text{stop}} - k_{\text{start}}}},$$

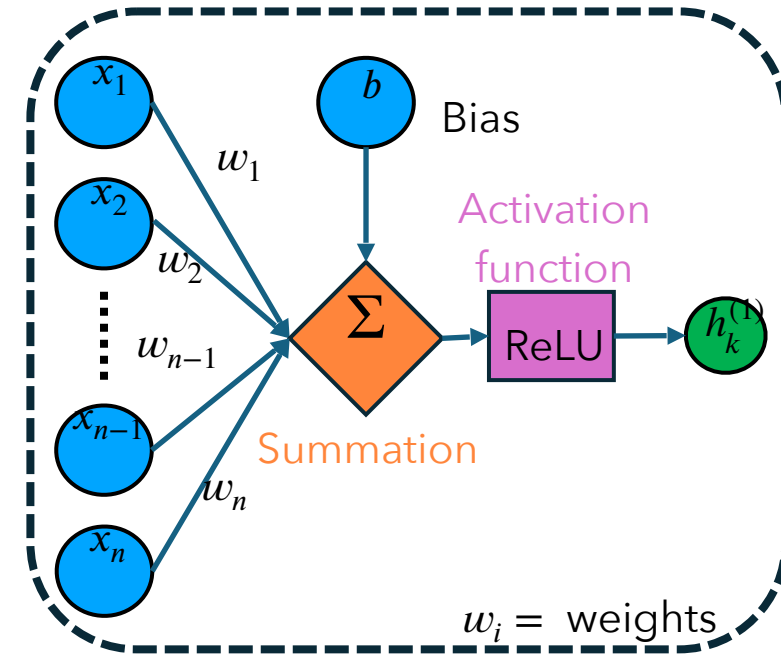
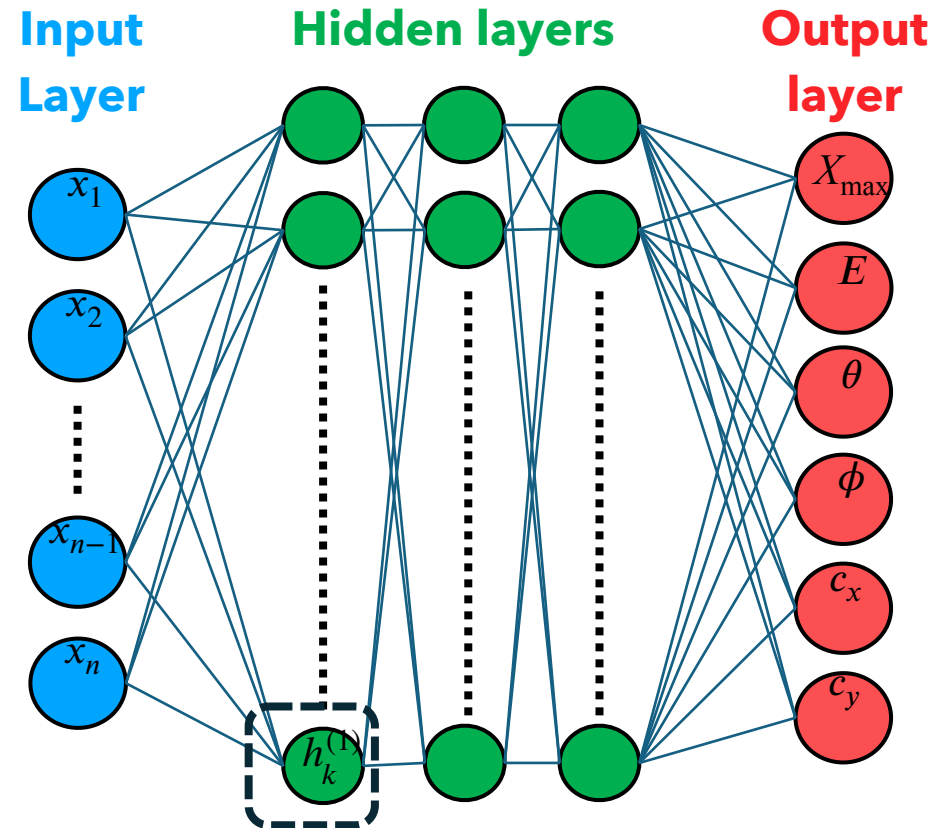
SNR threshold of 6



First guess I – Neural Networks

Resolutions of original FAST 3-Eye layout study

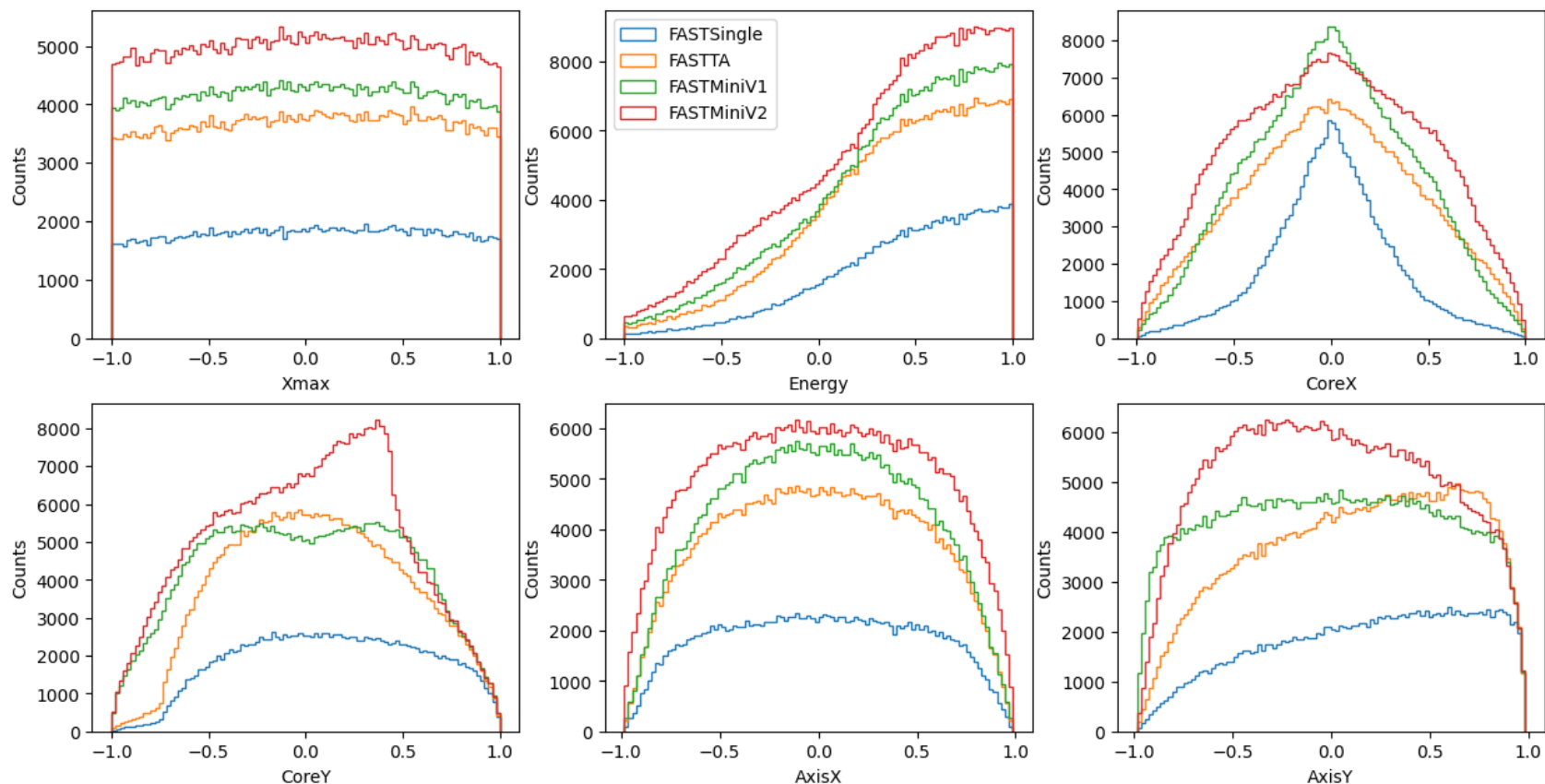
- Xmax: 30 g cm⁻²
- Energy: 8%
- Core: 465 m
- Arrival direction: 4.2°



First guess I – Neural Networks

Output Parameters

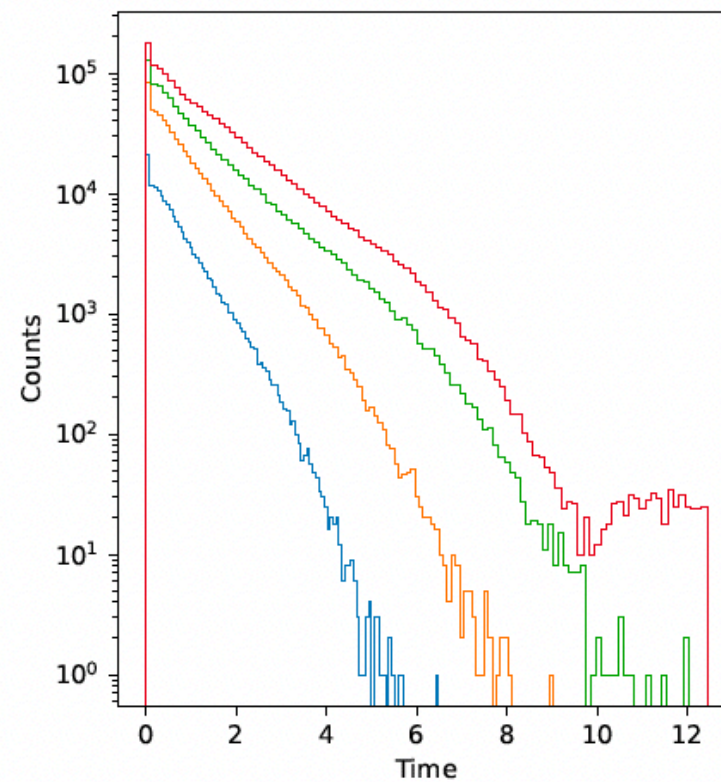
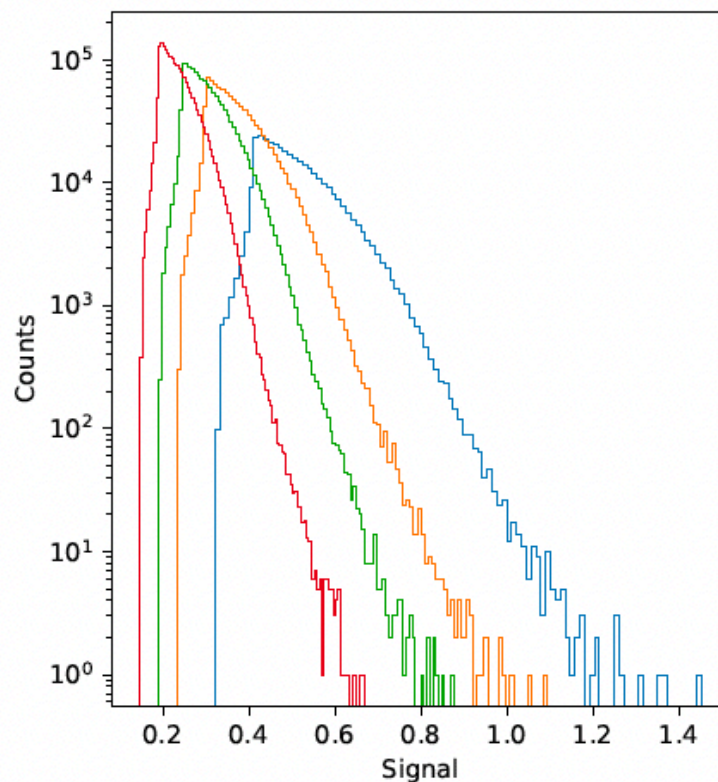
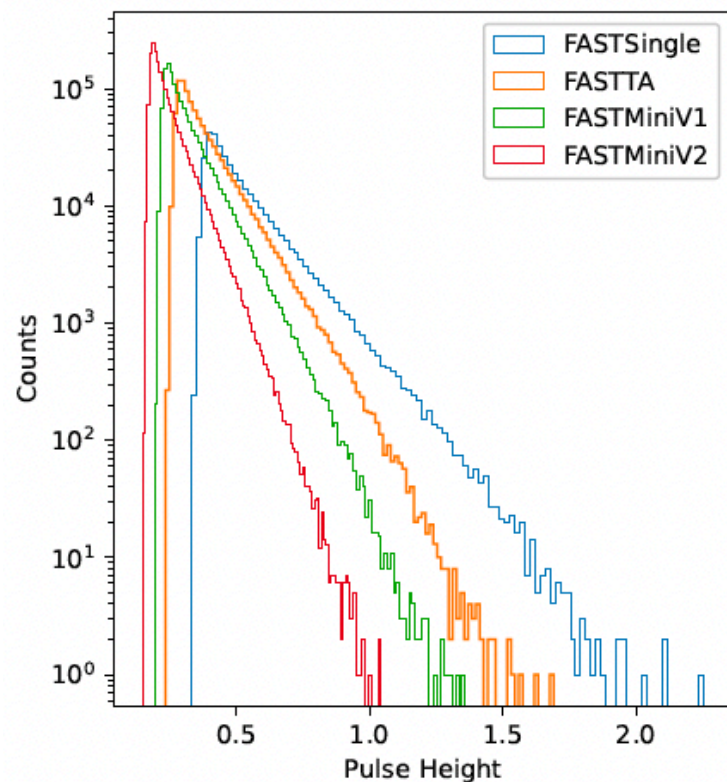
- Distributions of output (shower) parameters we want the models to learn – shapes of core/axis histograms due to layout geometry



First guess I – Neural Networks

Basic DNN

- Input parameter distributions



First guess I – Neural Networks

Basic DNN

- Feed-forward, deep neural network
 - ReLU activation function for hidden layers, Adam optimizer, MSE loss
- 3 inputs from each PMT with $\text{SNR} > 6$ (other PMT inputs set to 0):
 - Integrated signal –
$$S_i = \sum_{j=k_{\text{start}}}^{k_{\text{stop}}-1} \textcircled{S_j} \text{ signal in } j^{\text{th}} \text{ bin}$$

where k_{start} and k_{stop} are determined from maximum SNR region
 - Centroid time –
$$\bar{t}_i = \frac{\sum_{j=k_{\text{start}}}^{k_{\text{stop}}-1} S_j \textcircled{t_j} \text{ time of } j^{\text{th}} \text{ bin}}{\sum_{j=k_{\text{start}}}^{k_{\text{stop}}-1} S_j}$$
 - Pulse height -
$$h_i = \max(\{S_{k_{\text{start}}}, S_{k_{\text{start}}+1}, \dots, S_{k_{\text{stop}}-1}\})$$

$$\text{SNR} = \frac{S}{N} = \frac{\sum_{i=k_{\text{start}}}^{k_{\text{stop}}-1} \textcircled{S_i} \rightarrow S_i}{\sigma_{\text{nsb}} \sqrt{k_{\text{stop}} - k_{\text{start}}}}$$

First guess I – Neural Networks

Basic DNN

- Additional processing of inputs

- Integrated signal:

- Take log10 $\hat{S}_i = \log_{10}(S_i)$

- Divide by average total (logarithmic) signal over all even

$$\hat{\hat{S}}_i = \frac{\hat{S}_i}{\hat{S}_0}$$

- Centroid time

- Subtract earliest centroid time in event $\hat{t}_i = \bar{t} - \bar{t}_0$

- Divide by standard deviation of all centroid times in data set

$$\hat{\hat{t}}_i = \frac{\hat{t}_i}{\sigma_t}$$

Actual inputs

- Pulse height:

- Take log10 $\hat{h}_i = \log_{10}(h_i)$

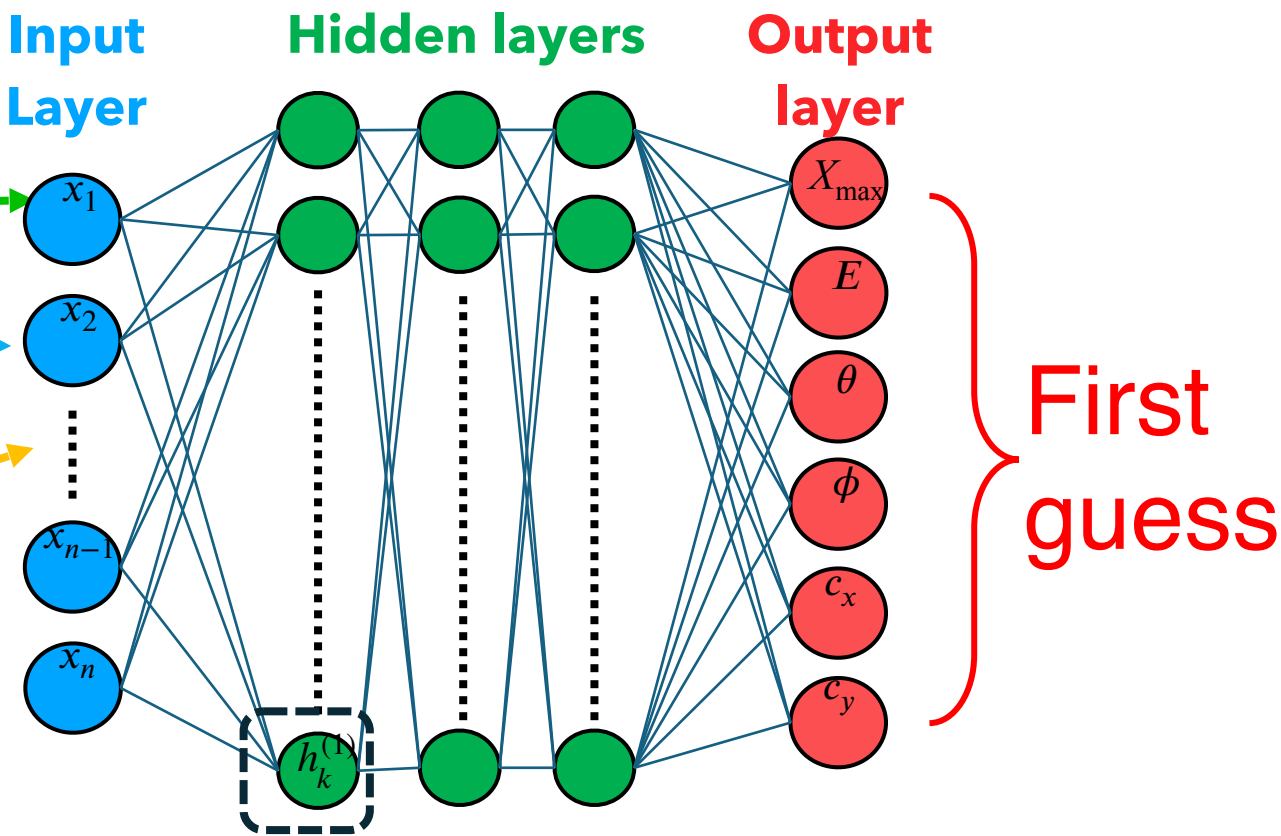
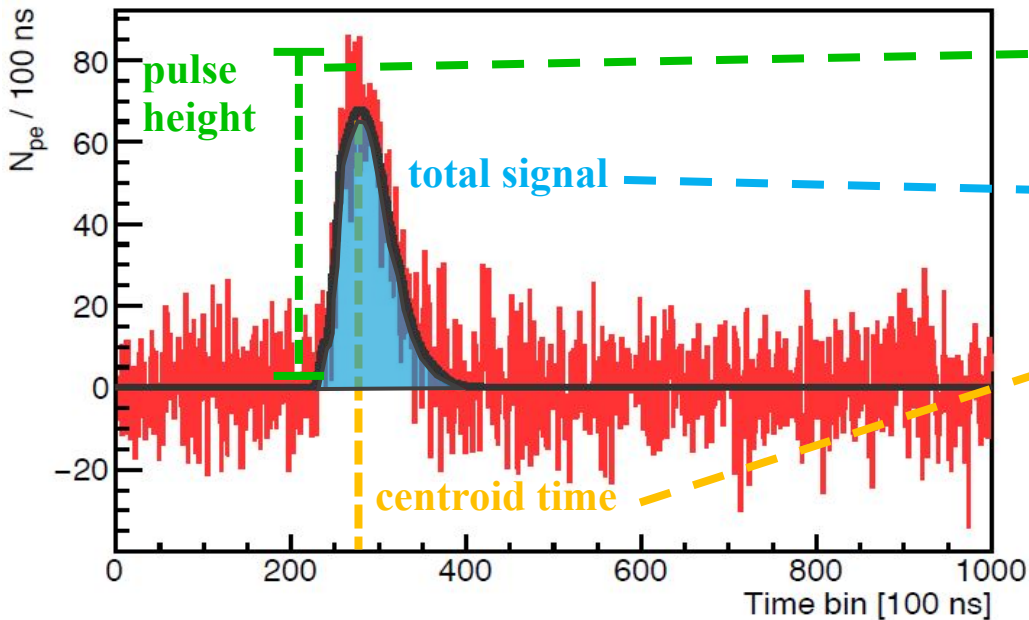
- Divide by average height over all even

$$\hat{\hat{h}}_i = \frac{\hat{h}_i}{\hat{h}_0}$$

First guess I – Neural Networks

Basic DNN

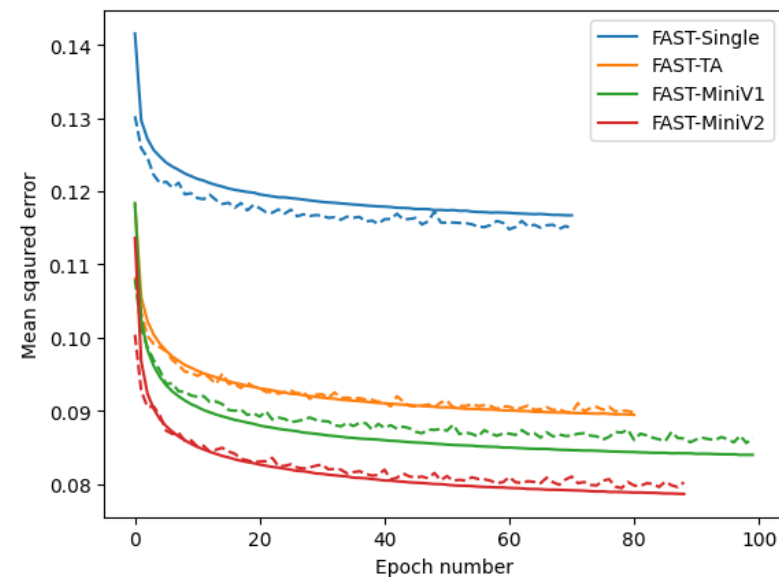
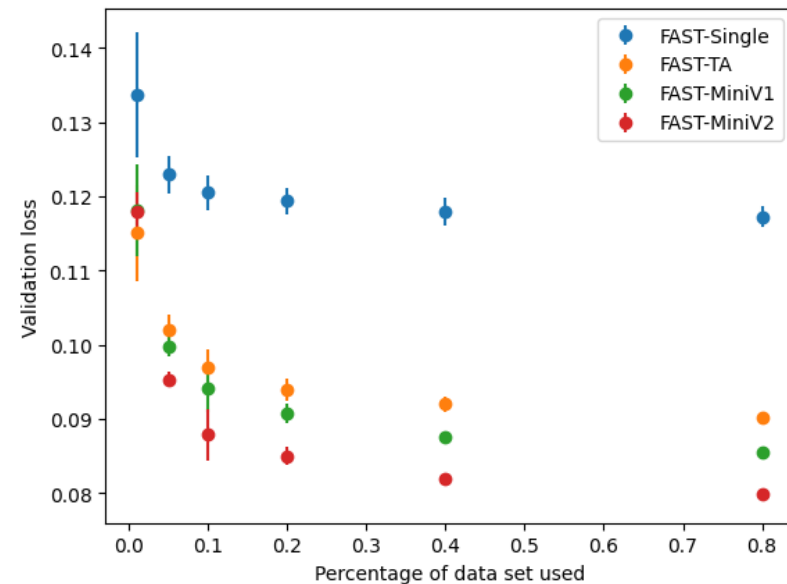
- Schematic



First guess I – Neural Networks

Basic DNN

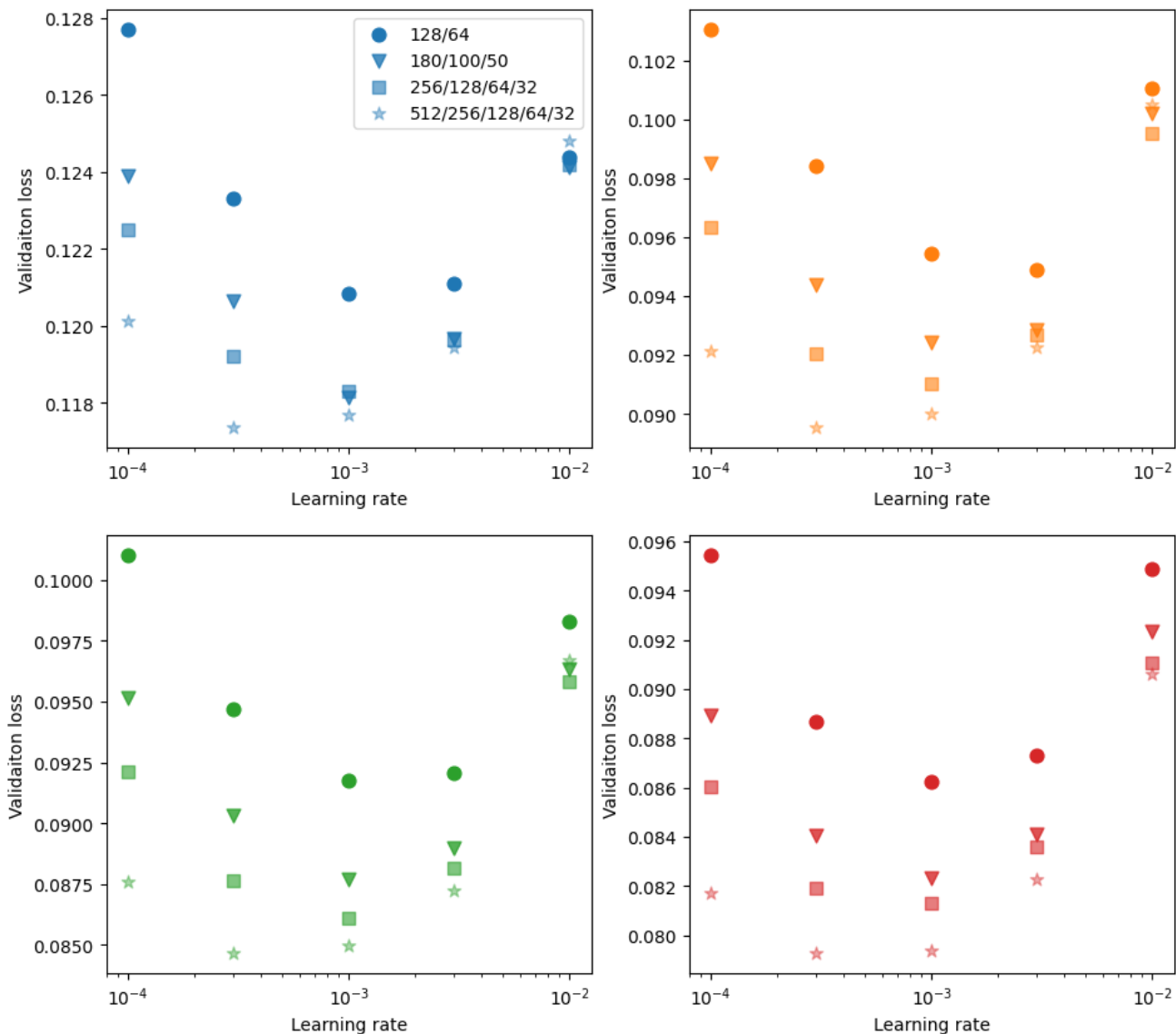
- Initial training setup
 - Learning rate 0.001
 - 90%:10% Train-Test split,
 - Epochs 100 (patience of 10)
 - Batch size of 64
 - Layer structure (Number of PMTs x 3)/ 128/64/6
- Check learning curves, validation loss vs. % of data set used



First guess I – Neural Networks

Basic DNN

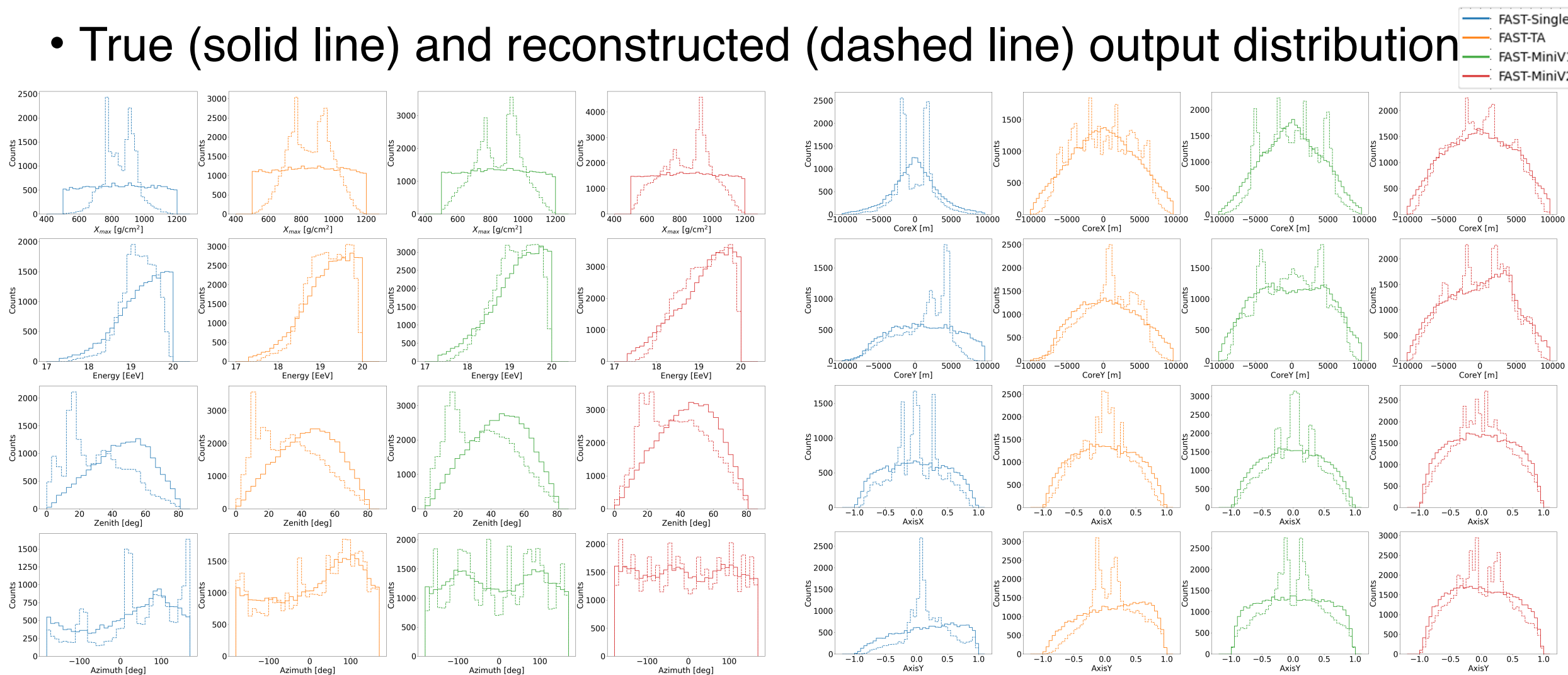
- Basic hyperparameter tuning
- Varying the learning rate and the number of layers/nodes in each layer
- Best result for each layout with 5-layer structure, 0.003 learning rate



First guess I – Neural Networks

Basic DNN: Results

- True (solid line) and reconstructed (dashed line) output distribution

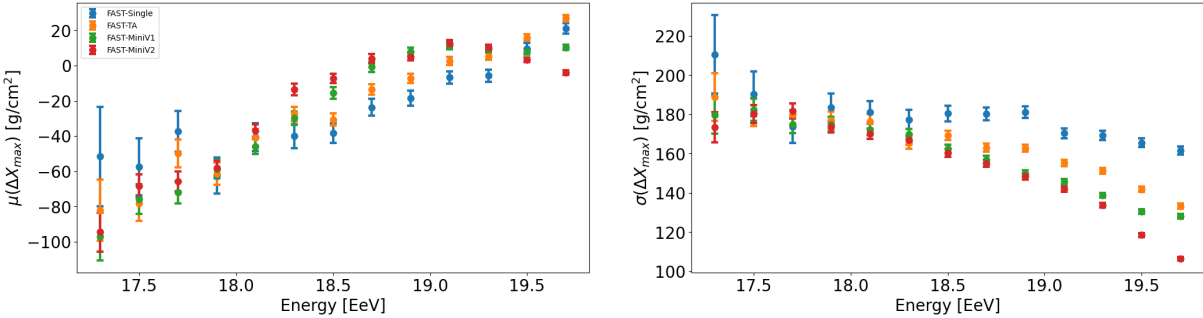


First guess I – Neural Networks

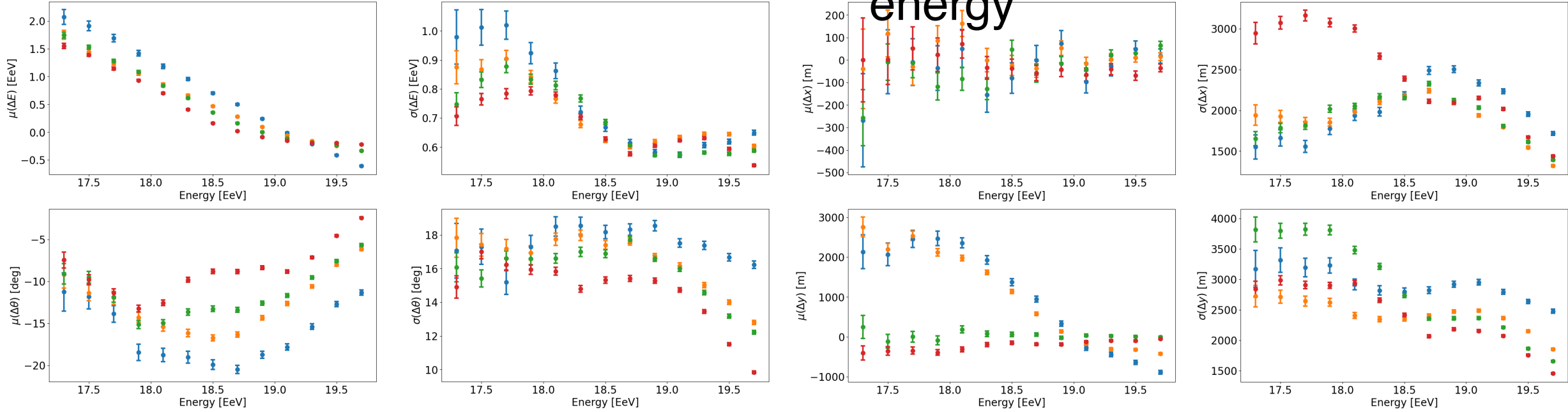
Basic DNN: Results

$$\Delta E = \ln(E_{\text{rec}}/E_{\text{true}})$$

- Biases and resolutions in each parameter as a function of energy.



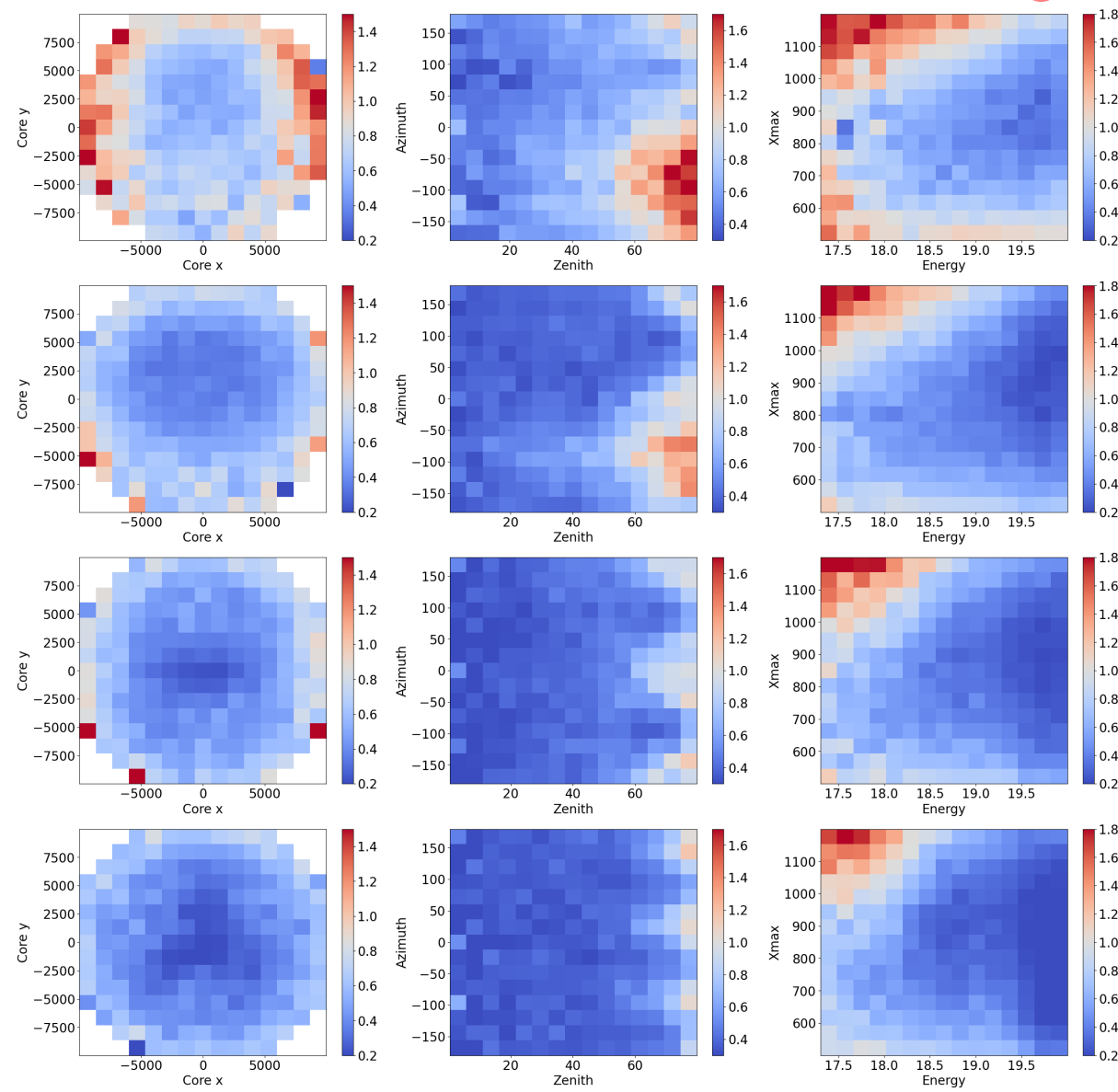
- Shape of some plots strange due to range of observable showers changing as function of energy



First guess I – Neural Networks

Basic DNN

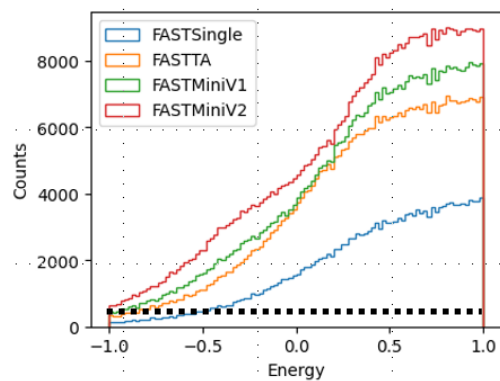
- Another diagnostic – 2D histograms of the average loss for slices in core location, arrival direction and $X_{\max}$ /energy
- Difficult regions
 - Low energy, high $X_{\max}$ - not present in data so OK
 - Showers coming from behind telescopes – as seen before
 - Edge of array



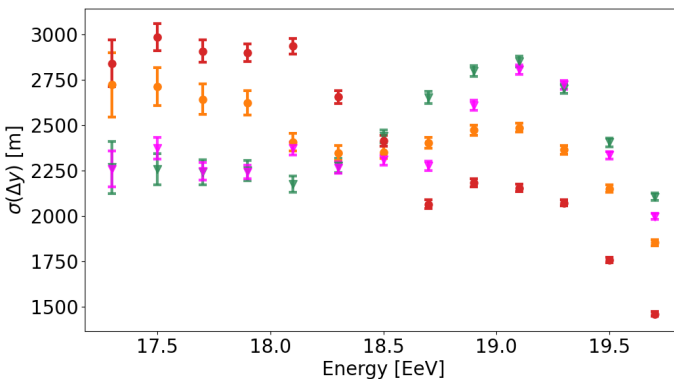
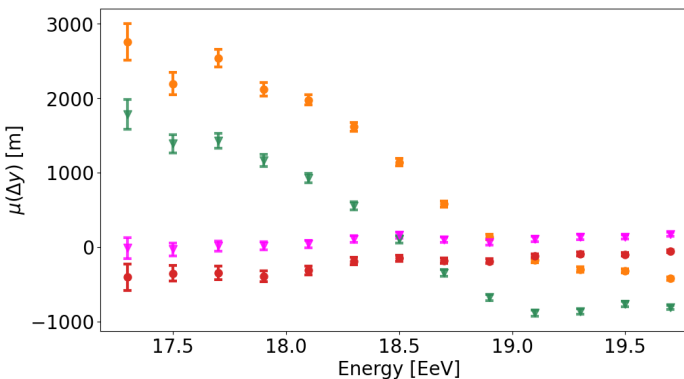
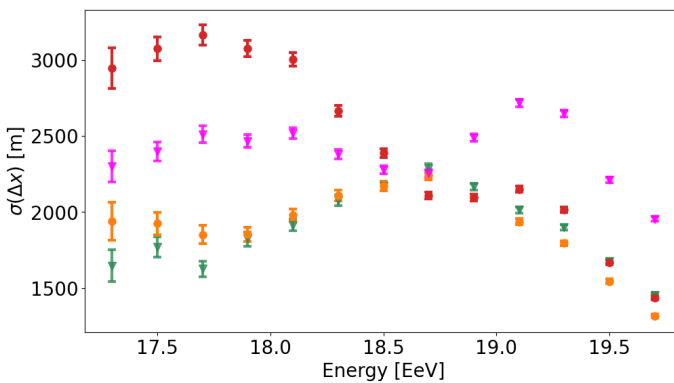
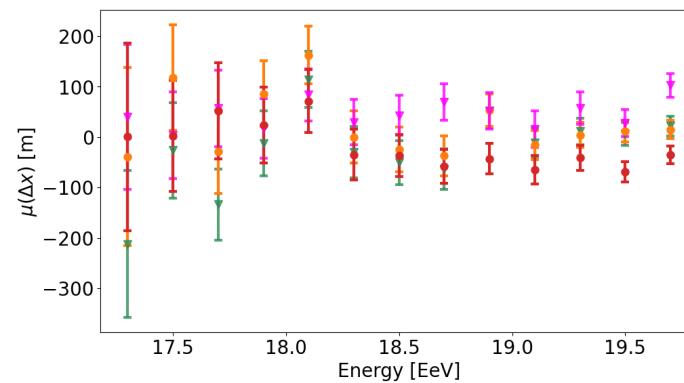
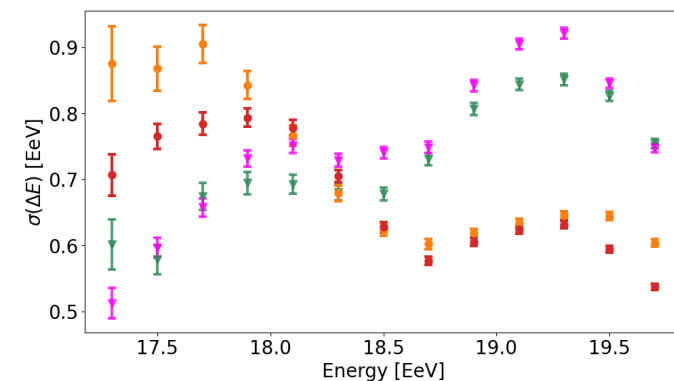
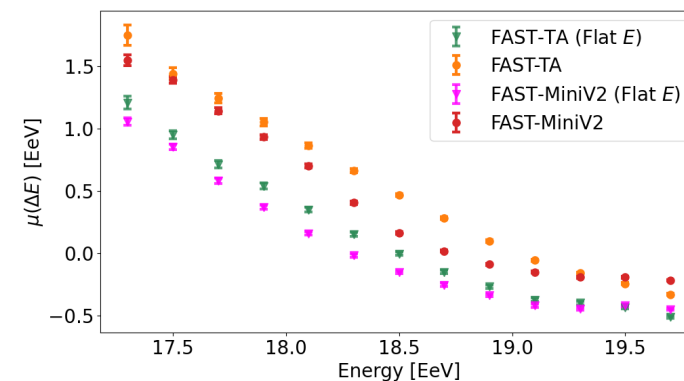
First guess I – Neural Networks

Basic DNN: Results

- Strong energy dependence seen in previous results – maybe due to uneven distribution of energies used in training



- Train on a flat distribution of energies →



First guess I – Neural Networks

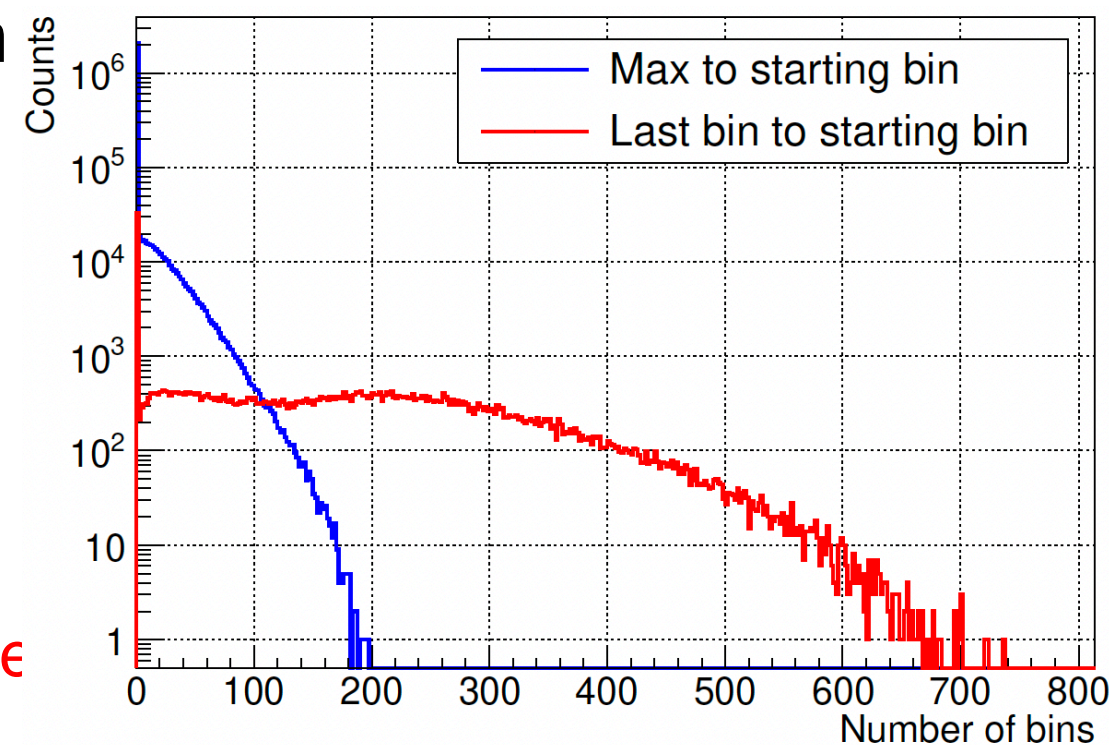
Basic DNN: Summary

- Better results with more telescopes (as expected)
- More data would only give limited improvement
- Tendency to guess towards center of distributions
- Could introduce cuts, separate events by number of PMTs triggered
 - However probably more efficient to try and make better model...

First guess I – Neural Networks

TSFEL DNN:

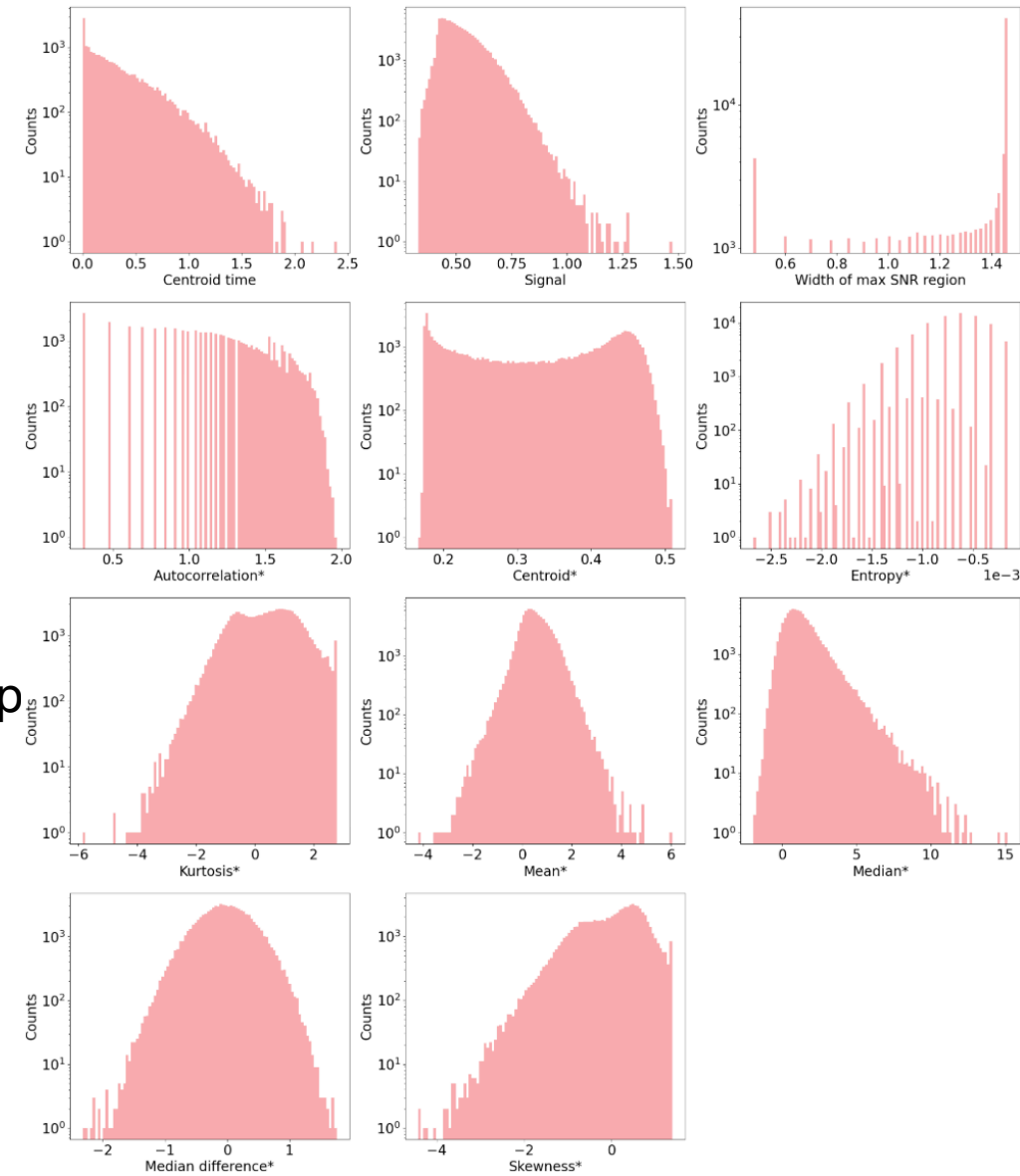
- Extract temporal and statistical features from 600 bin segment of each trace
 - Between ($t_{\text{first}} - 150, t_{\text{first}} + 450$)
- Trace segment determined by analyzing traces with no noise in simulations
 - Difference between first and last bin above 5 p.e.
 - Difference between max bin of earliest triggered PMT and first bin



First guess I – Neural Networks

TSFEL DNN:

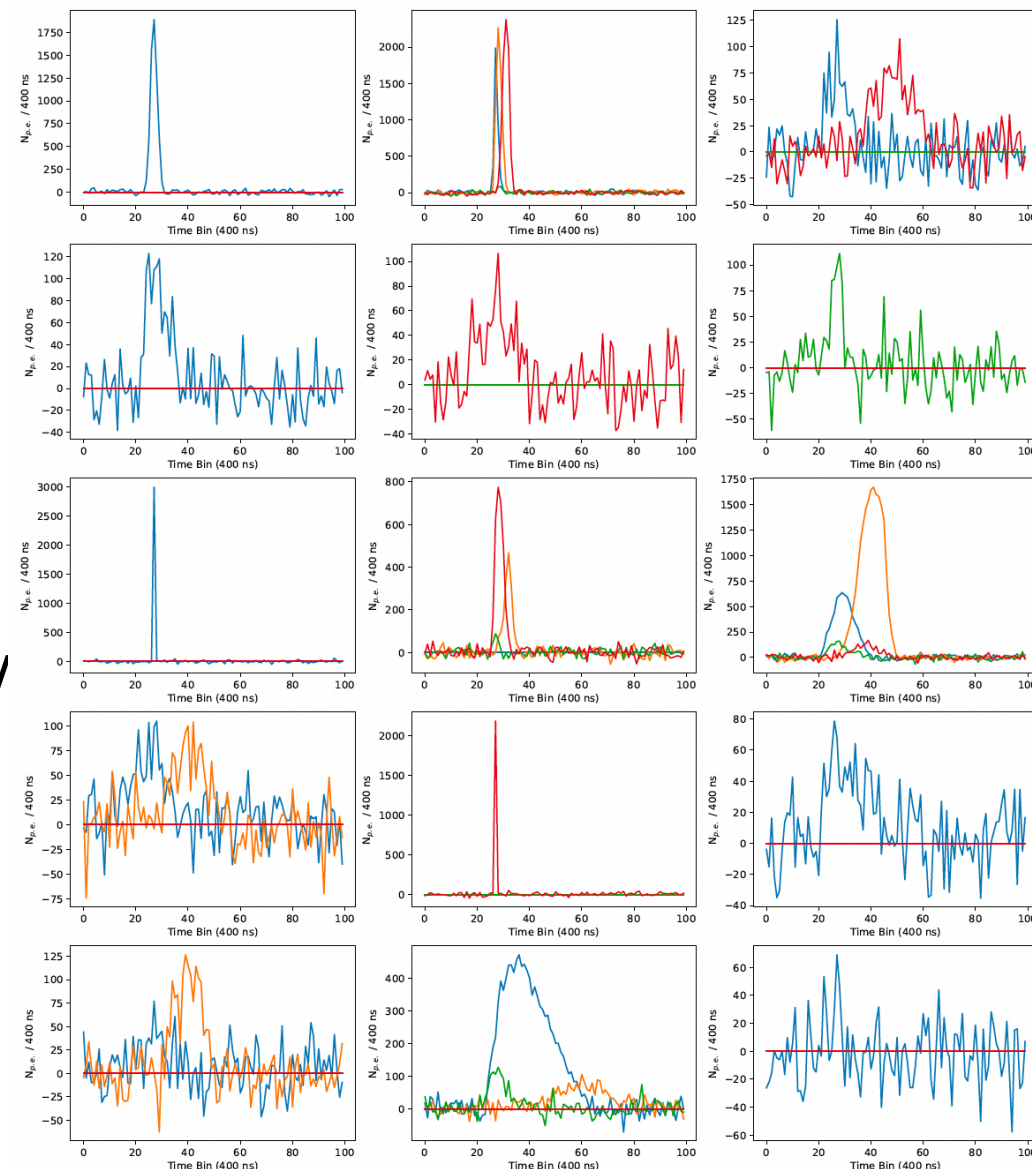
- Remove features which don't vary between traces
- Apply $\hat{x} = \begin{cases} \log_{10}(|x|) & x \neq 0 \\ 0 & x = 0 \end{cases}$ to remaining feat.
- Add features from Basic DNN & k_{stop} - k_{stop}
- Remove features with correlation > 0.95
- Left with 11 features $\rightarrow$ (* from TSFEL)



First guess I – Neural Networks

LSTM Network:

- Use same 600 bin segments
 - Cut first 10 and last 40 bins, re-bin by factor of 4, gives 100 bin traces (400 ns/bin)
- Input traces directly to network
 - Each PMT trace processed by same LSTM layer (extract same features – 64/PMT)
 - Structure is
 - $n\text{PMTs trace inputs} \rightarrow \text{LSTM layer} \rightarrow (64 * n\text{PMTs}) \text{ nodes} \rightarrow 64 \text{ nodes} \rightarrow \text{Output layer}$



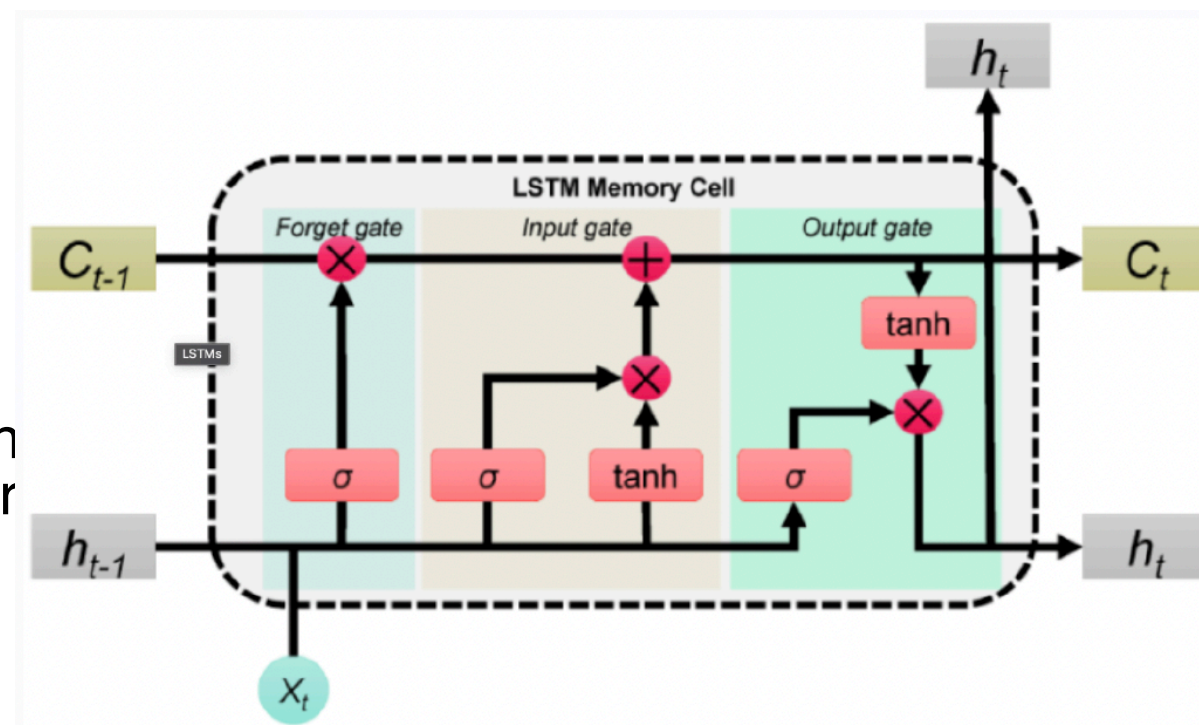
First guess I – Neural Networks

LSTM Network

- Hidden state H_t (short term memory)
- Internal state C_t (long term memory)
- Processing trace $x = \{x_1, x_2, \dots, x_n\}$
 - At each time step t , x_t & C_{t-1} & H_{t-1} are combined to give C_t and H_t
 - Updated using three different gates which use sigmoid/tanh activation functions. For a gate g

$$\vec{I}_g = U_g \vec{h}_{t-1} + W_g \vec{x}_t + \vec{b}_g$$

- **Goal:** Learn the matrices U_g & W_g and bias terms b_g for each gate



TSFEL DNN Performance

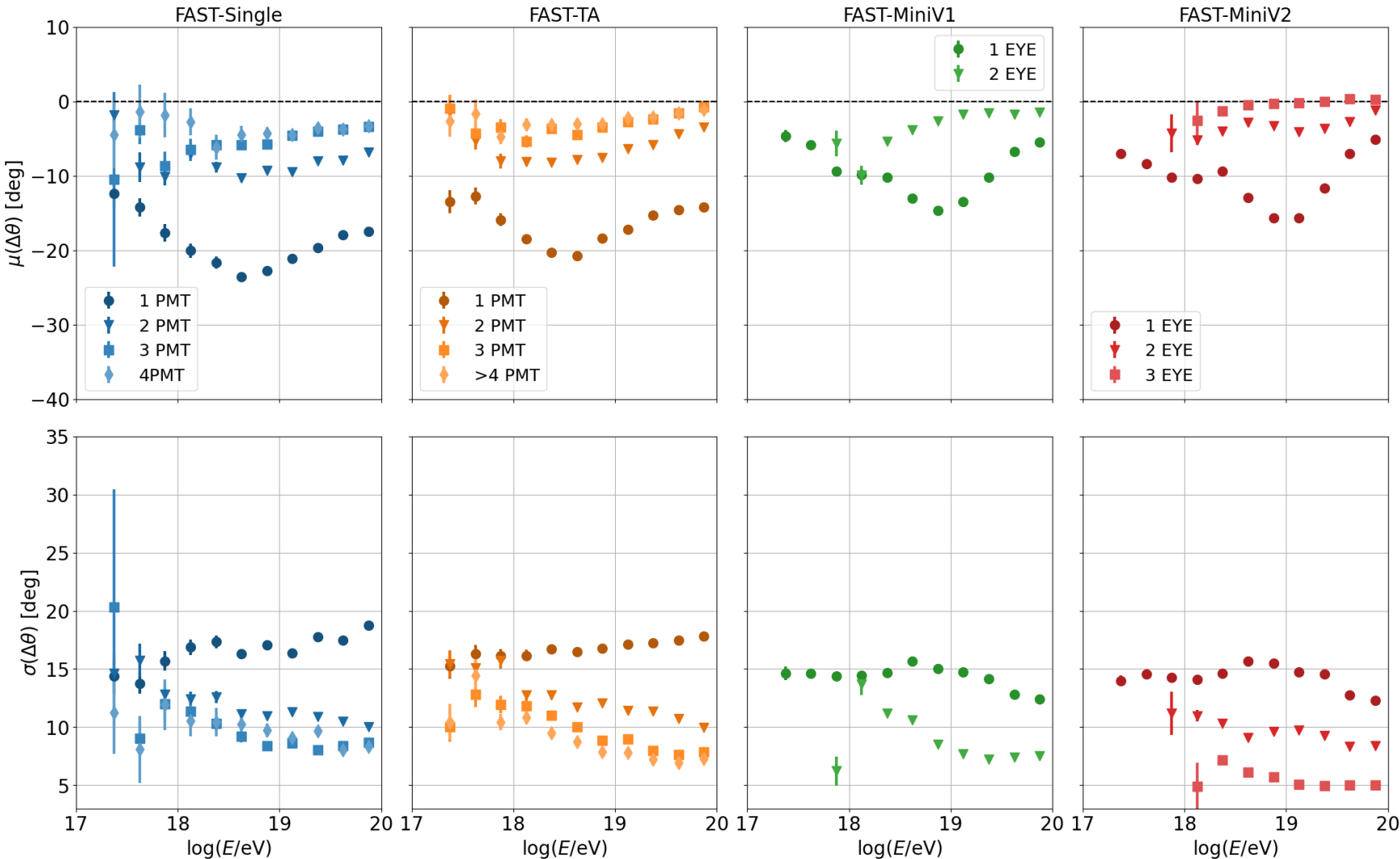
Biases and resolutions: Zenith

Biases

(mean of distribution of differences between true and reconstructed values)

Resolutions

(width of distribution of differences between true and reconstructed values)



TSFEL DNN Performance

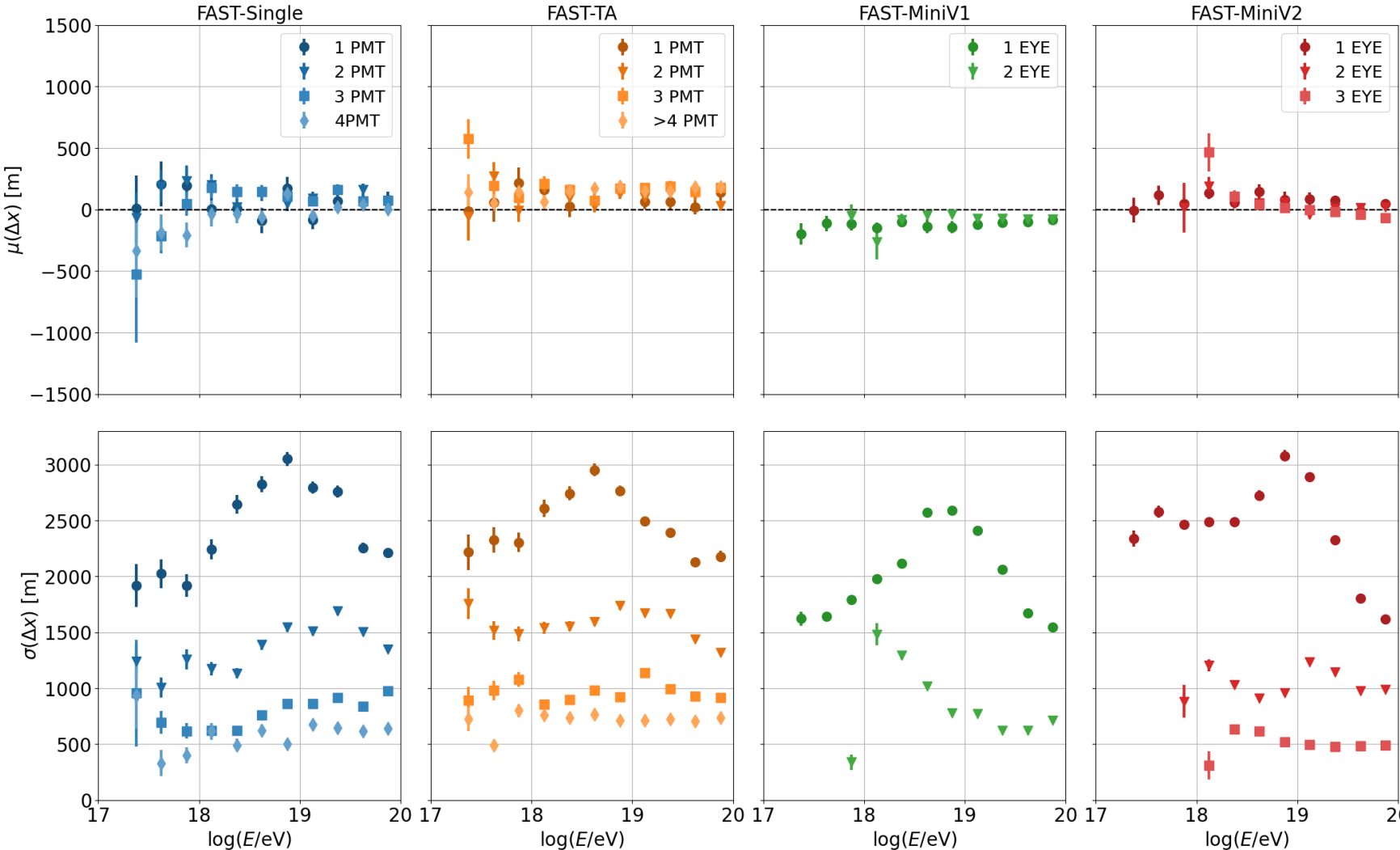
Biases and resolutions: Core x

Biases

(mean of distribution of differences between true and reconstructed values)

Resolutions

(width of distribution of differences between true and reconstructed values)



TSFEL DNN Performance

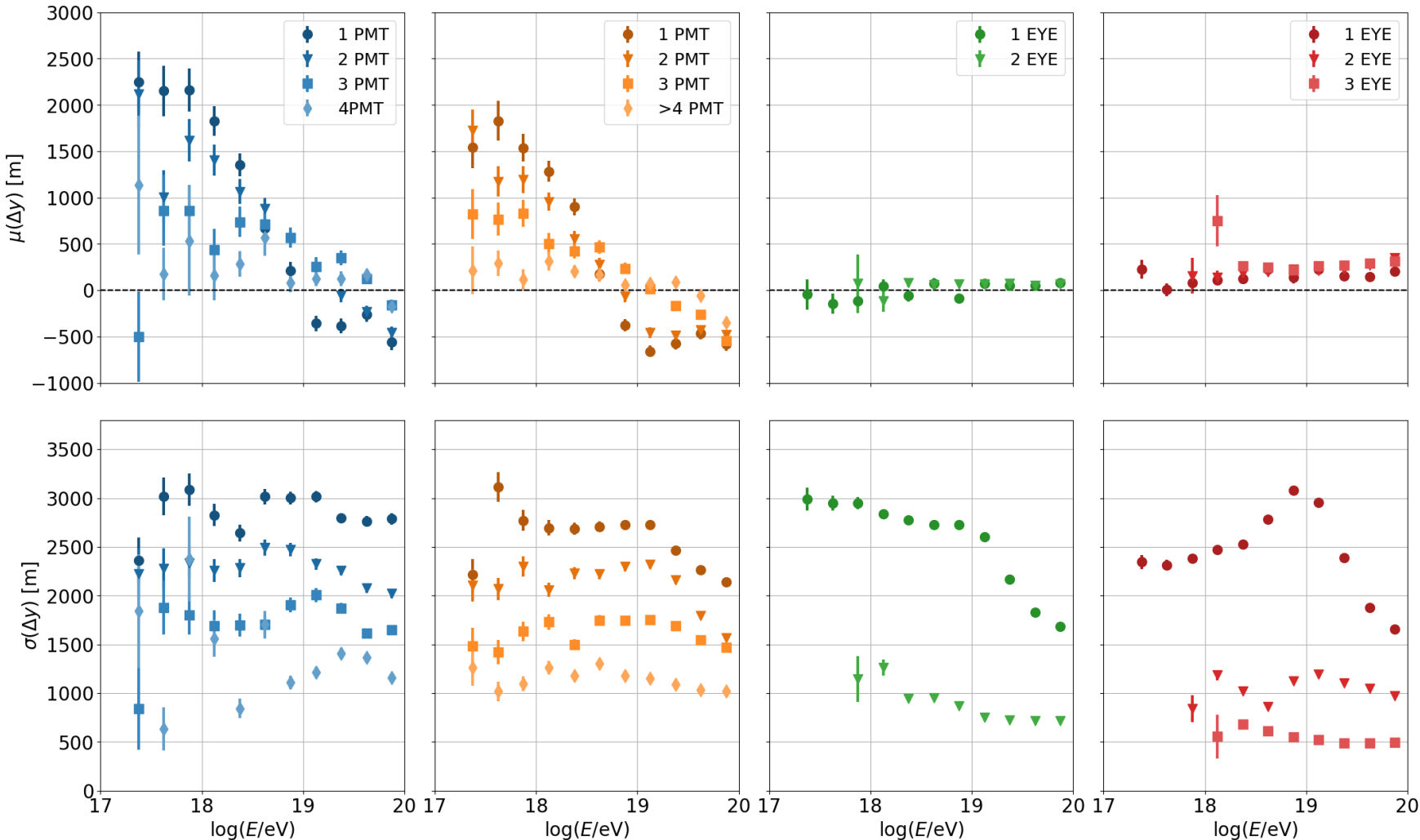
Biases and resolutions: Core y

Biases

(mean of distribution of differences between true and reconstructed values)

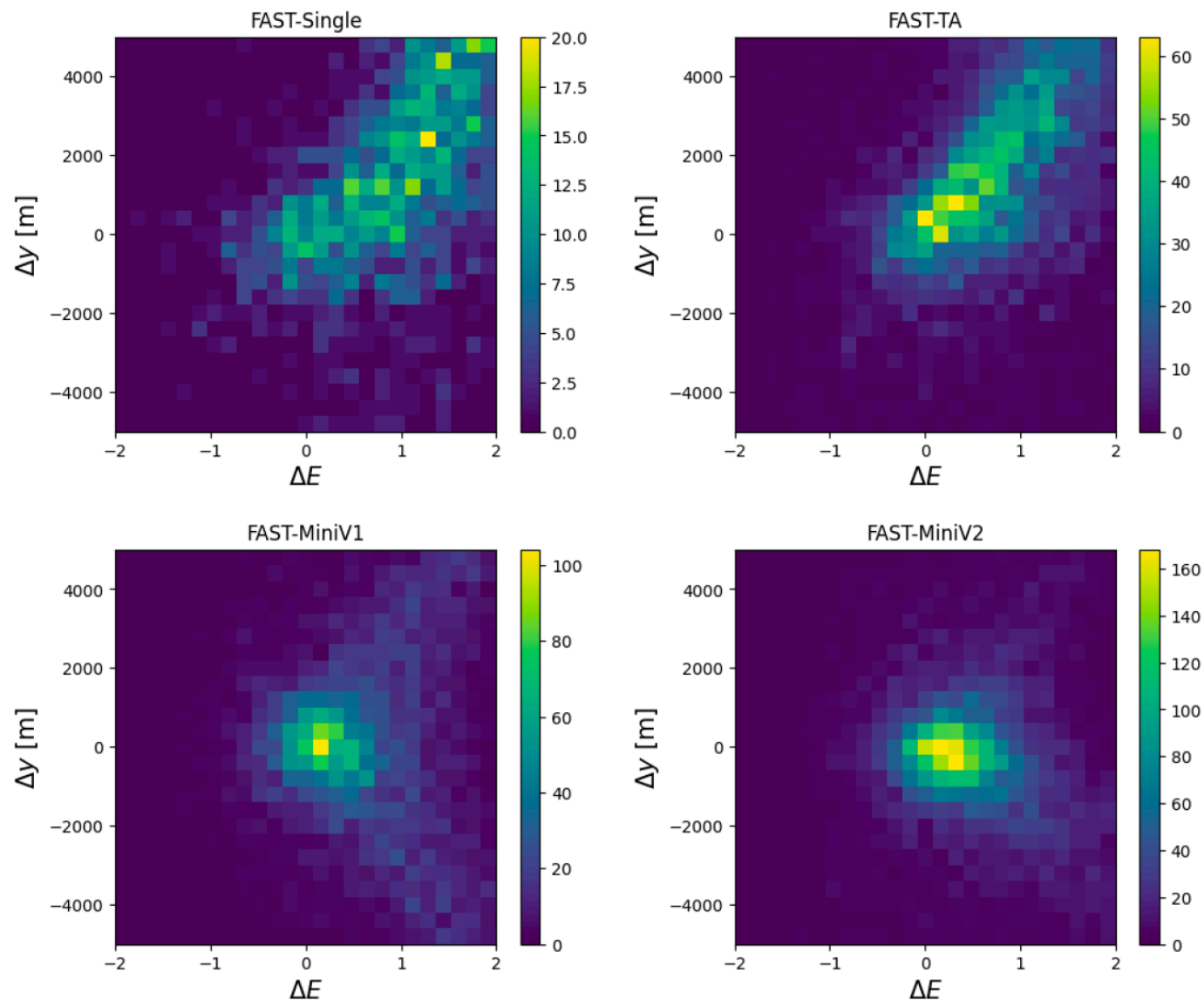
Resolutions

(width of distribution of differences between true and reconstructed values)



First Guess I - Neural Networks

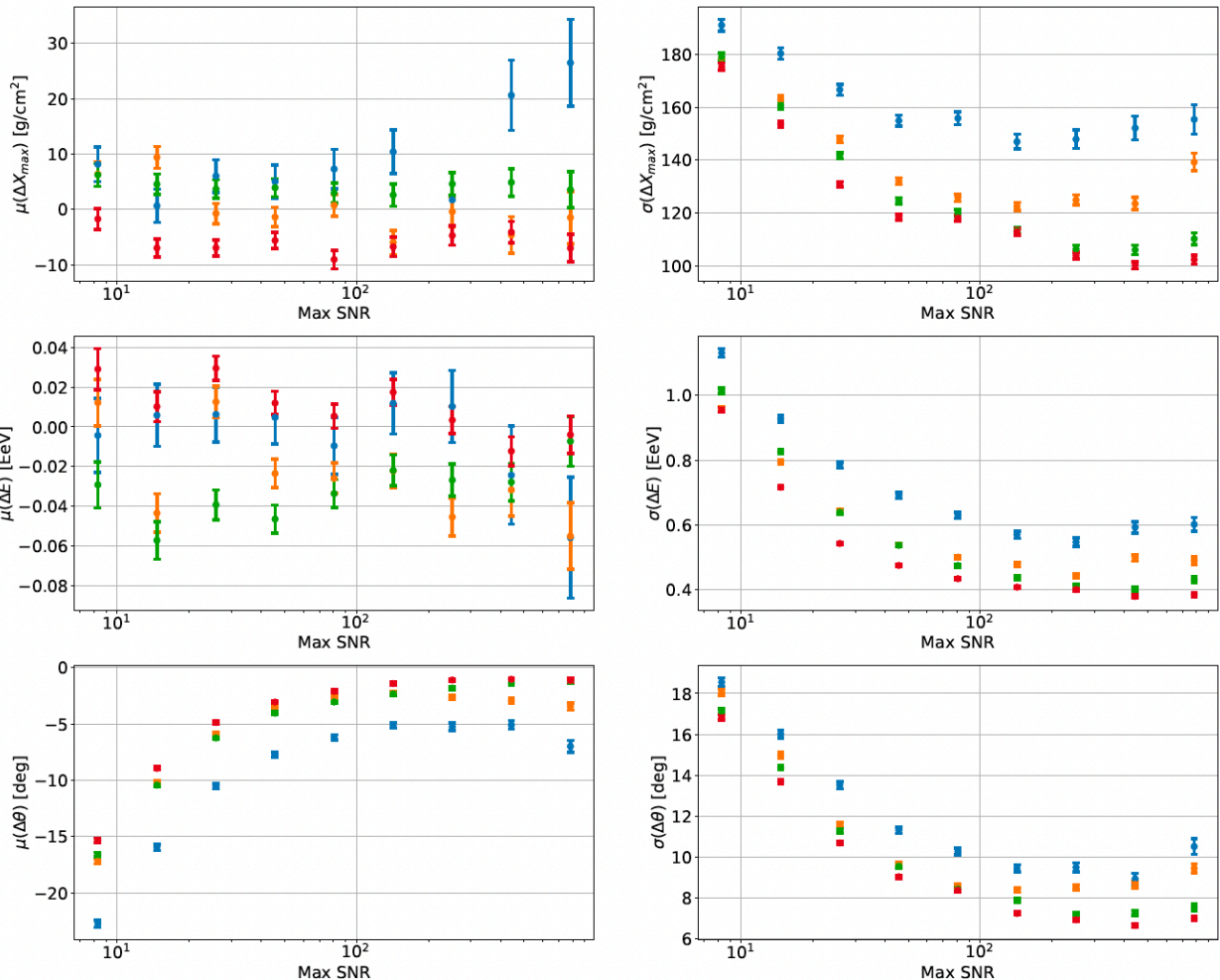
Degeneracy in reconstructed core position and energy



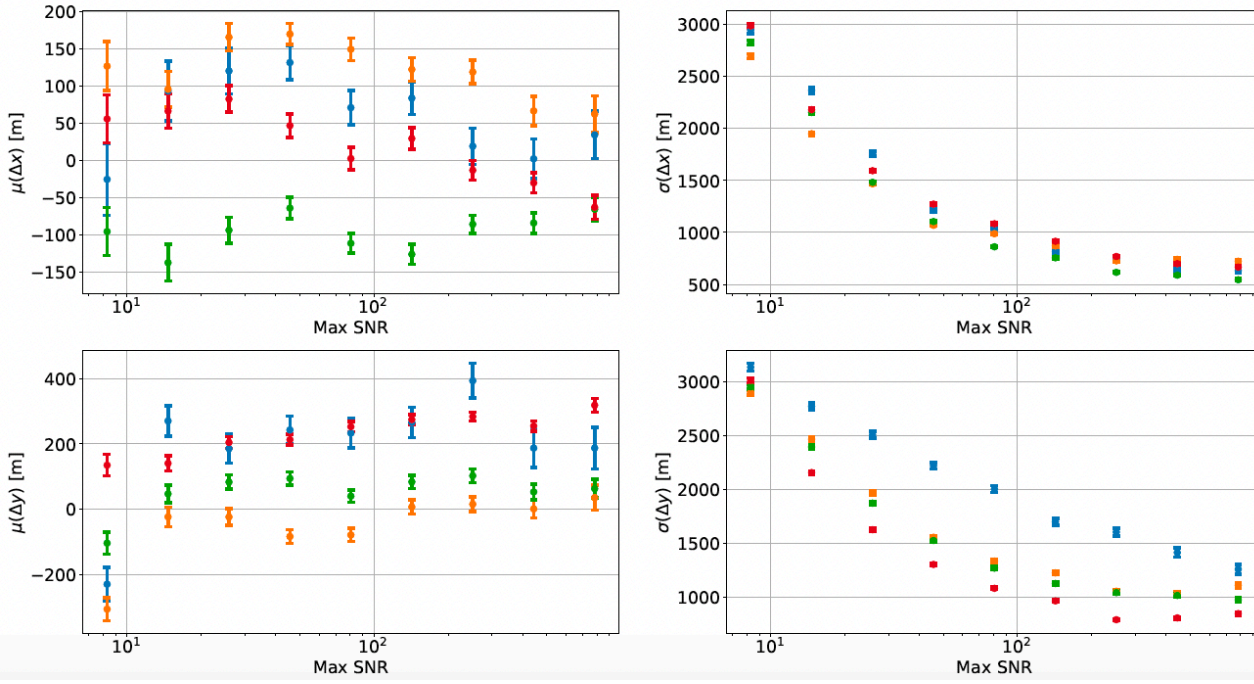
First guess I – Neural Networks

TSFEL DNN Performance:

- Biases and resolutions as function of max SNR



- Behaves more like expected (range of shower parameters doesn't change as function of max SNR)



First guess I – Neural Networks

TSFEL DNN: Summary

- With only 1 or 2 pixels, can't predict parameters very well
- Need 3 or more pixels in one eye or stereo observation
 - 100 g cm⁻²/40%/ 750 m/8° for 2-Eye stereo observation with FAST-MiniV1
 - 75 g cm⁻²/30%/ 750 m/6.5° for 3-Eye stereo observation with FAST-MiniV2

>10¹⁹eV

- 2-Eye stereo pointing at each other better than 2-Eye stereo in triangle
- Future Improvements: Filtering to reduce noise in traces/transformer architecture (?), add telescope location/pointing (maybe help with showers observed in stereo?)

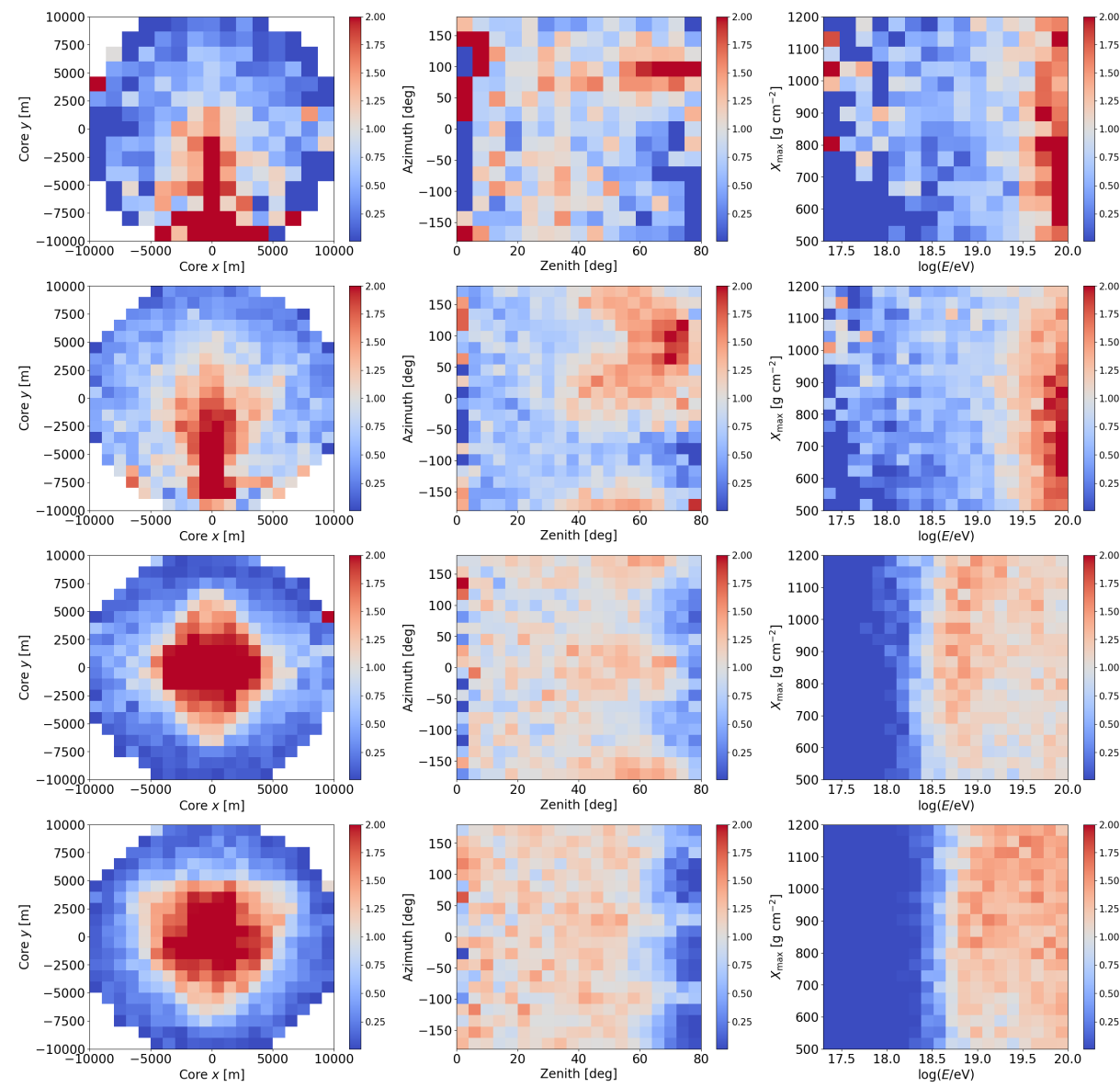
First guess I – Neural Networks

If only 3 or 4 pixels / stereo....:

- Ratio between normalised number of events expected to be observed with (top to bottom)

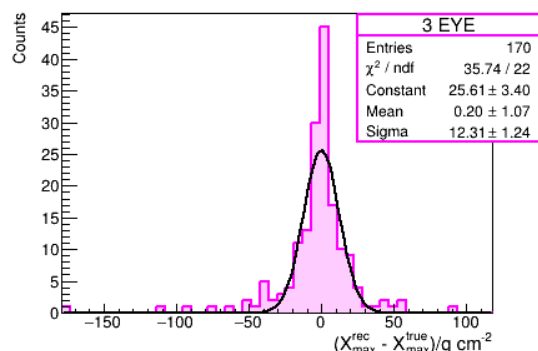
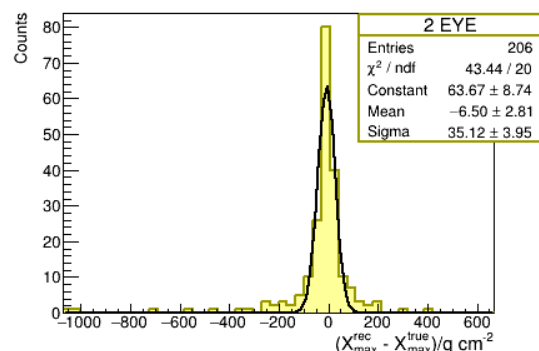
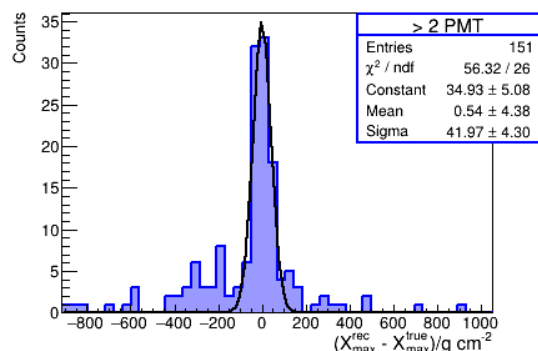
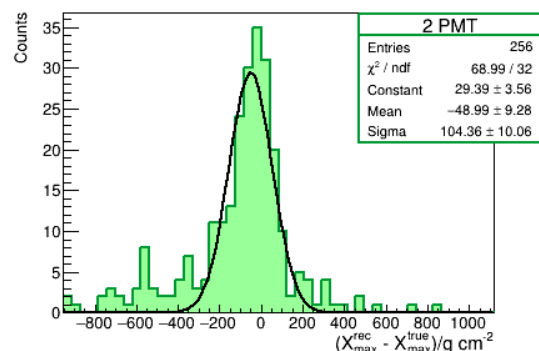
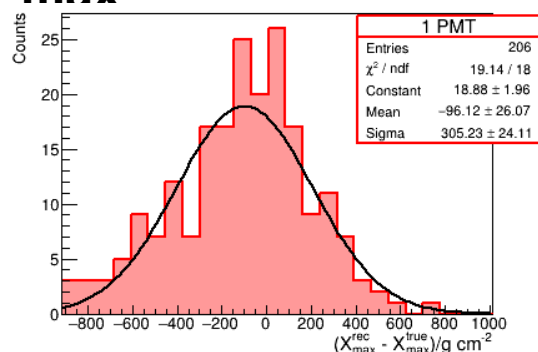
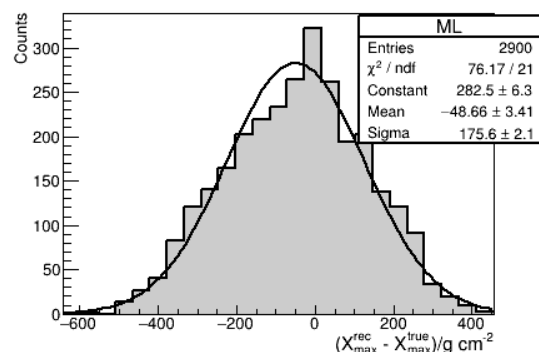
4 pixels / >4 pixels / 2 Eyes / 3 Eyes

and normalised number of events expected to be observed with at least 1 pixel

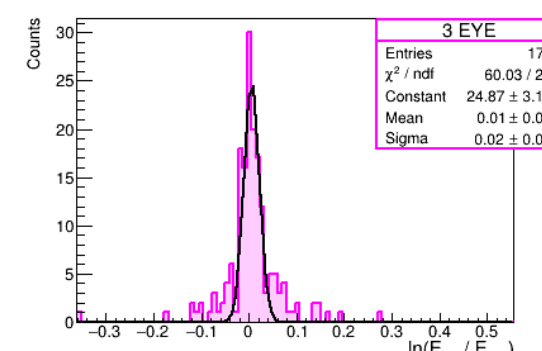
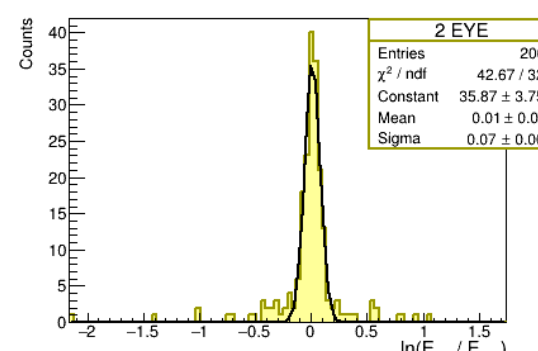
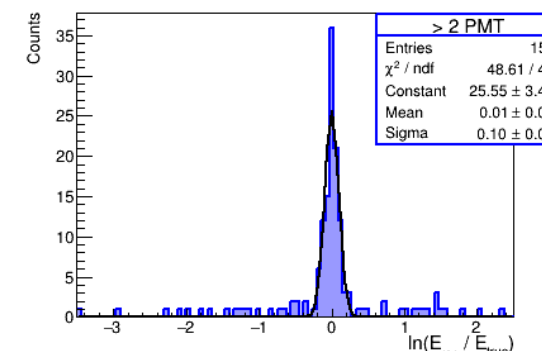
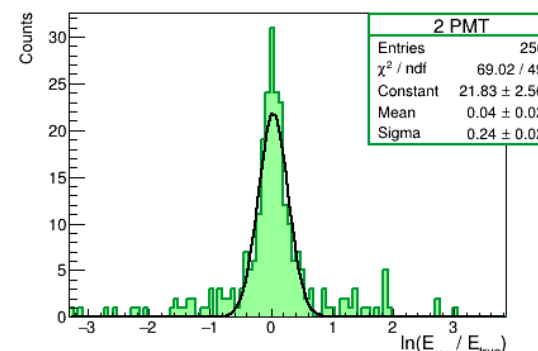
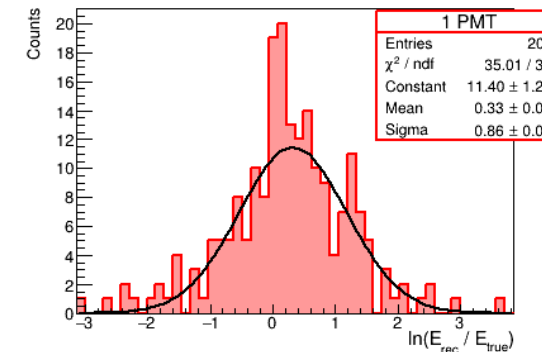
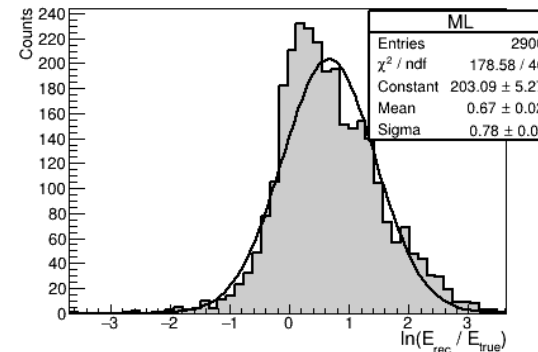


TSFELN DNN + TDR: FASTMiniV2

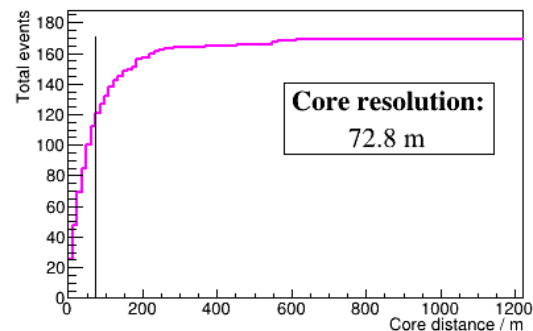
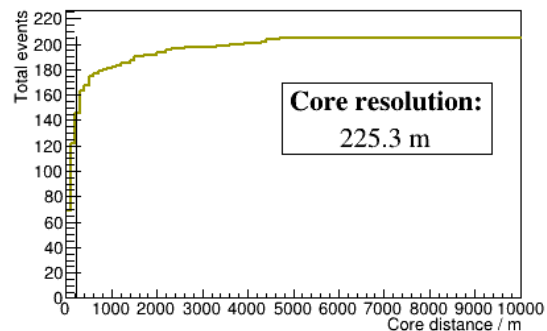
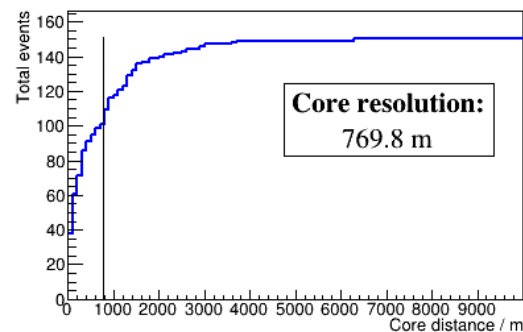
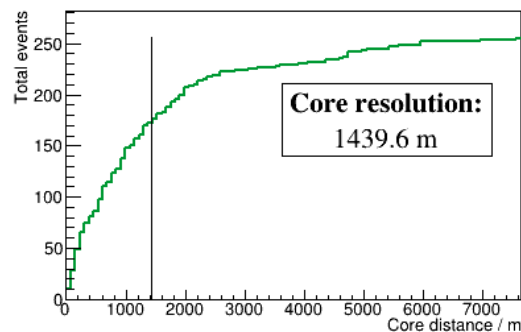
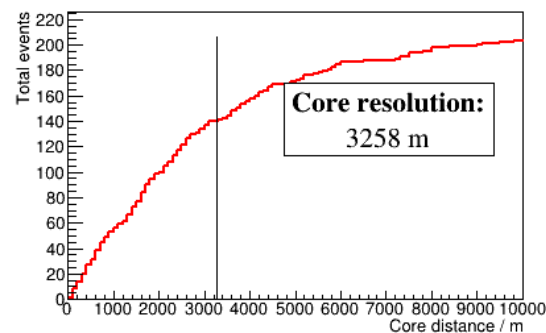
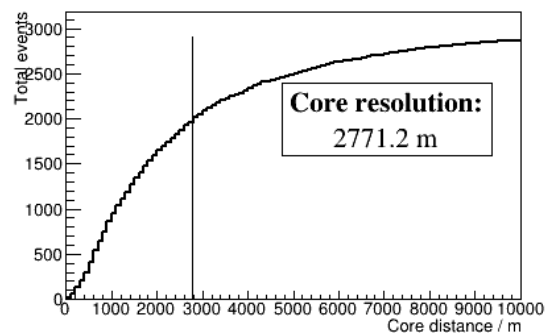
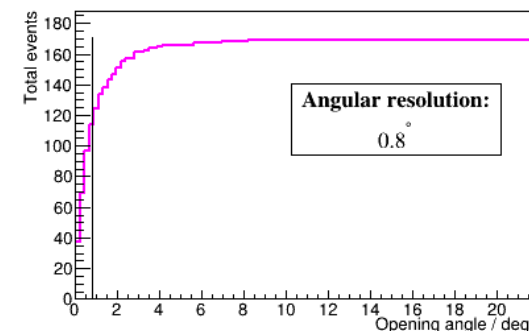
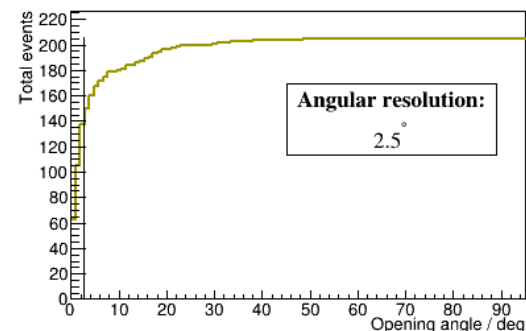
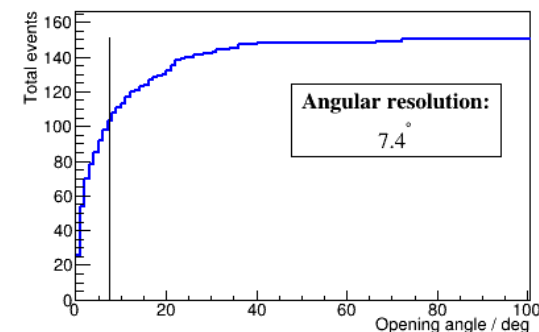
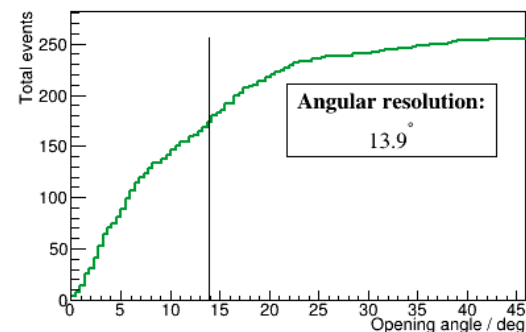
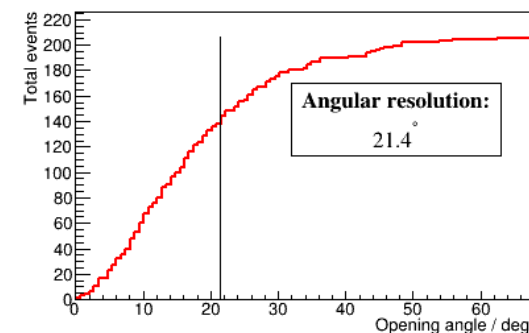
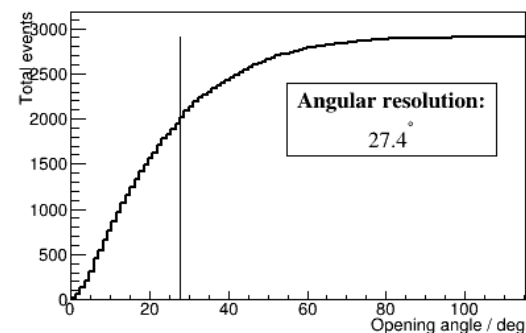
$X_{\max}$



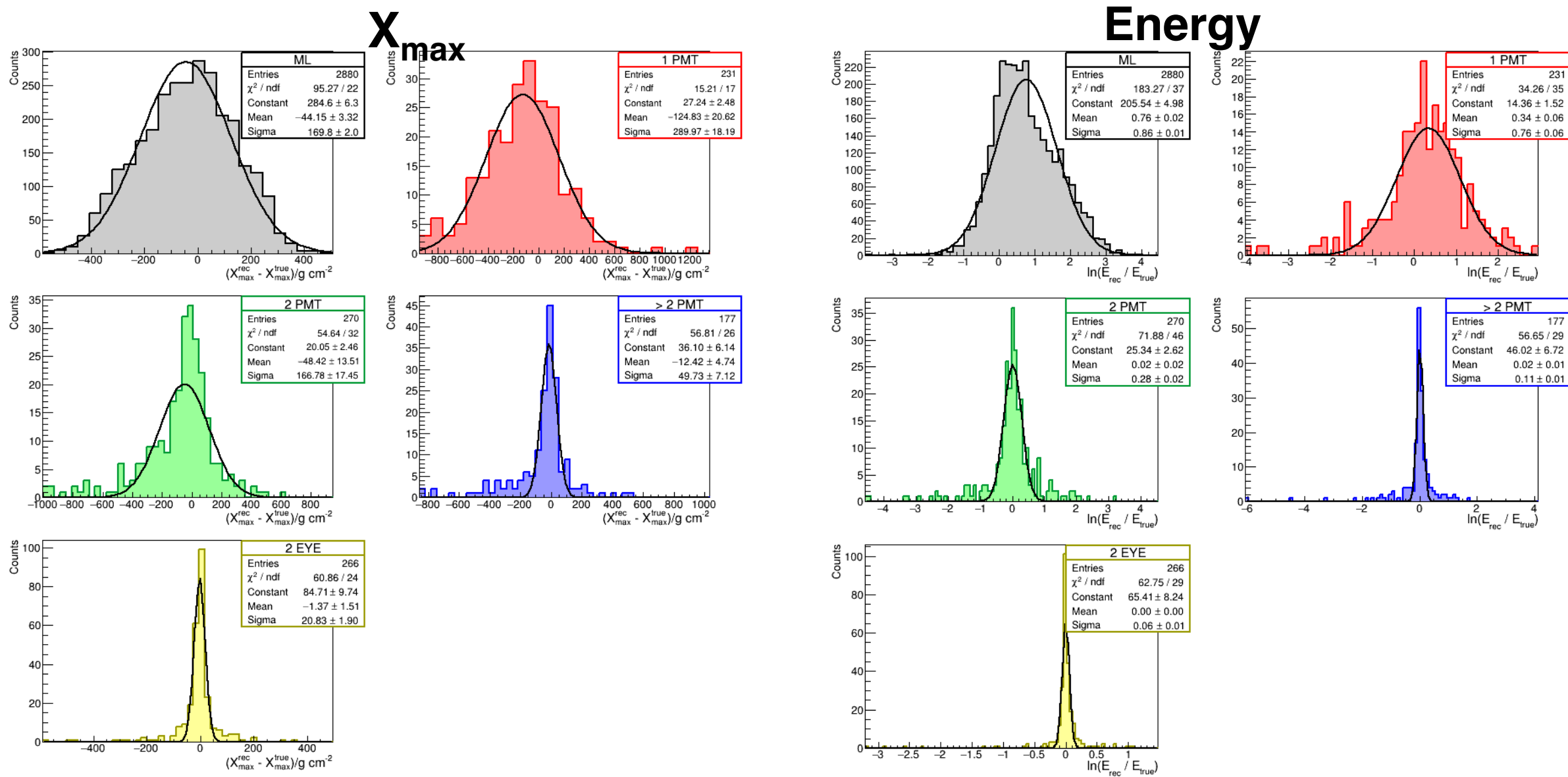
Energy



TSFELN DNN + TDR: FASTMiniV2

Core**Angular**

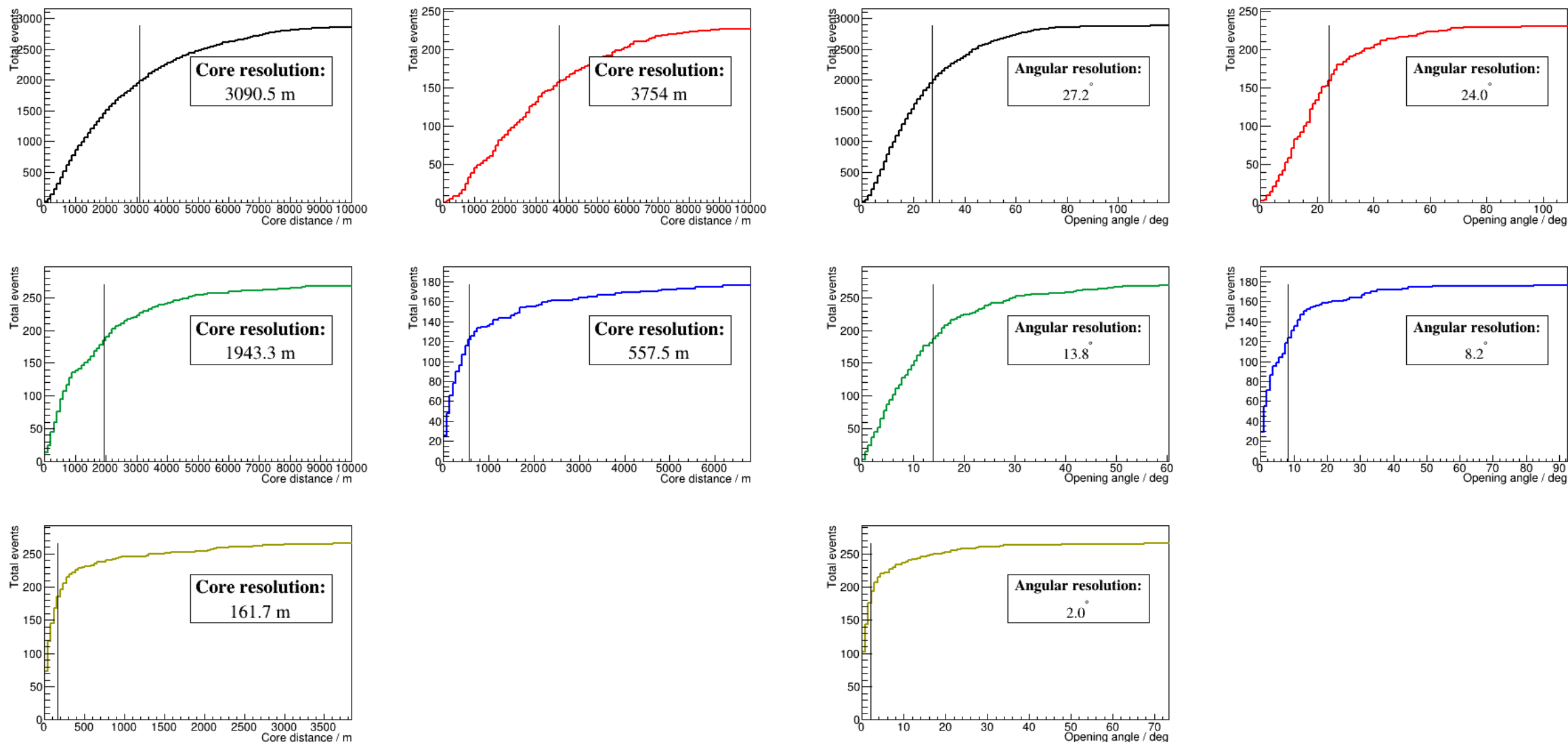
TSFELN DNN + TDR: FASTMiniV1



TSFELN DNN + TDR: FASTMiniV1

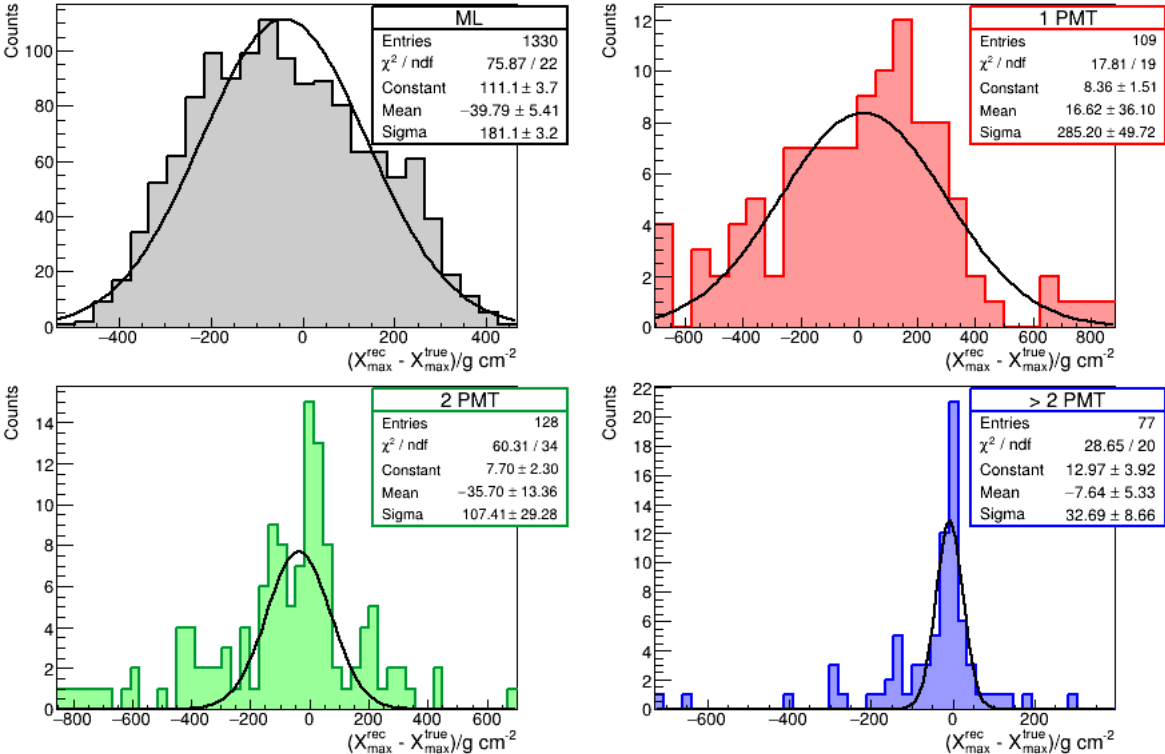
Core

Angular

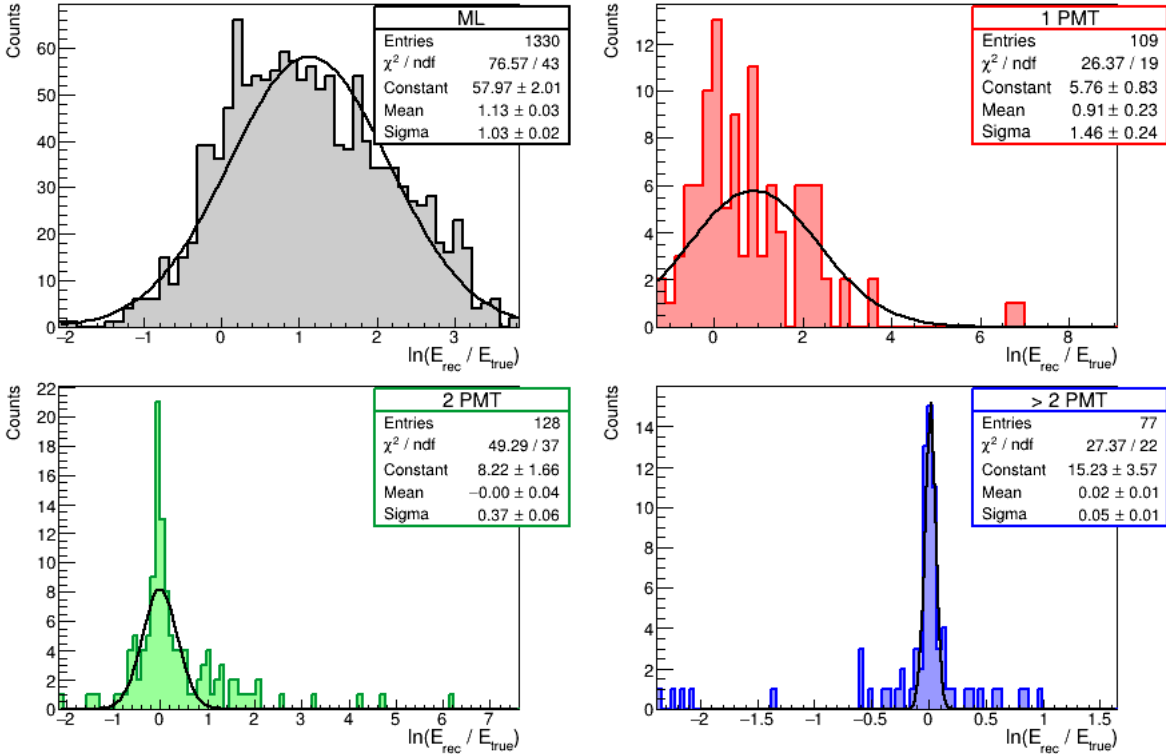


TSFELN DNN + TDR: FAST-Single

X_{max}



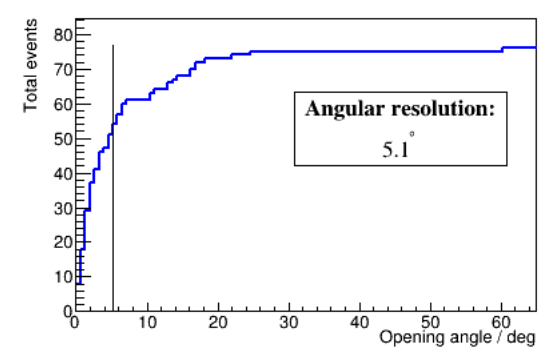
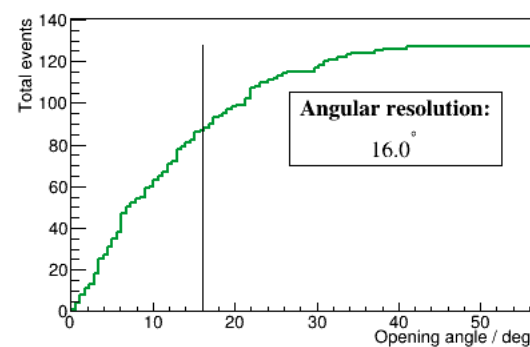
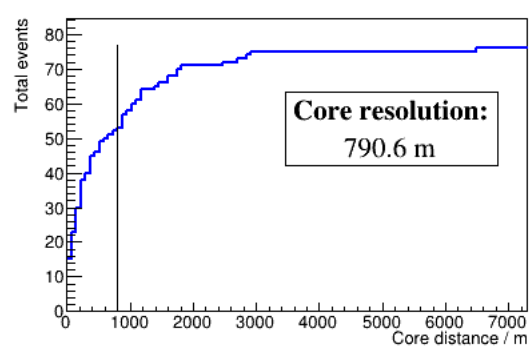
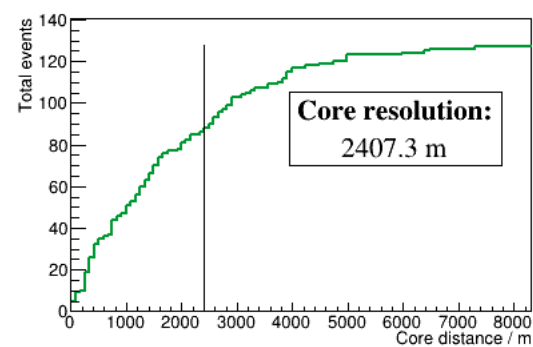
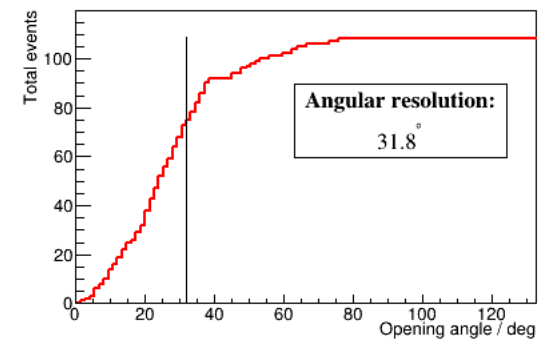
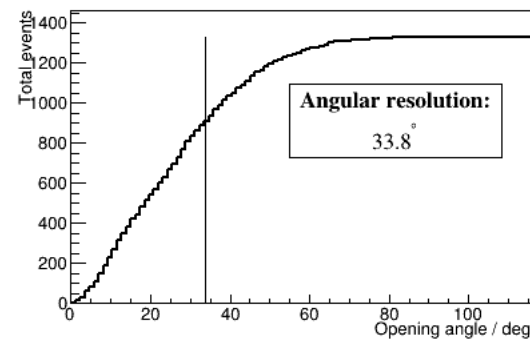
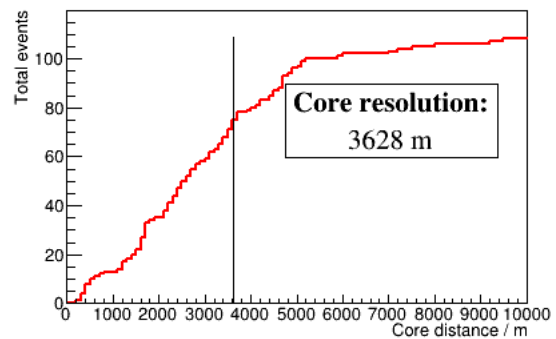
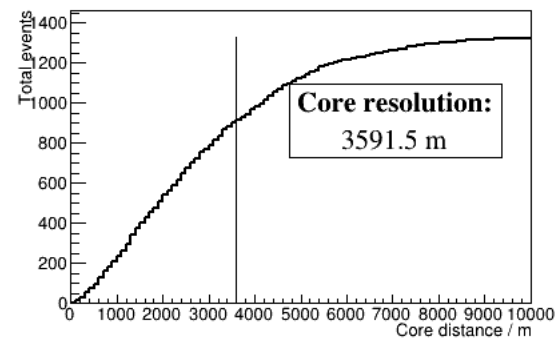
Energy



TSFELN DNN + TDR: FAST-Single

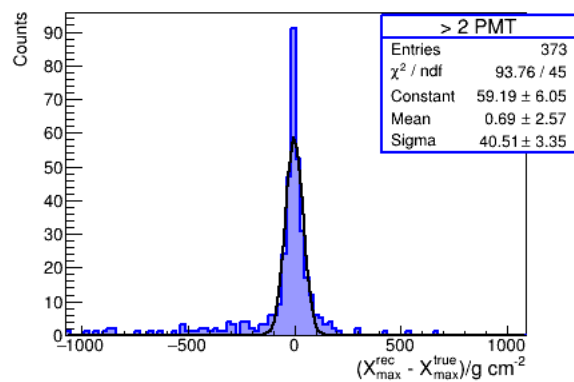
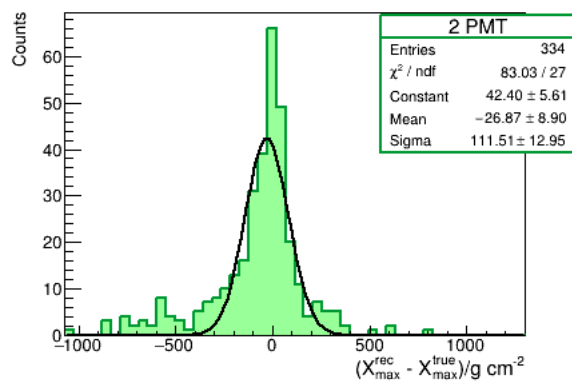
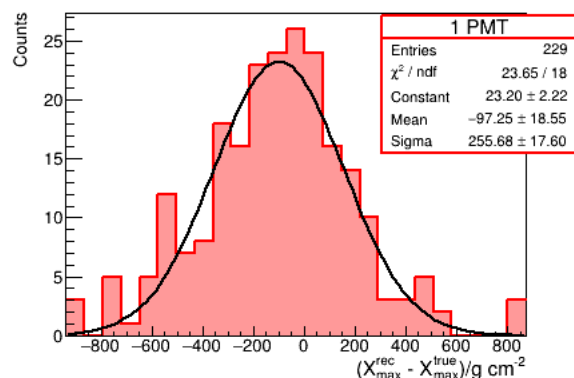
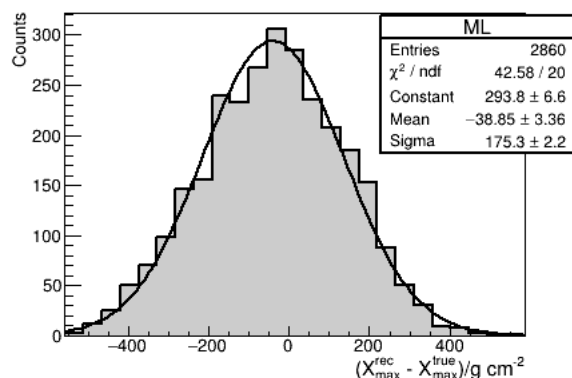
Core

Angular

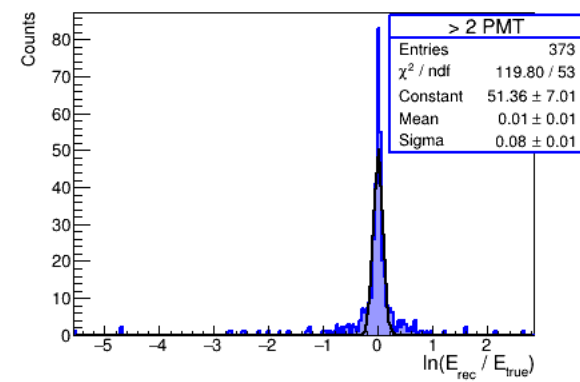
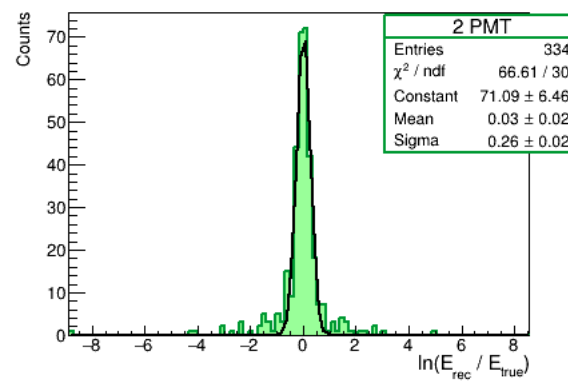
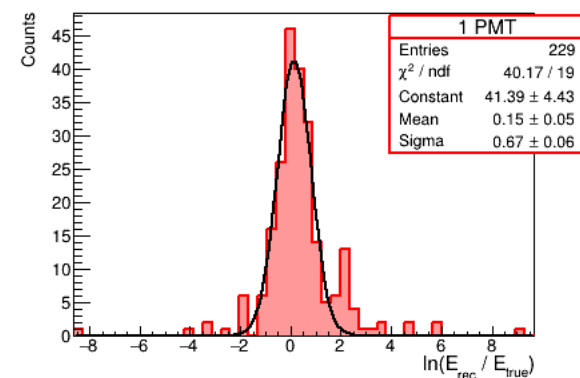
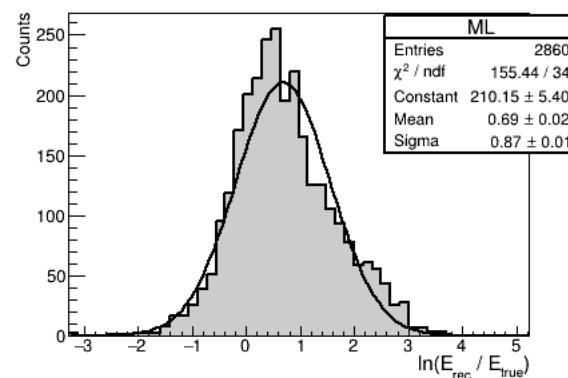


TSFELN DNN + TDR: FAST-TA

$X_{\max}$

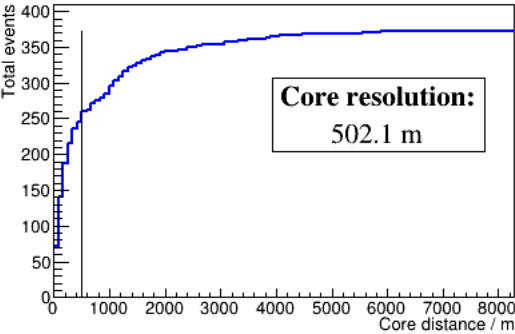
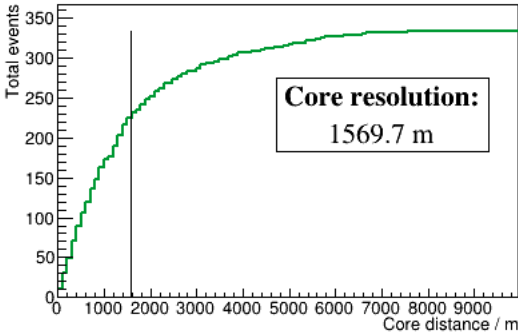
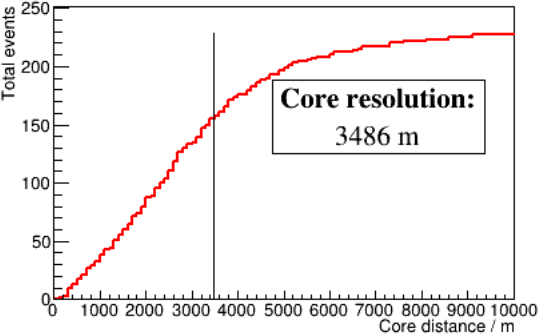
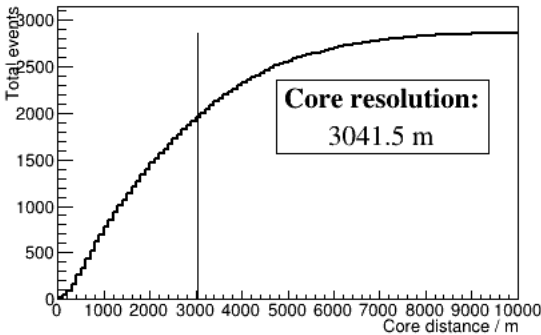


Energy

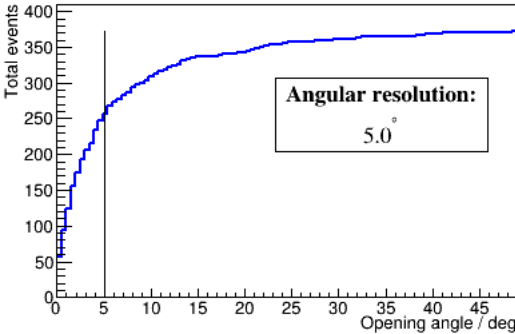
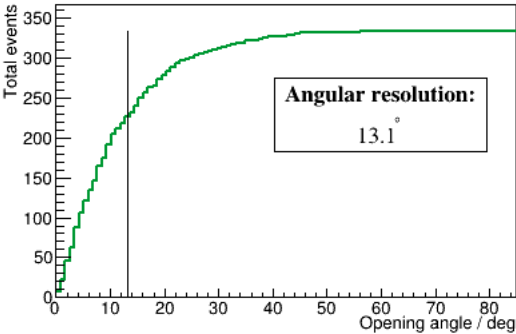
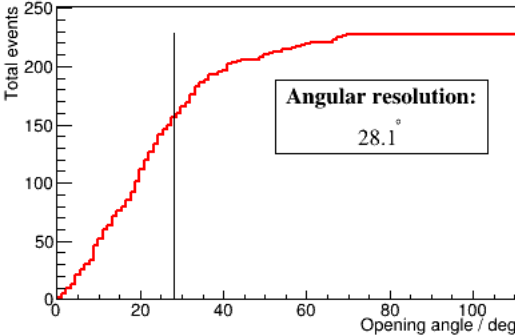
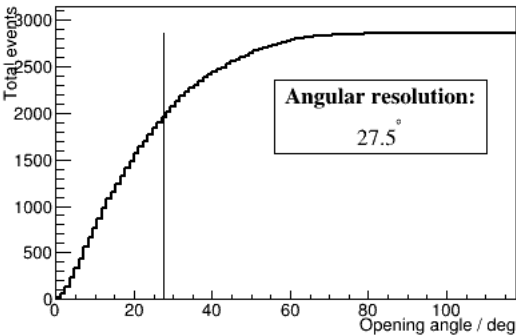


TSFELN DNN + TDR: FAST-TA

Core

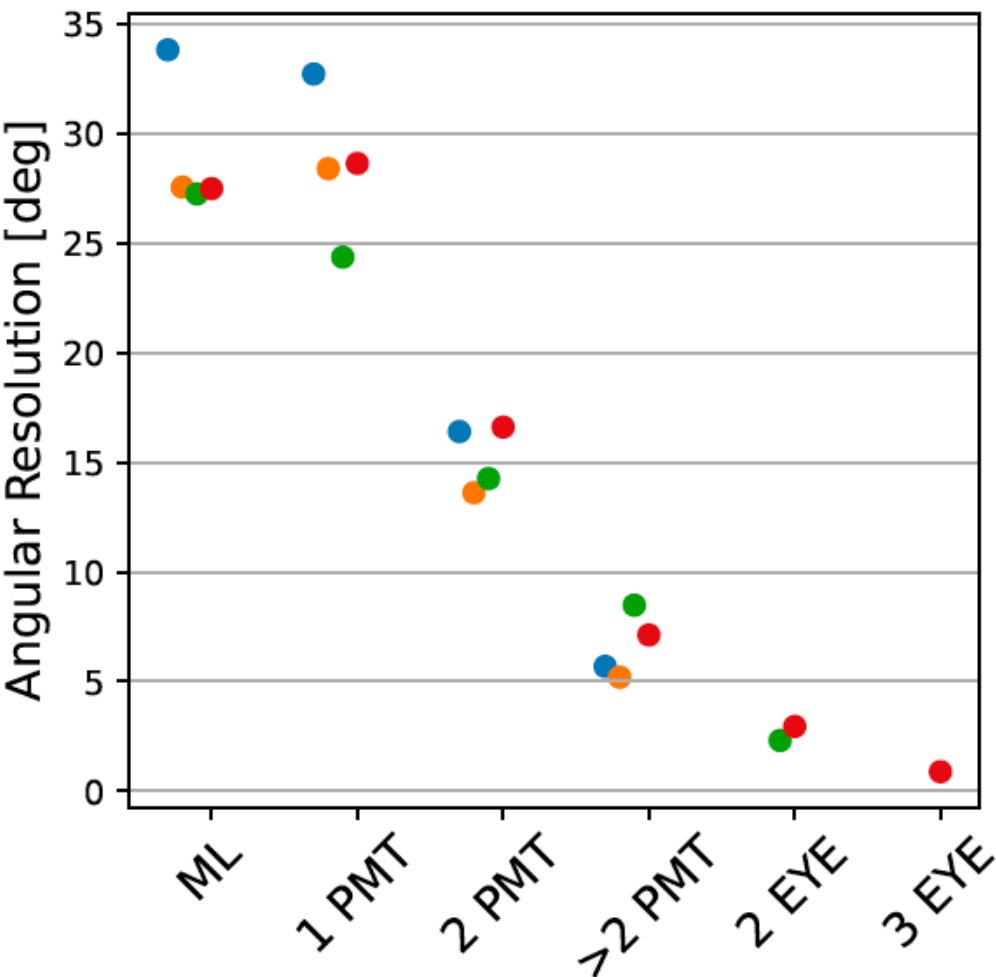
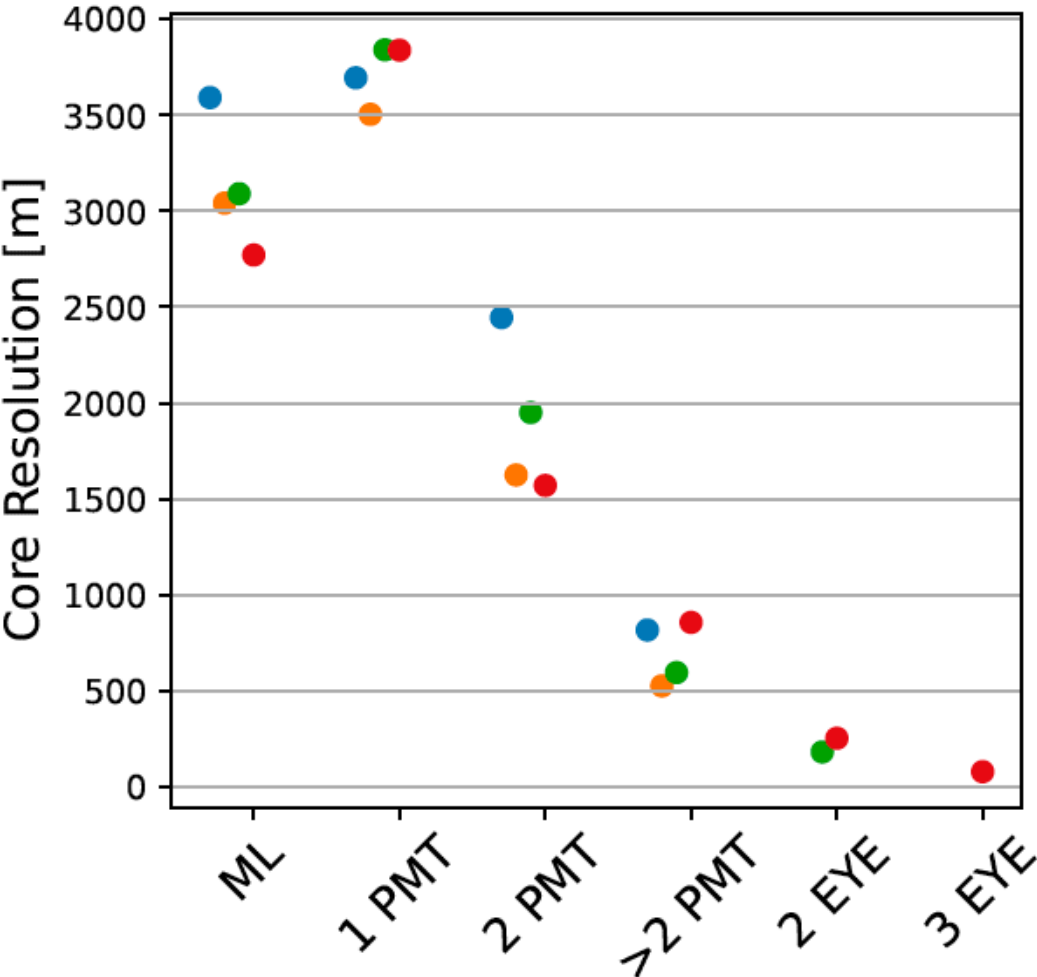


Angular



TSFELN DNN + TDR: Summary

Geometry



First guess I – Neural Networks

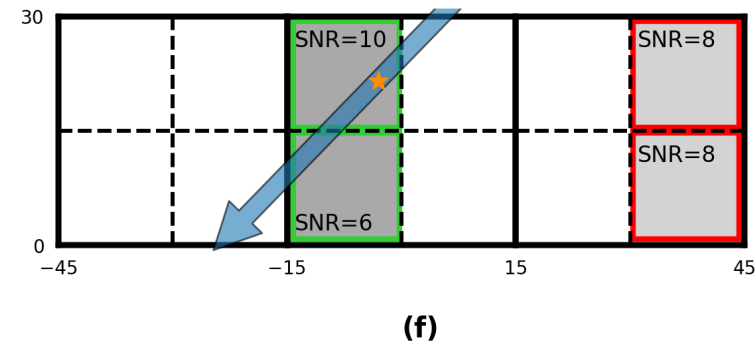
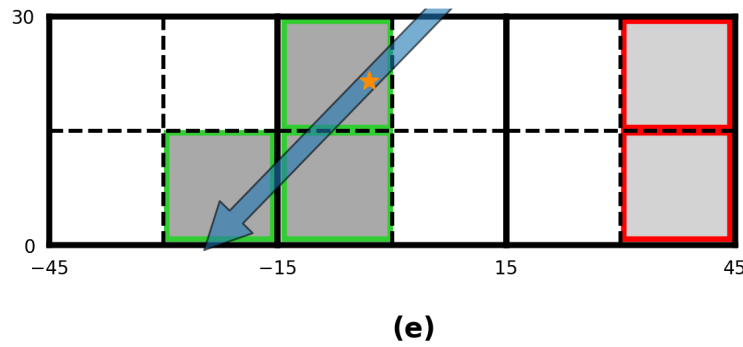
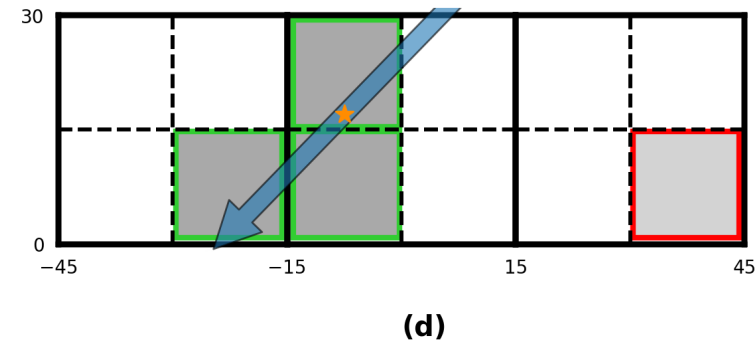
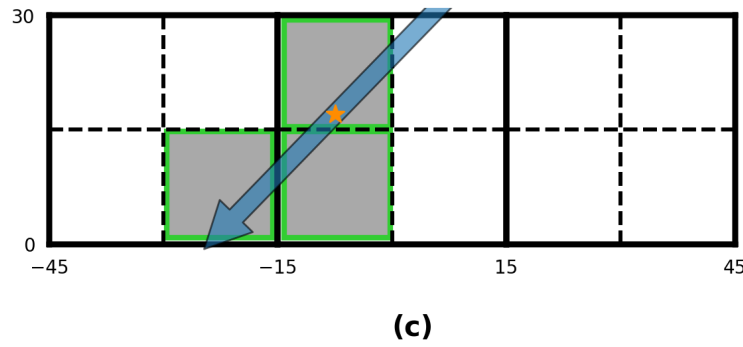
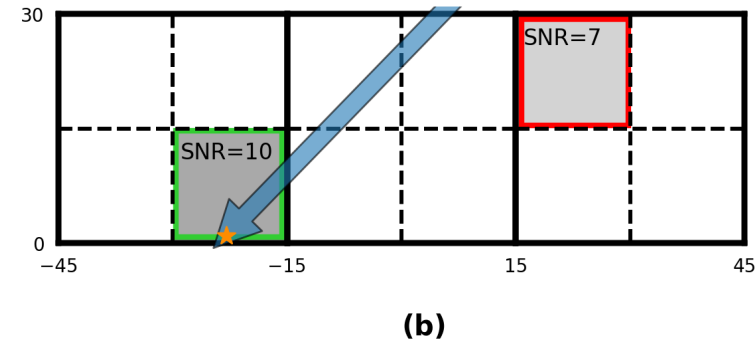
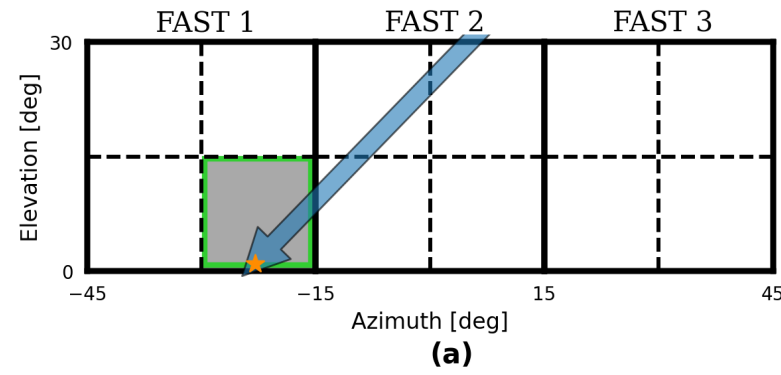
Shortcomings

- Need to train different models for different layouts
 - Future FAST layout will have slightly different distances between eyes (surely)
 - Current model does not generalize to such a case (would be systematic error)
- No uncertainty estimate given
 - Would be useful to know the confidence in first guess
 - Assess whether TDR can be reliably applied
- Models which can handle these cases can be built (graphneural networks/Bayesian networks)— but would require (probably) a lot more data/computation and model complexity

First guess II – Template Method

Additional triggering:

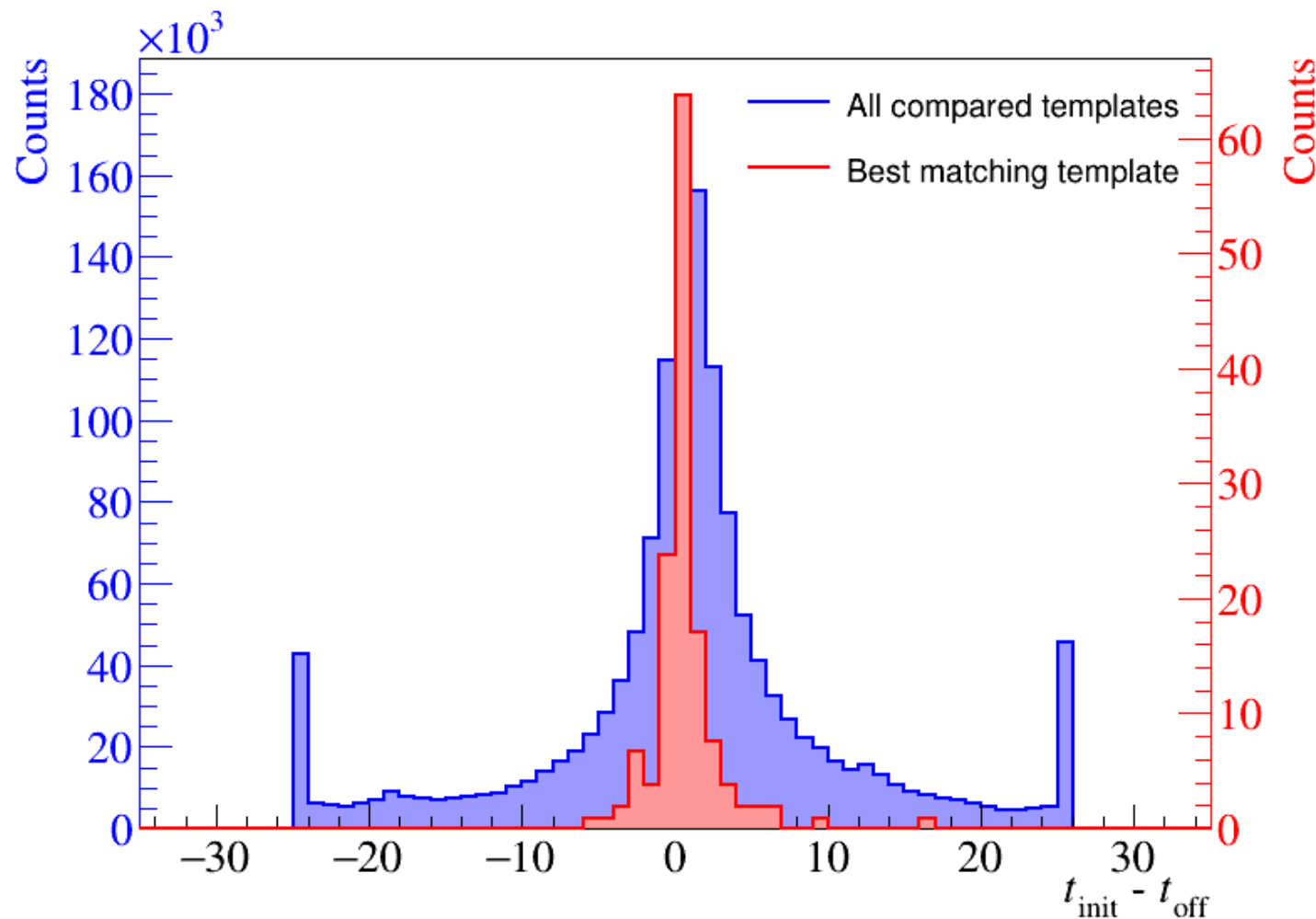
- With only threshold trigger, can get false positives in data due to baseline fluctuations, anthropogenic noise
- Introduce timing condition and pixel grouping condition



First guess II – Template Method

Finding the best t_{off} :

- How many values around t_{init} should we test?
- Take sample of simulated showers, try testing out to ± 25 bins
- Red shows offset for best matching template – general within ± 5



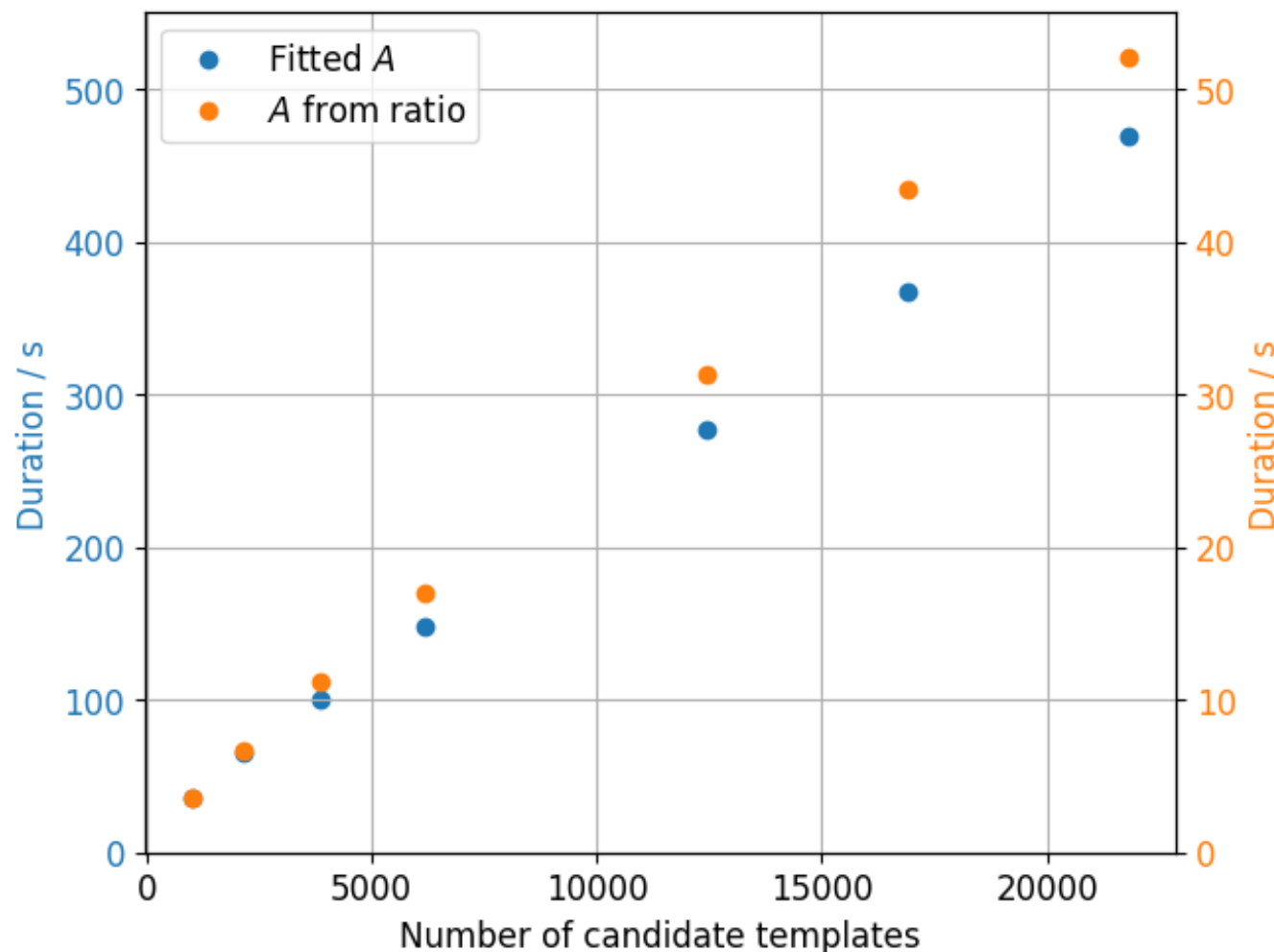
Template Comparison

Finding the best t_{off} and A :

- Ideally would **fit** A at each t_{off} , however **takes too long**
- Instead **estimate** A **from ratio** of signals

$$A_{\text{test}} = \frac{\sum_i S_i^{\text{data}}}{\sum_i S_i^{\text{template}}}$$

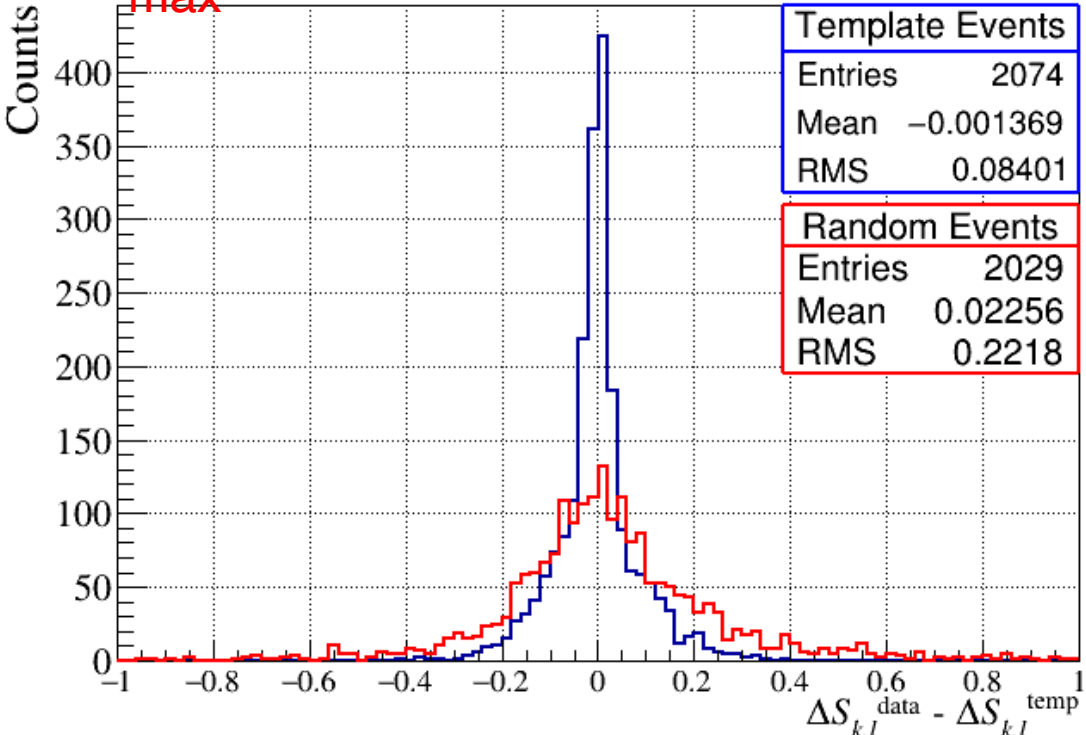
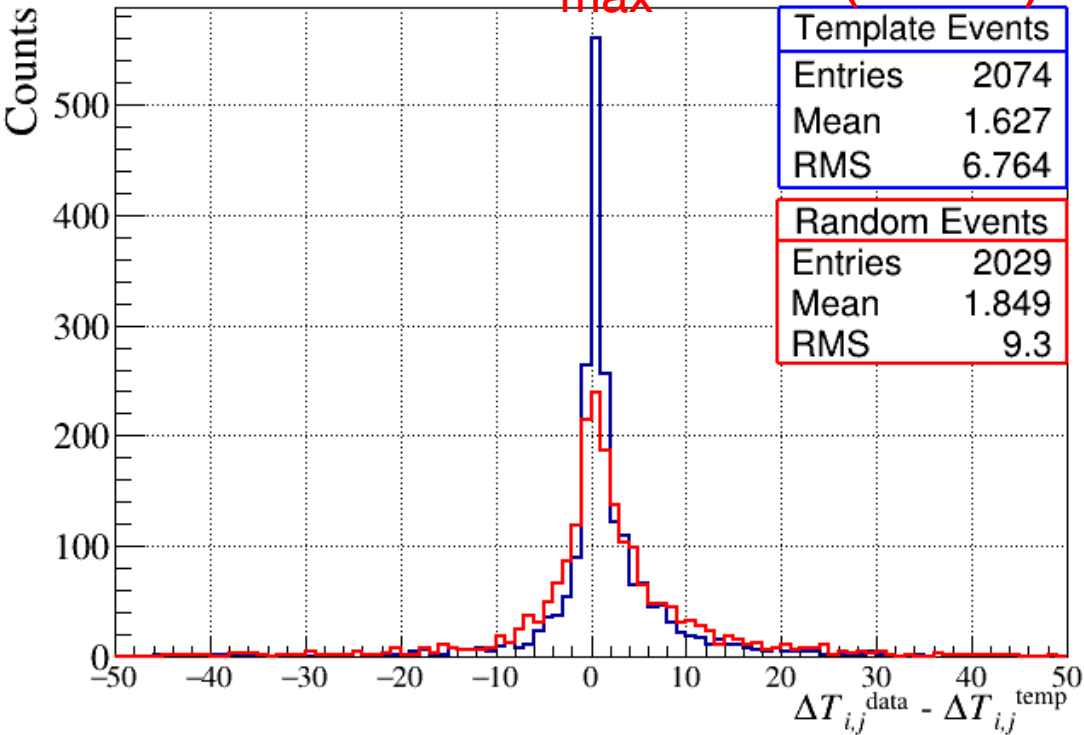
- Scale likelihood value over all tested t_{off} / A



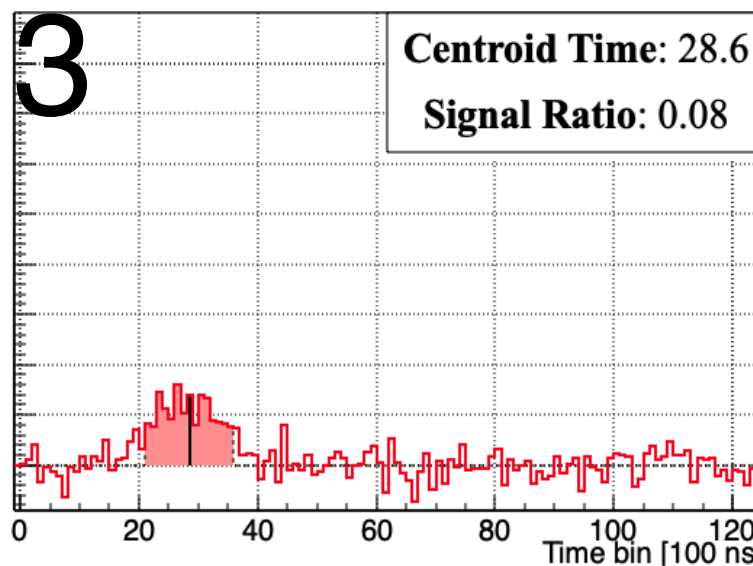
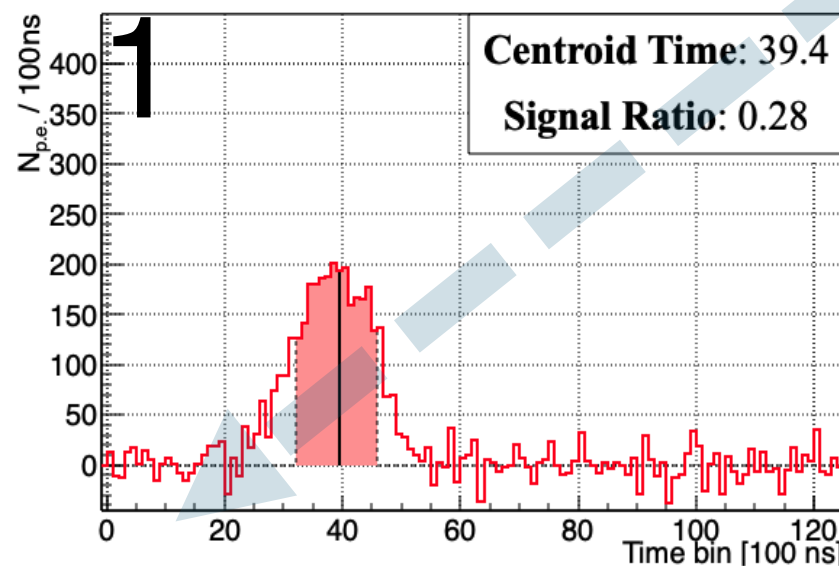
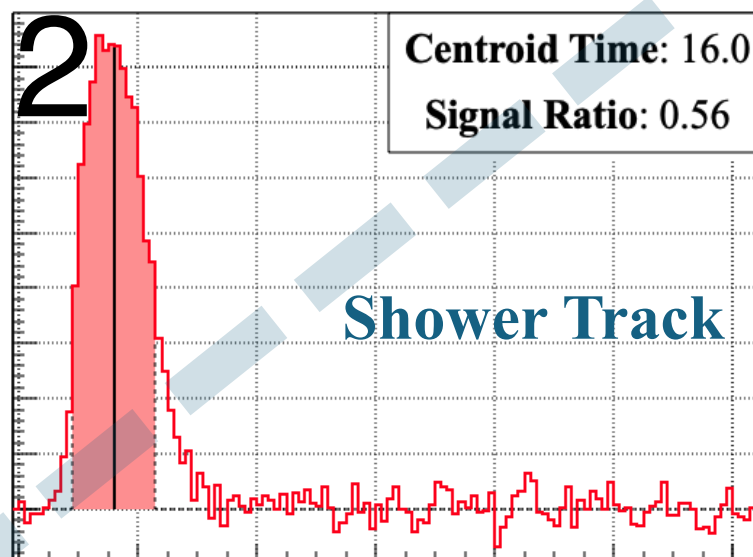
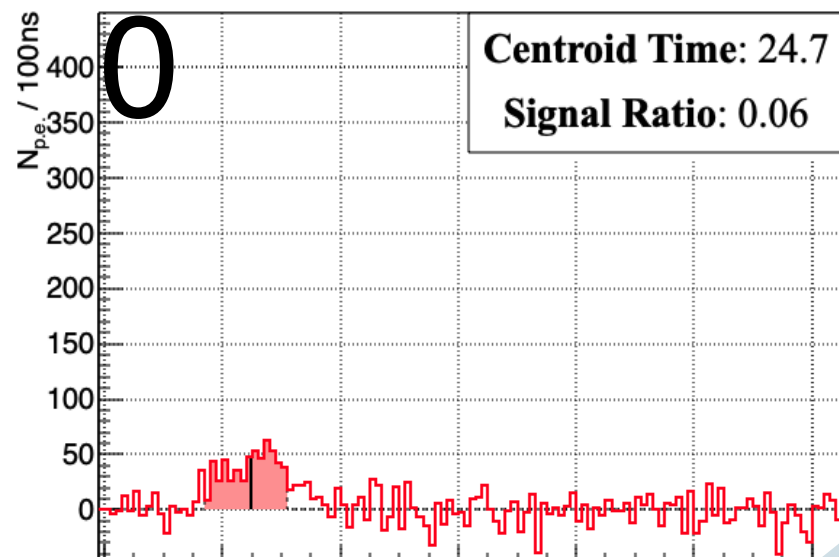
Template Selection

What should $T_{\max}$ and $S_{\max}$ be?

- Take sample of templates $\rightarrow$ add noise $\rightarrow$ calculate $\Delta T_{i,j}^{\text{data}}$ and $\Delta S_{k,l}^{\text{data}}$ for triggered PMTs, and compare to original $\Delta T_{i,j}^{\text{temp}}$ and $\Delta S_{k,l}^{\text{temp}}$
- Choose values of $T_{\max} = 10$ (1 ms) and $S_{\max} = 0.2$



Template Selection



$$\Delta T_{2,0}^{\text{data}} = 8.7 \quad \Delta S_{0,3}^{\text{data}} = 0.02$$

$$\Delta T_{0,3}^{\text{data}} = 3.9 \quad \Delta S_{3,1}^{\text{data}} = 0.2$$

$$\Delta T_{3,1}^{\text{data}} = 10.8 \quad \Delta S_{1,2}^{\text{data}} = 0.28$$

Select all templates with

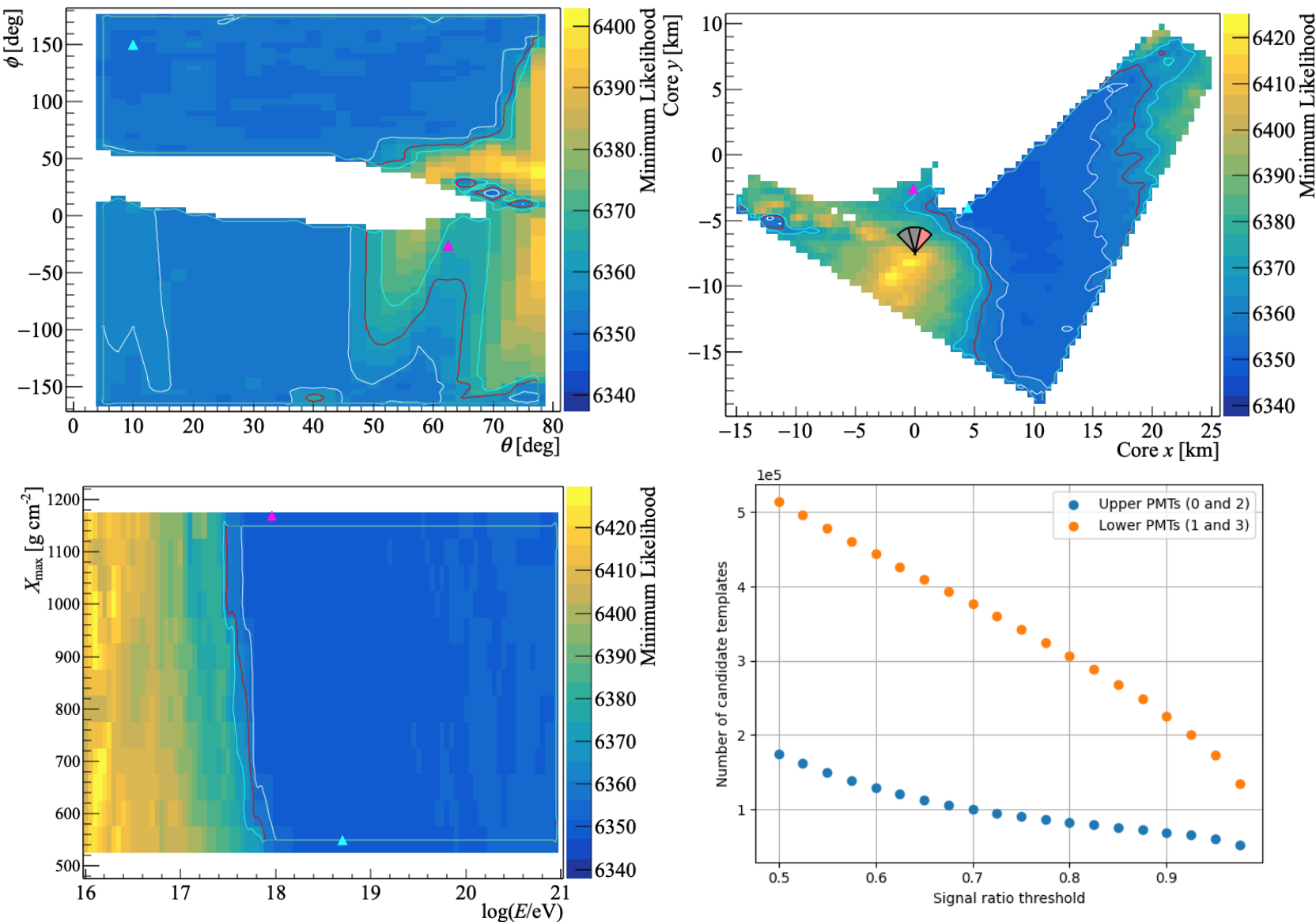
$$\Delta T_{i,j}^{\text{temp}} \quad \Delta S_{k,l}^{\text{temp}}$$

with

$$|\Delta T_{i,j}^{\text{data}} - \Delta T_{i,j}^{\text{temp}}| < T_{\text{max}}$$

$$|\Delta S_{k,l}^{\text{data}} - \Delta S_{k,l}^{\text{temp}}| < S_{\text{max}}$$

One triggered PMT

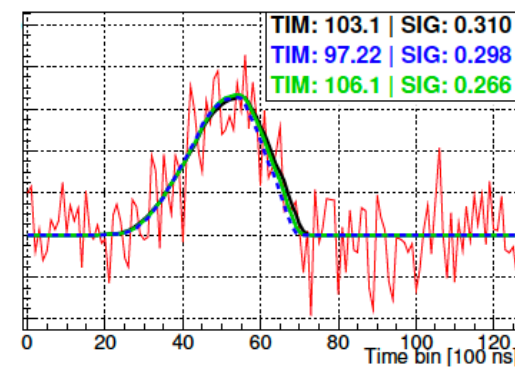
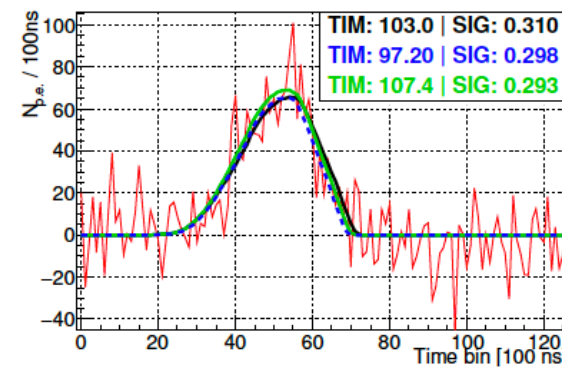
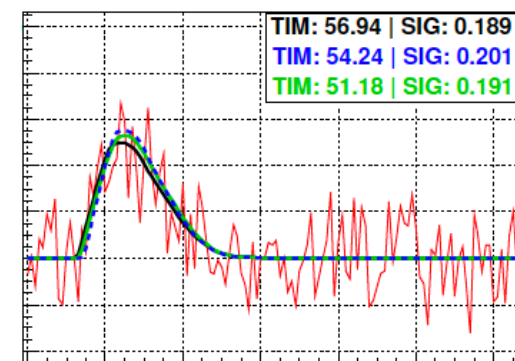
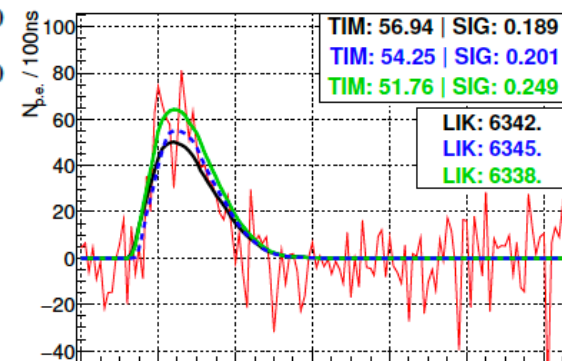
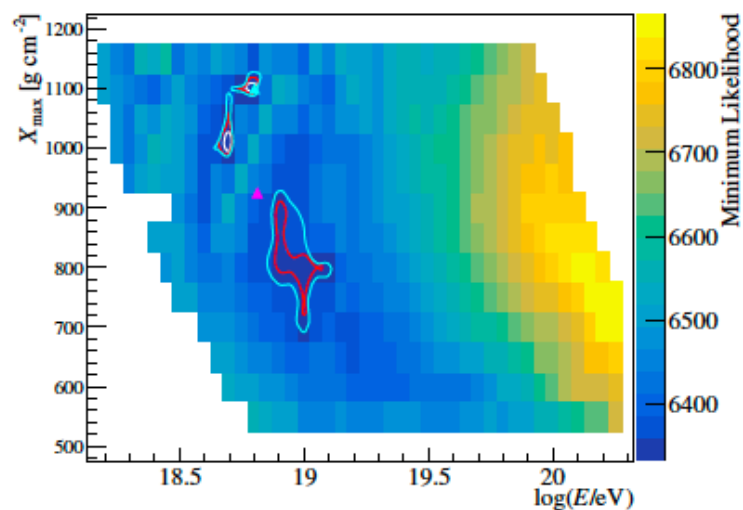
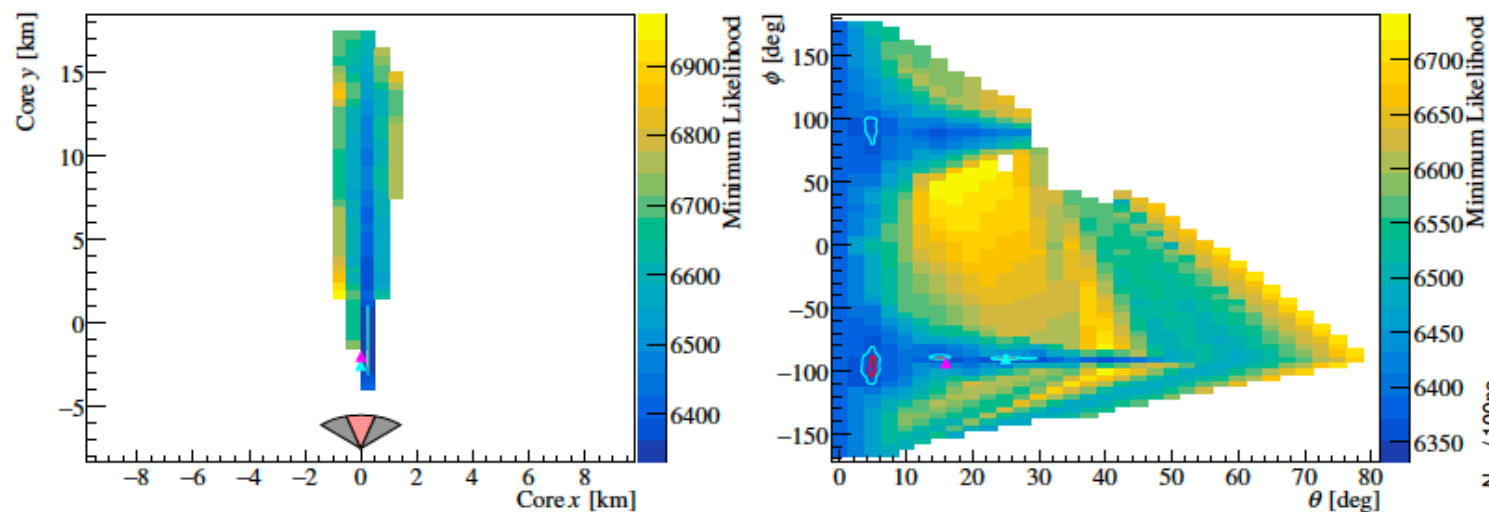


Extreme degeneracy by itself

However, can still help to constrain parameters

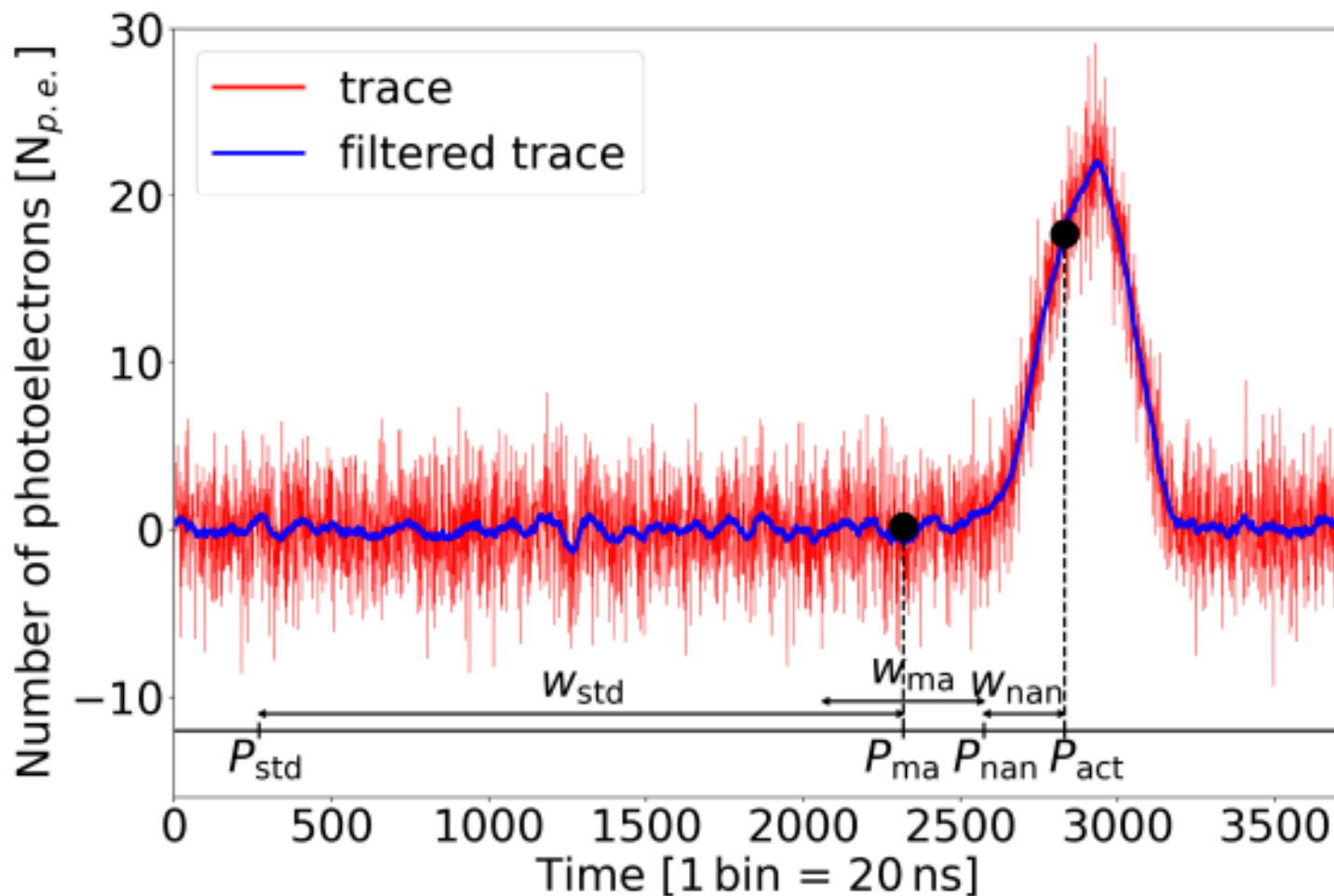
Compare templates with a signal ratio > 0.5 in triggered PMT and only try $t_{\text{off}} = t_{\text{init}}$

Template Example



Coincidence data analysis

New signal search algorithm



- Filter trace with finite impulse filters

- Calculate

$$SNR_j(i) = \frac{F_j(i) - T_{BG}(i)}{\sigma_{F_j}}$$

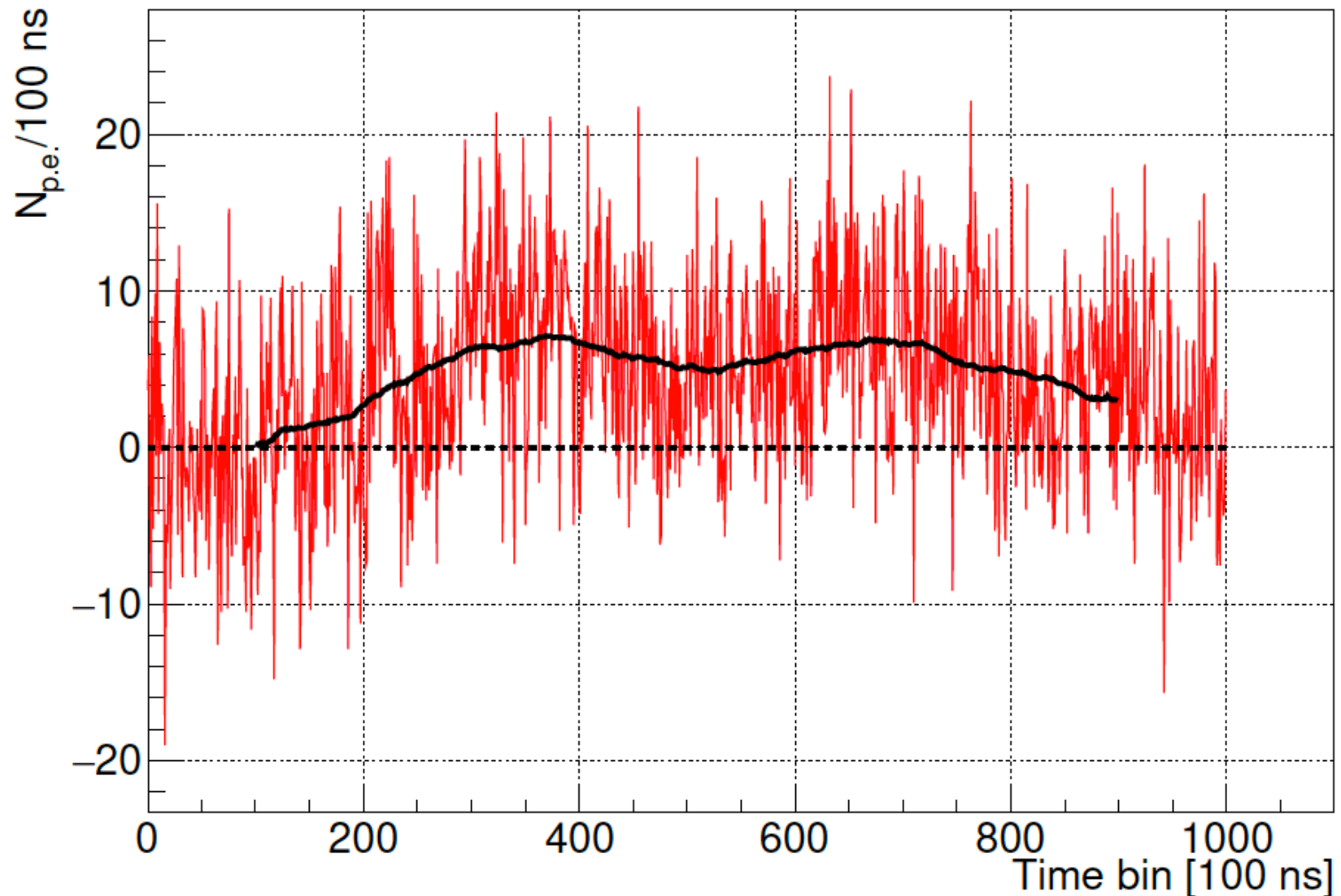
Signal baseline
↓

for each bin for each filter

- If SNR above threshold for that filter, PMT is triggered

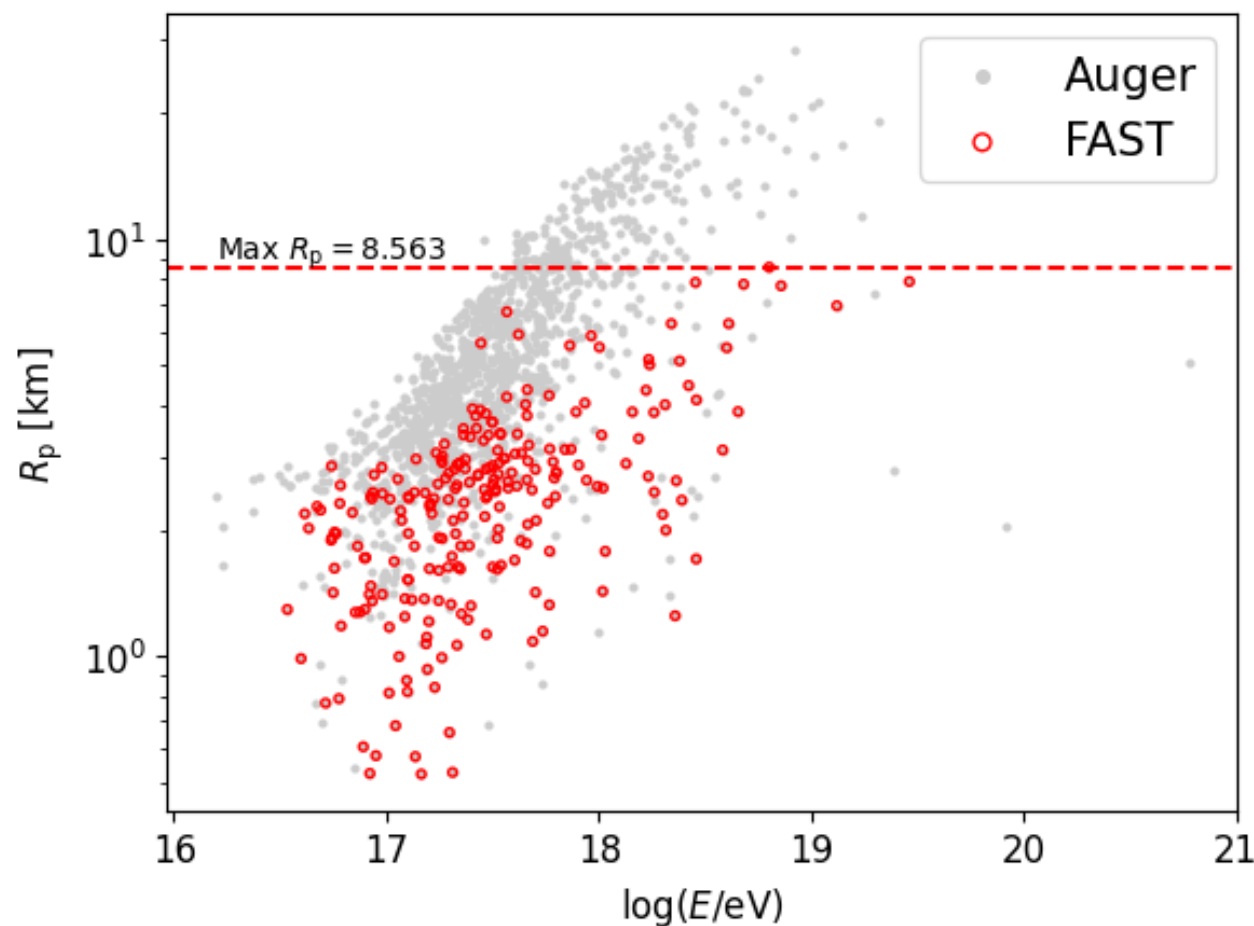
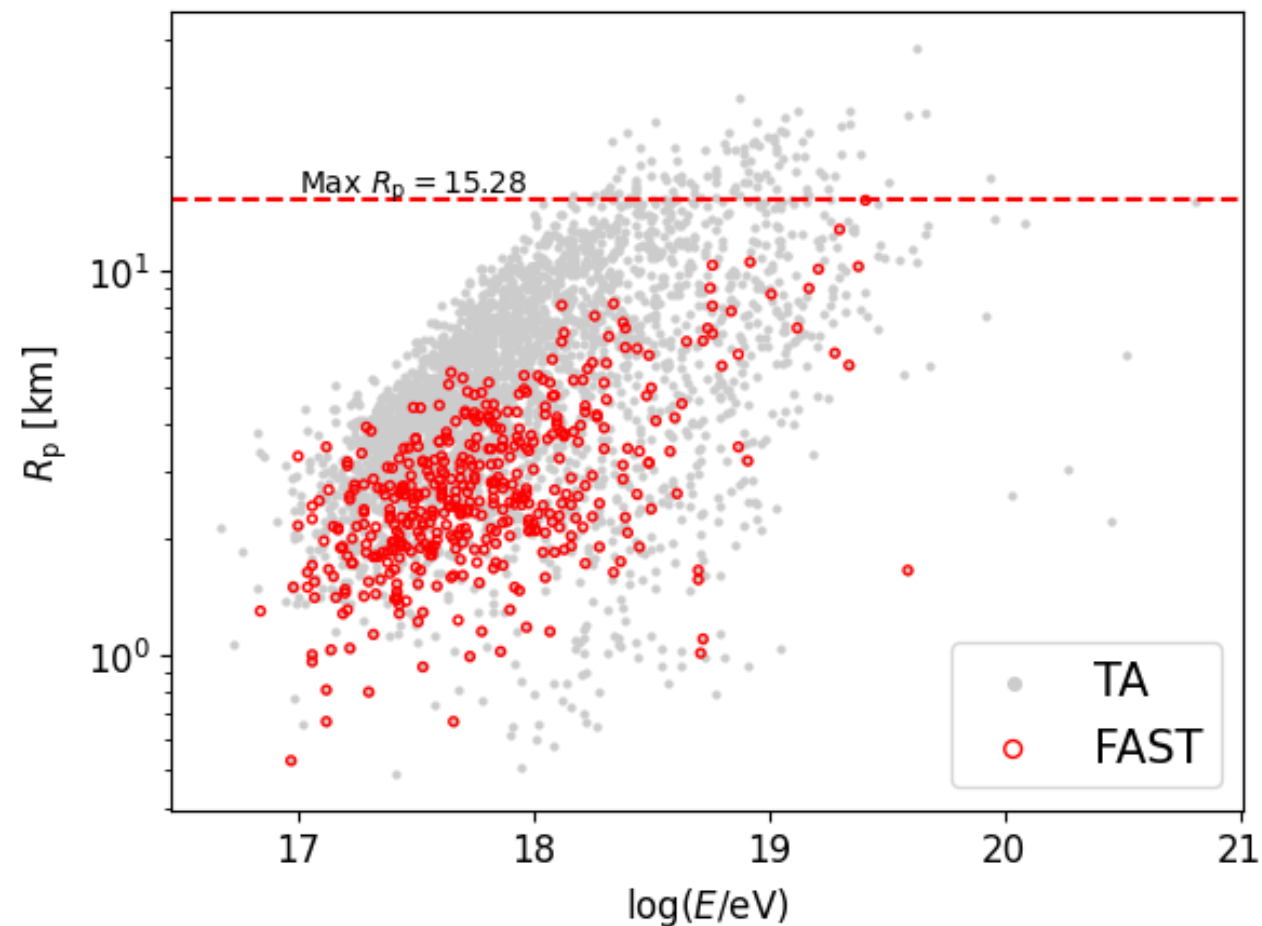
Baseline Fluctuations

Without accounting for
baseline fluctuations –
many false triggers



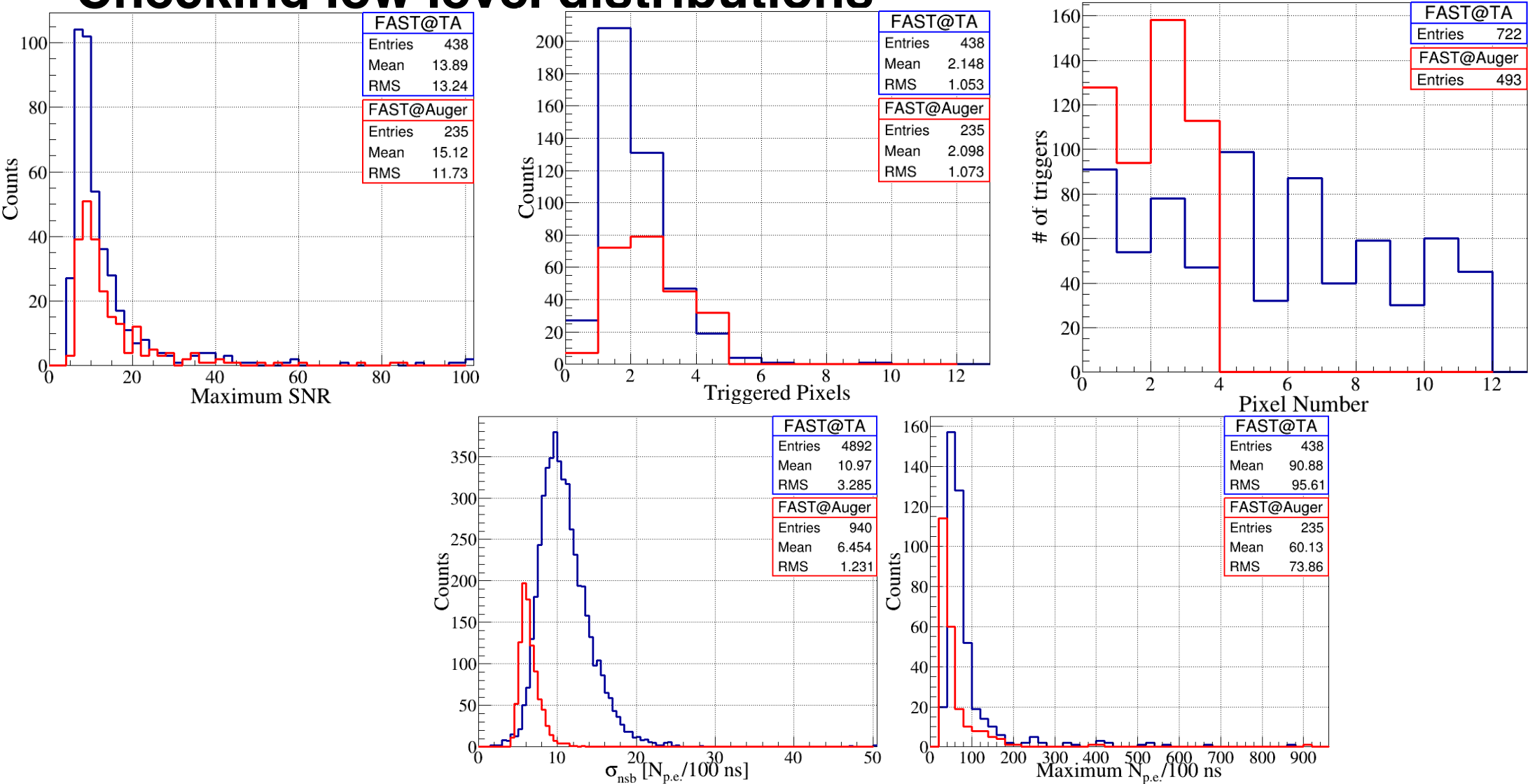
Coincidence data analysis

Energy vs Rp



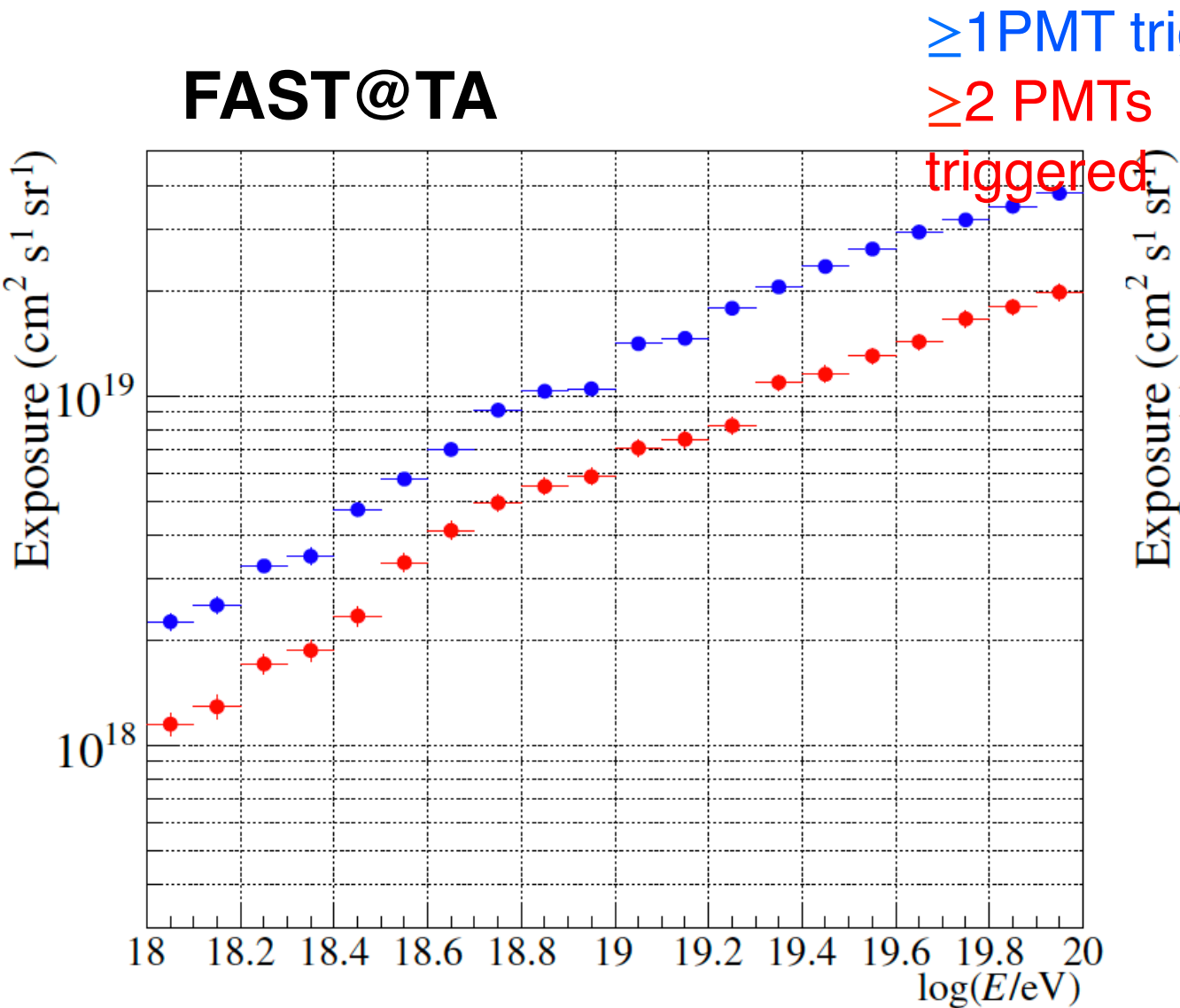
Coincidence data analysis

Checking low level distributions

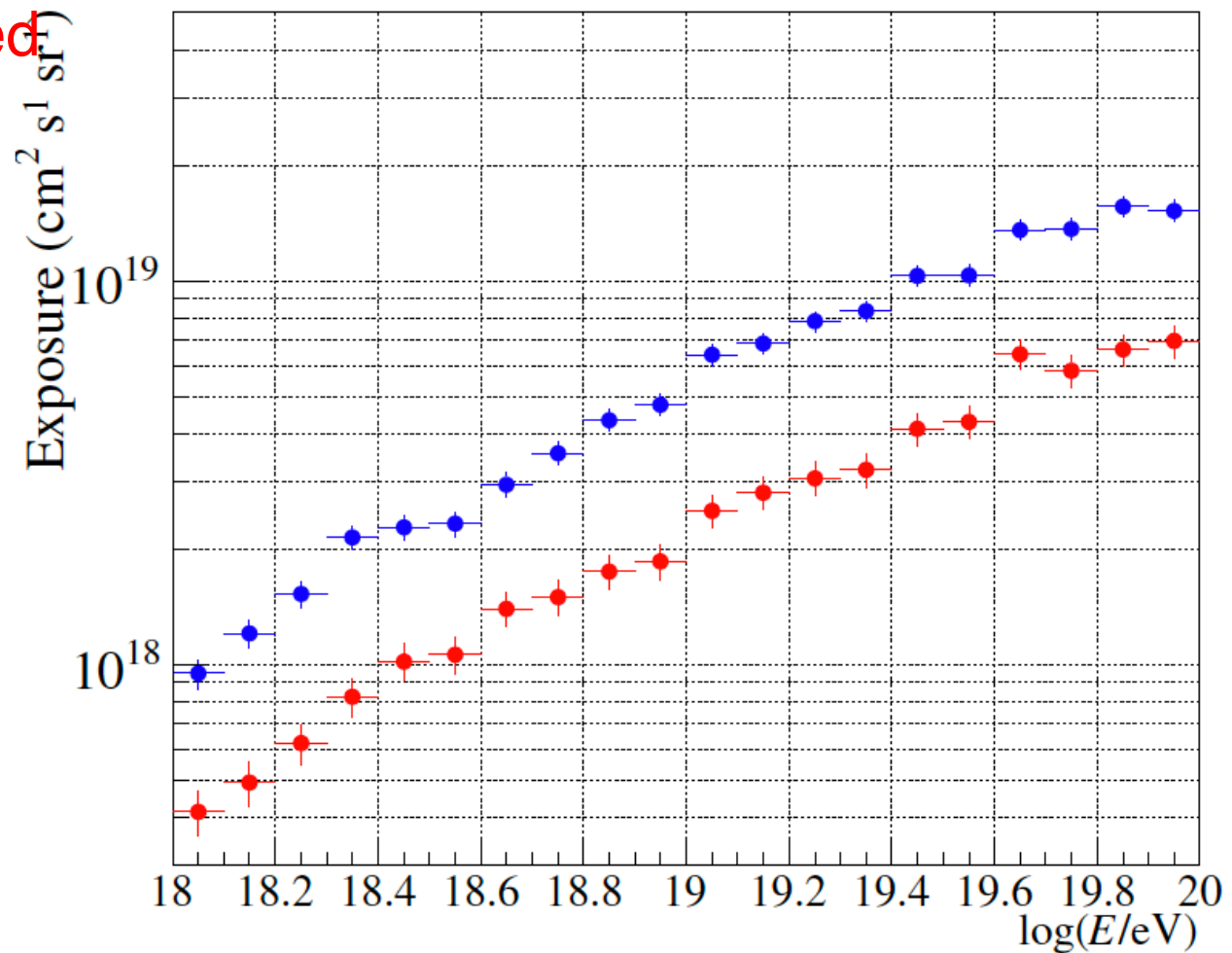


MC/Data – Exposure calculation

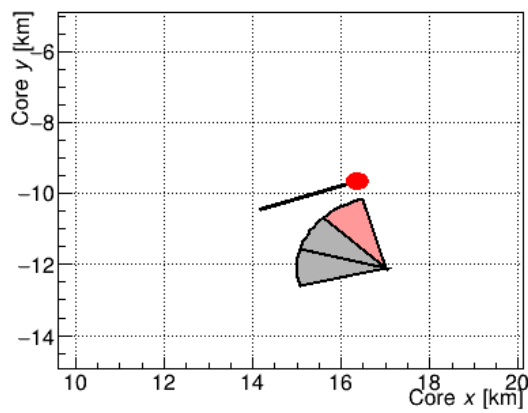
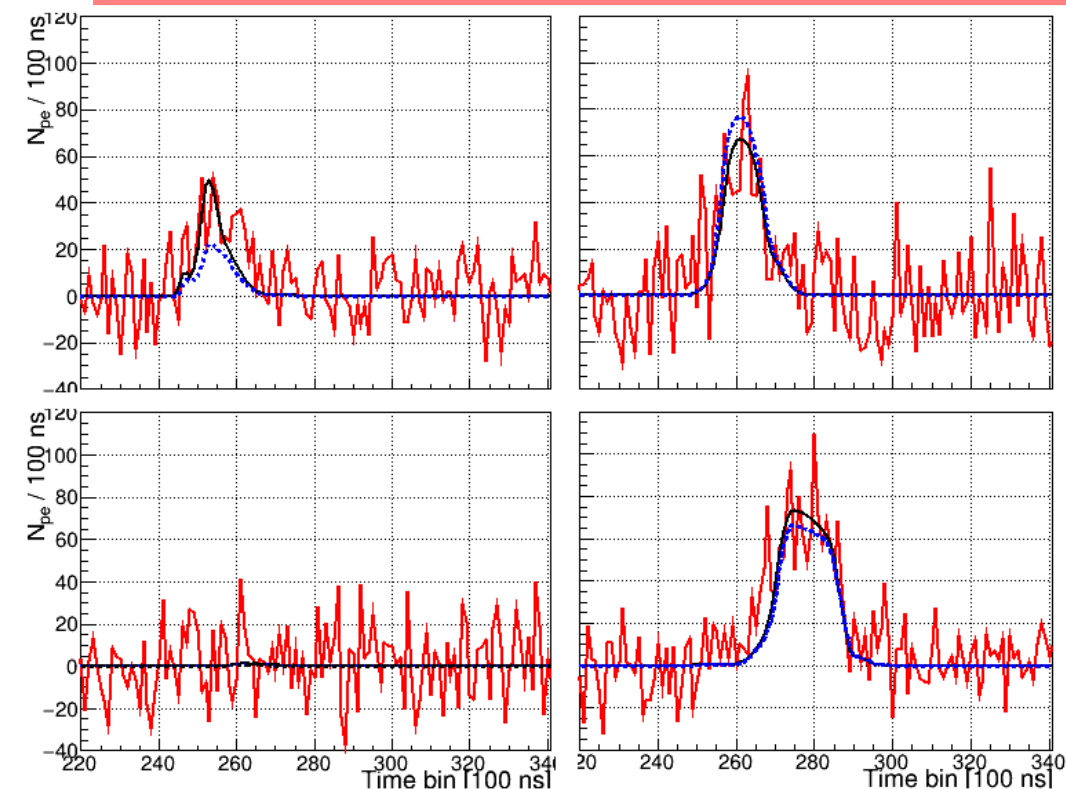
FAST@TA



FAST@Auger

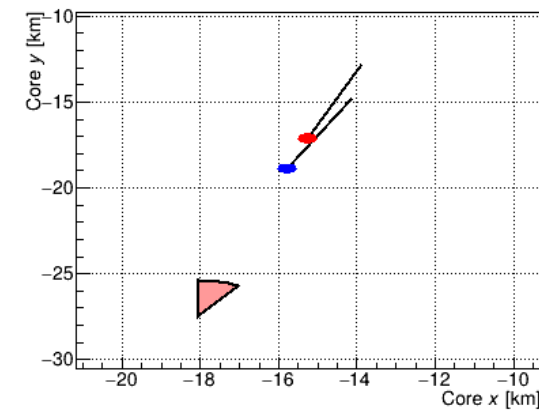
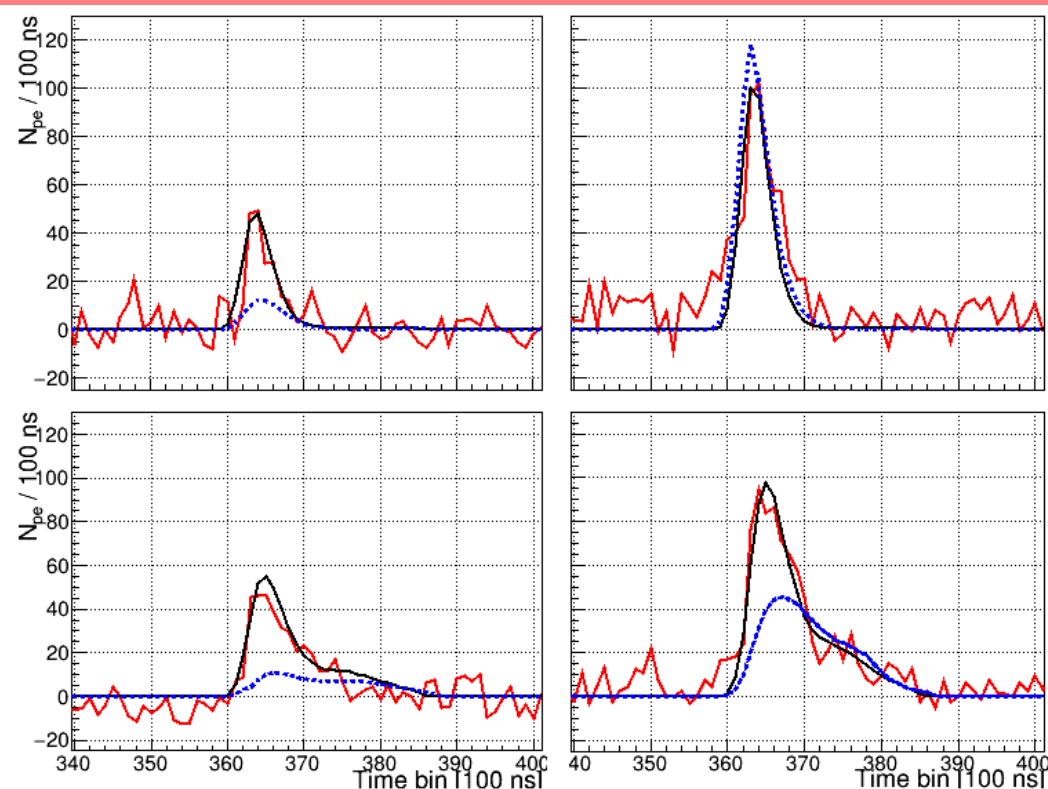


Example Reconstructions



RECONSTRUCTION RESULTS

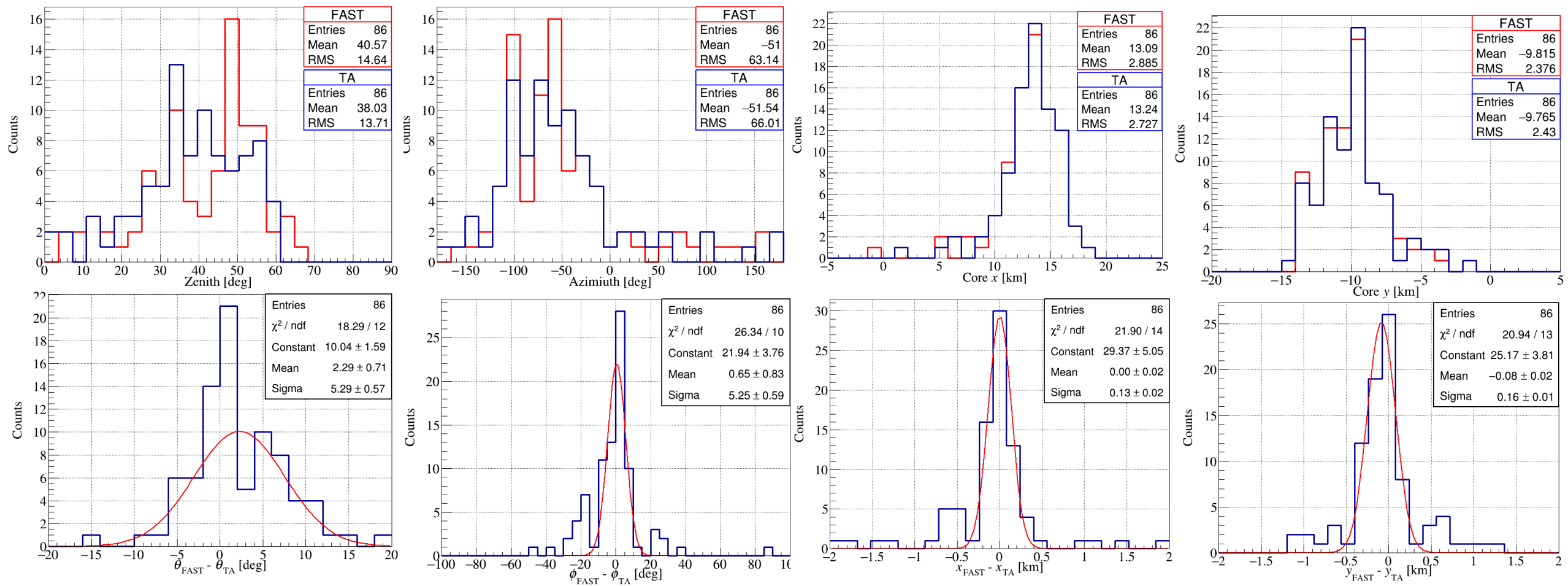
$X_{\max}$: 802 ± 71 (796) [g cm^{-2}]
 E : 17.9 ± 17 (17.92) [$\log(E/\text{eV})$]
 θ : 29.5 ± 1.4 (28.2) [$^\circ$]
 ϕ : 199.1 ± 4.6 (-160.2) [$^\circ$]
 x : 16352 ± 33 (16360) [m]
 y : -9675 ± 57 (-9650) [m]



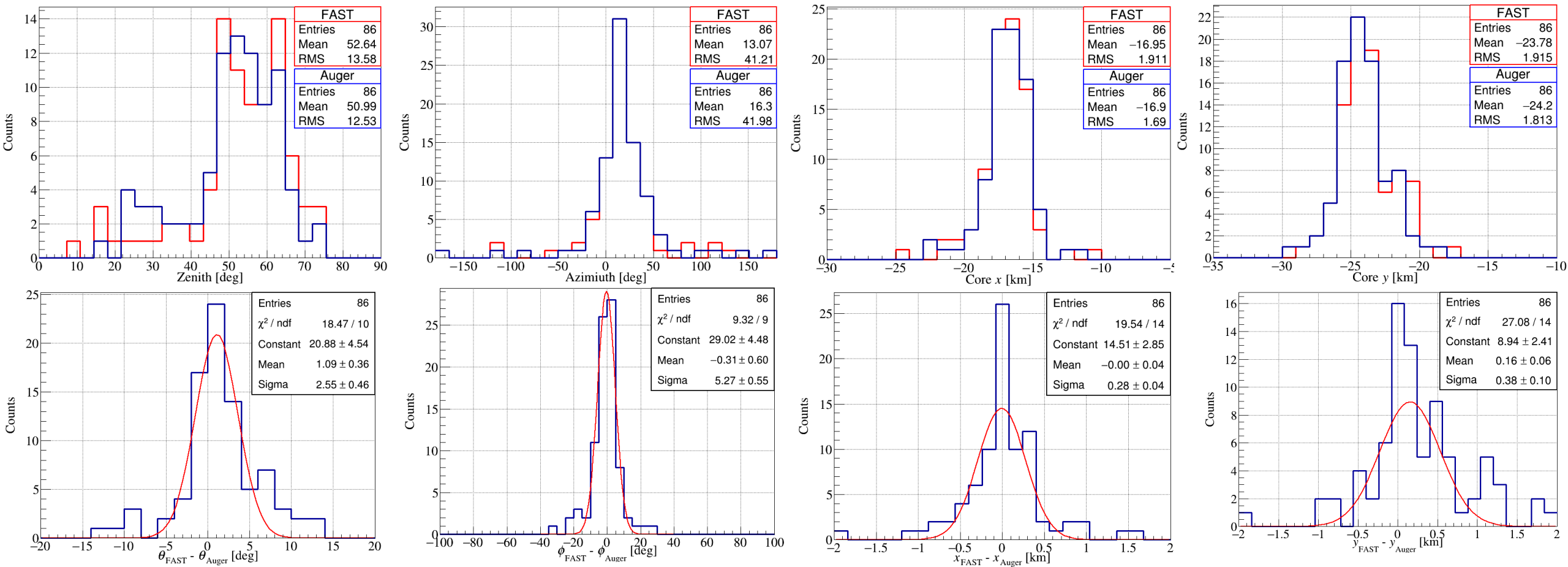
RECONSTRUCTION RESULTS

$X_{\max}$: 828 ± 91 (864.8) [g cm^{-2}]
 E : 18.8 ± 18.3 (18.4589) [$\log(E/\text{eV})$]
 θ : 65.6 ± 0.5 (62.842) [$^\circ$]
 ϕ : 72.4 ± 1 (68.062) [$^\circ$]
 x : -15267 ± 338 (-15795) [m]
 y : -17107 ± 820 (-18862) [m]

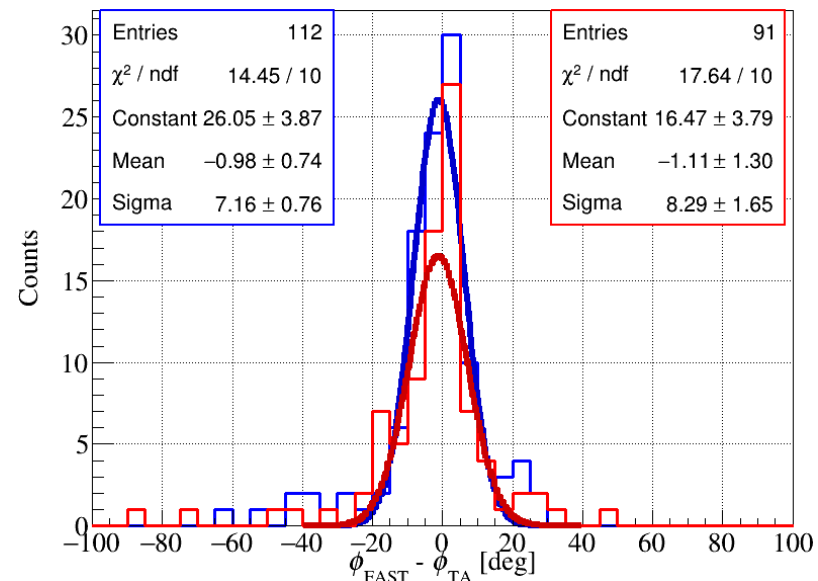
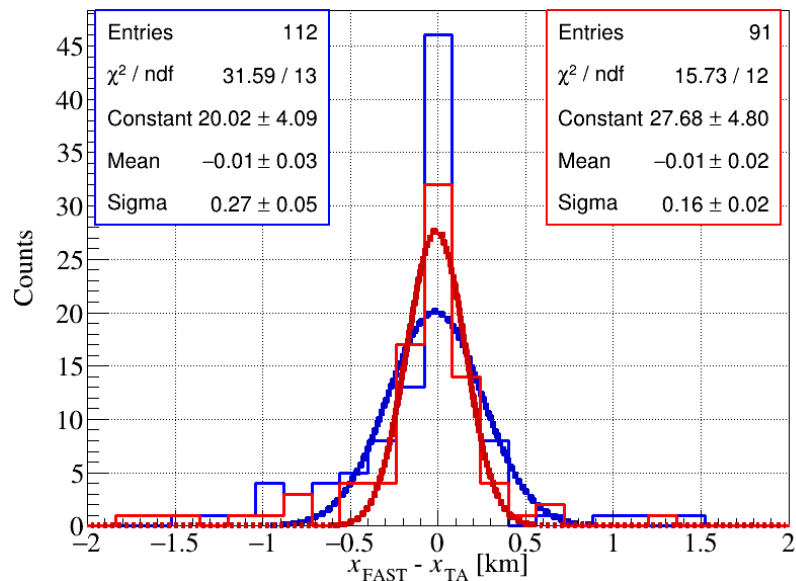
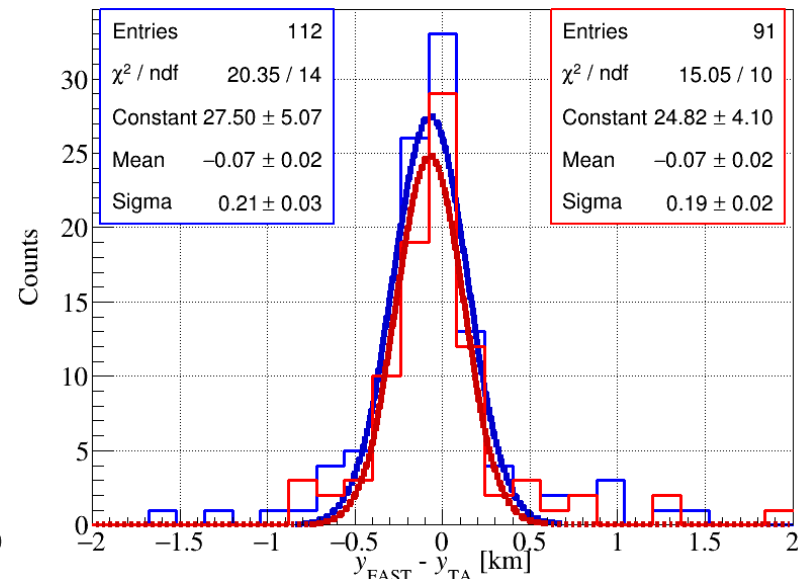
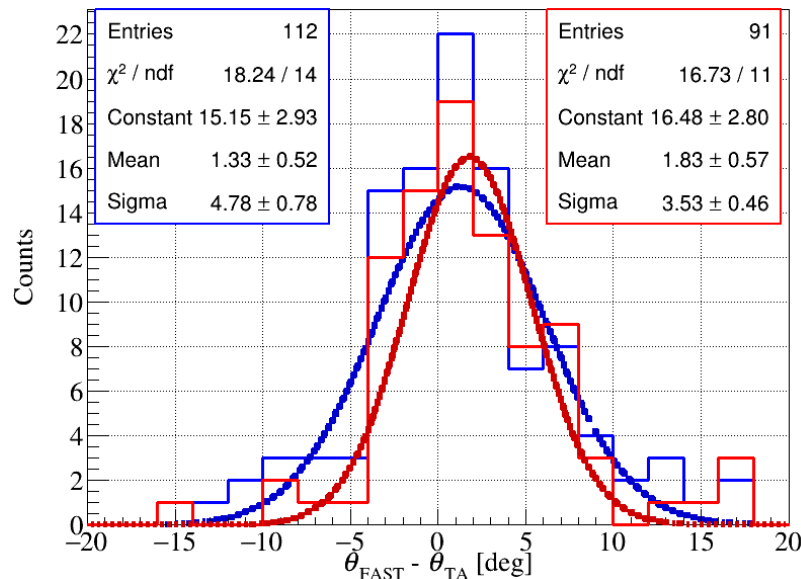
Initial Reconstruction: FAST@TA



Initial Reconstruction: FAST@Auger



New maps reconstruction

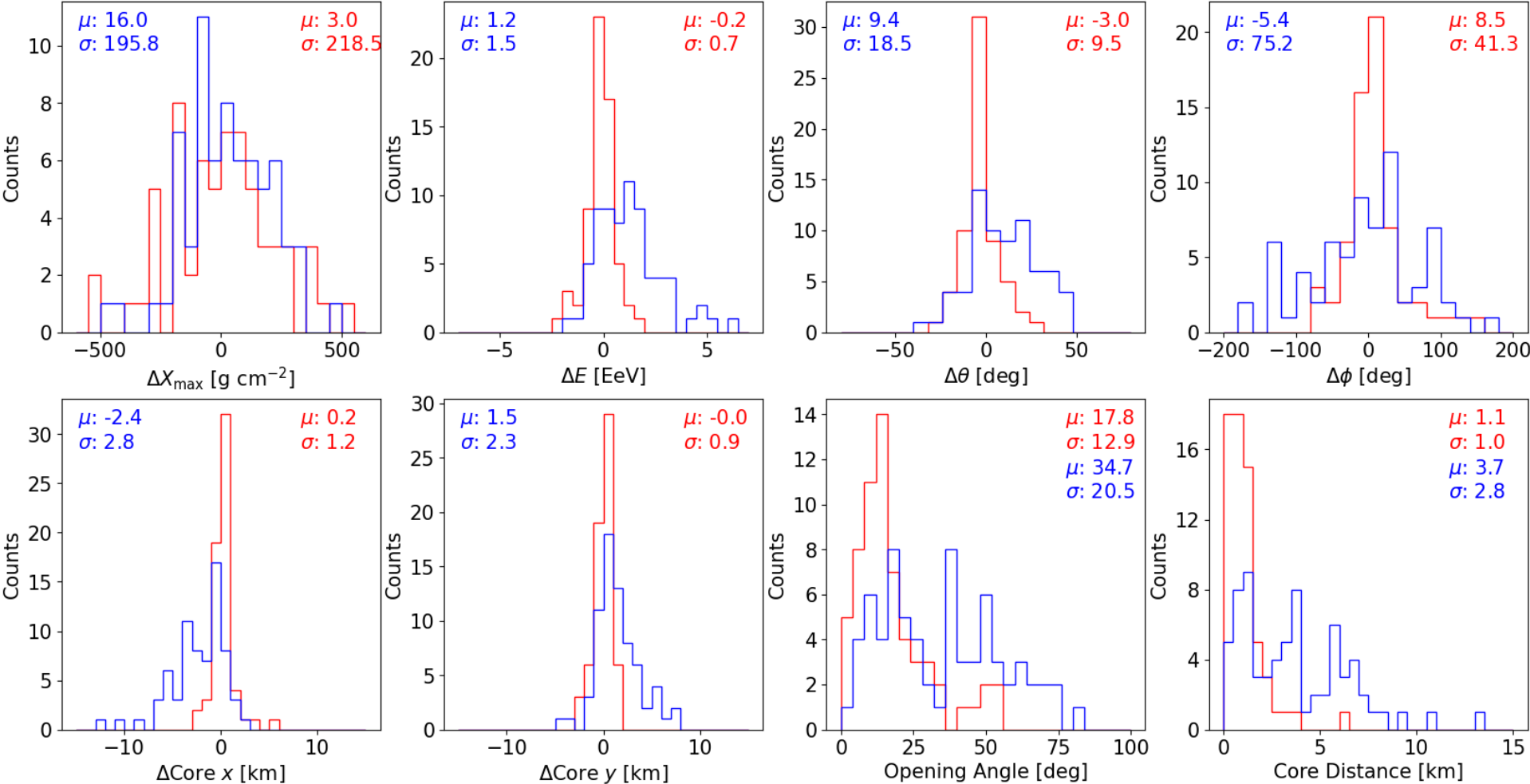


FAST Only Recon. (TSFEL DNN)

FAST@TA

Blue = TSFEL DNN – TA / Auger

Orange = (TSFEL DNN + TDR) – TA / Auger

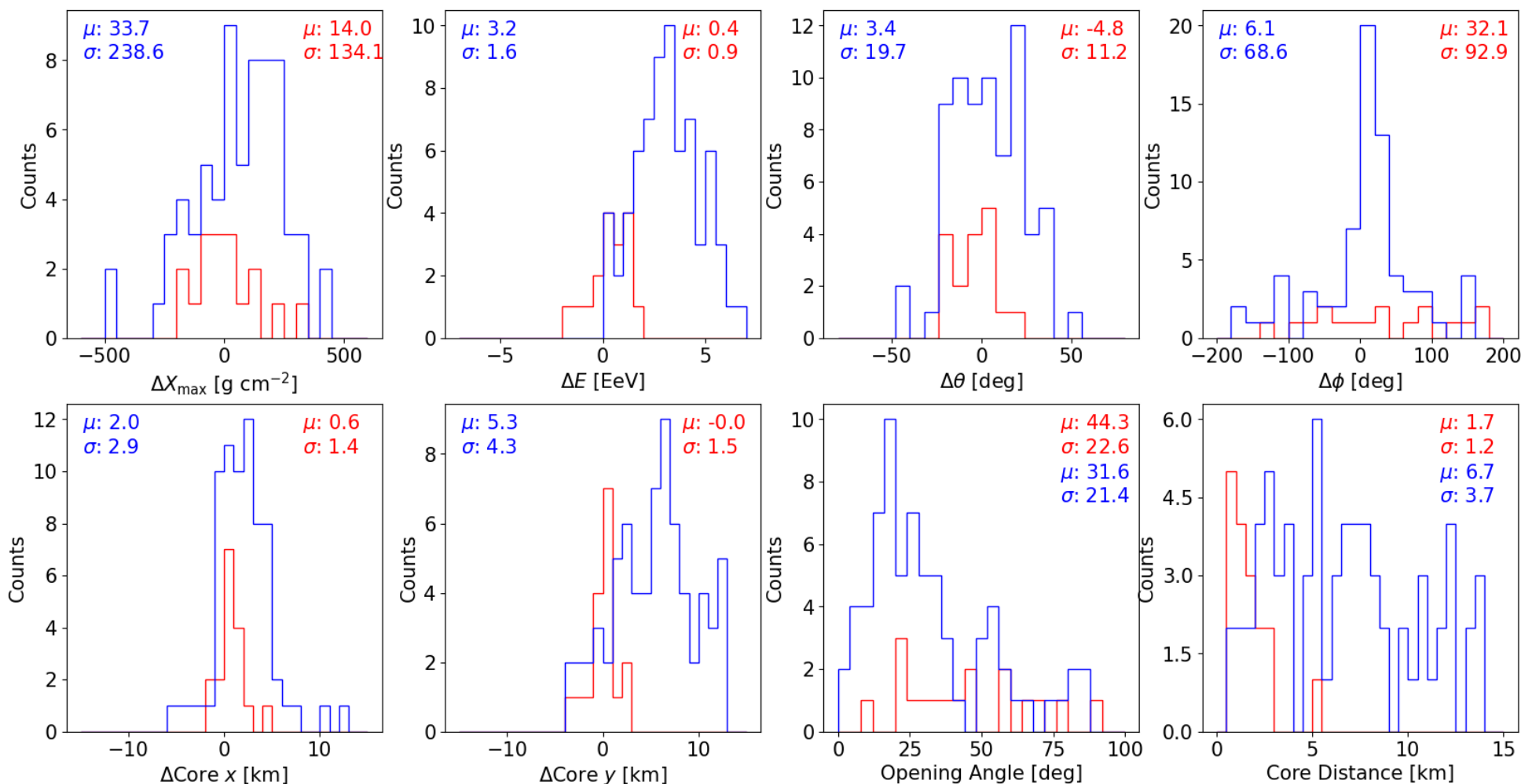


FAST Only Recon. (TSFEL DNN)

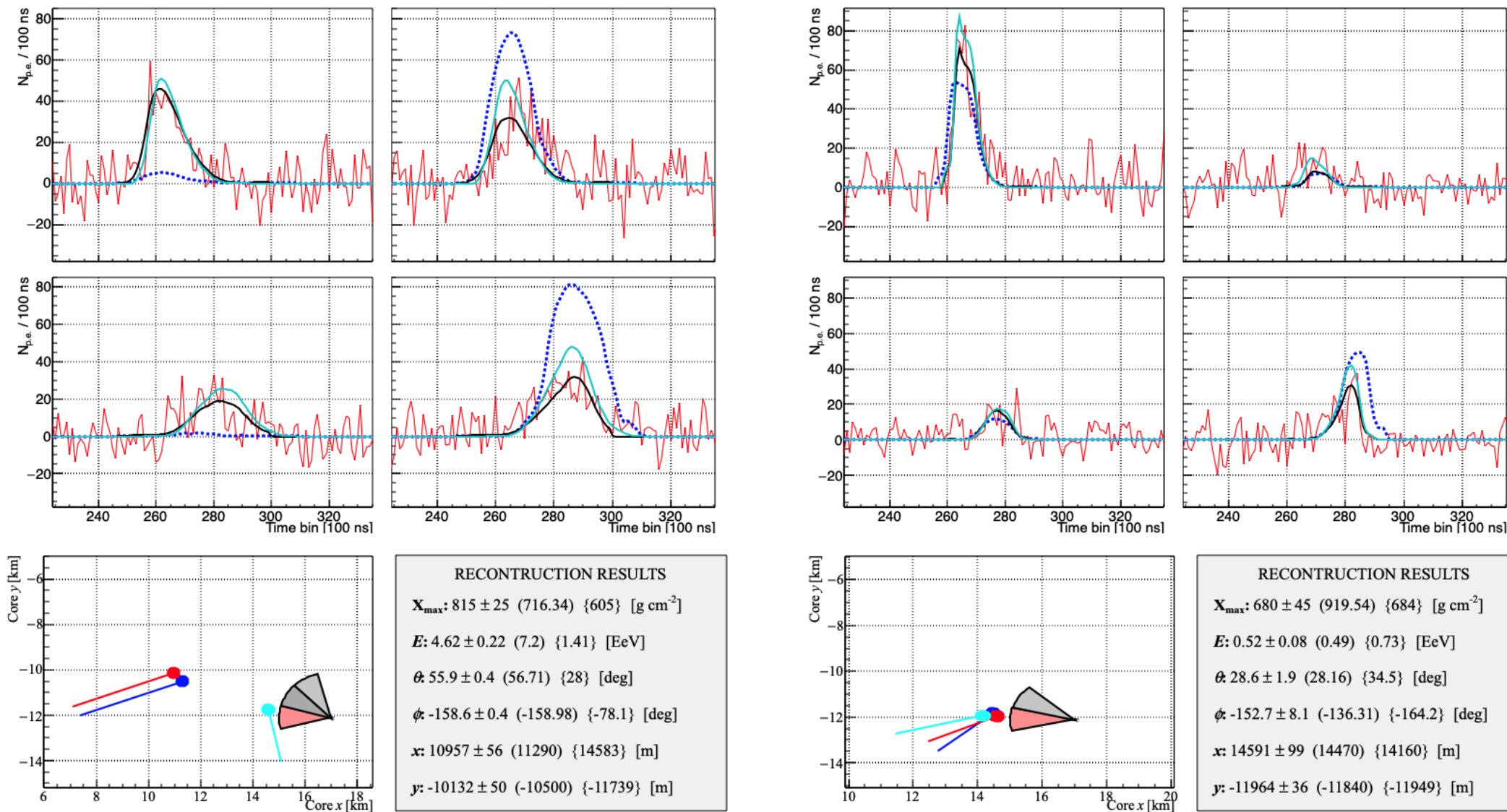
FAST@Auger

Blue = TSFEL DNN – TA / Auger

Orange = (TSFEL DNN + TDR) – TA / Auger



FAST Only Recon. (TSFEL DNN)

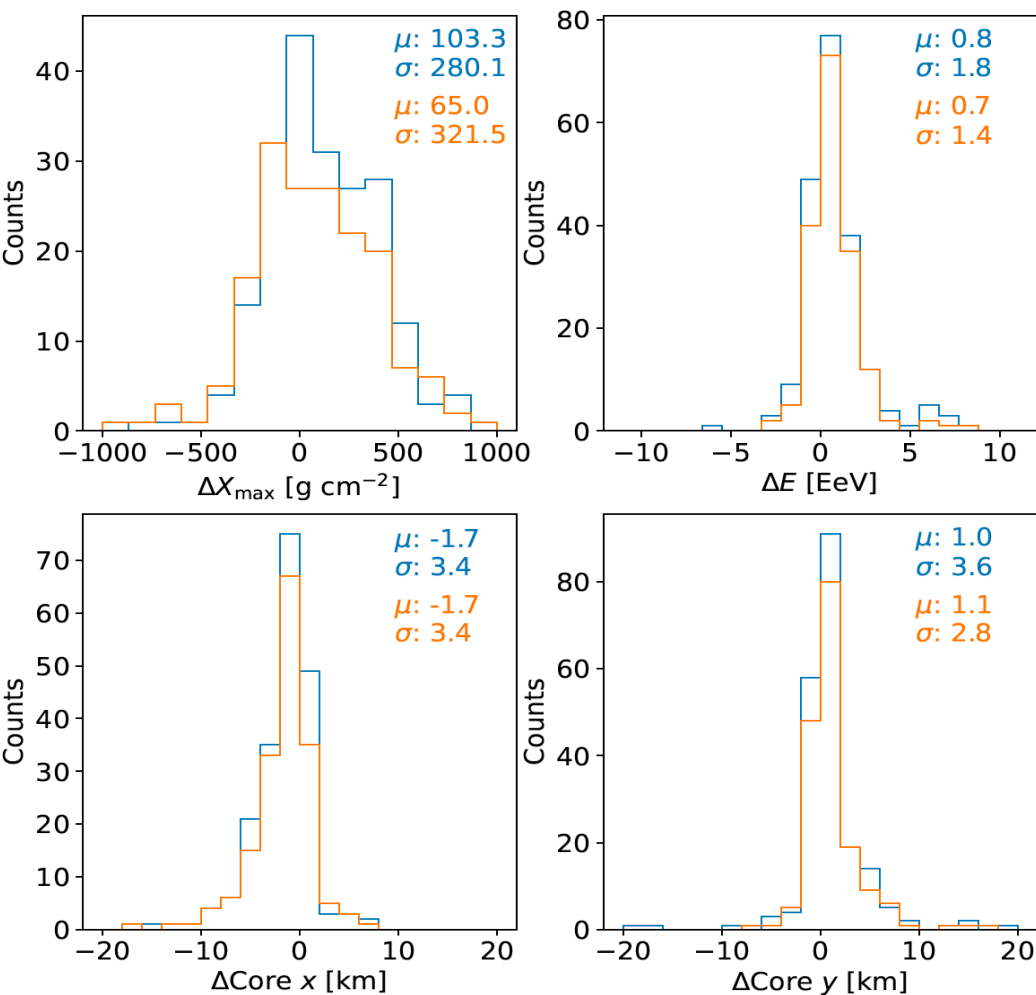


Template Method + TDR

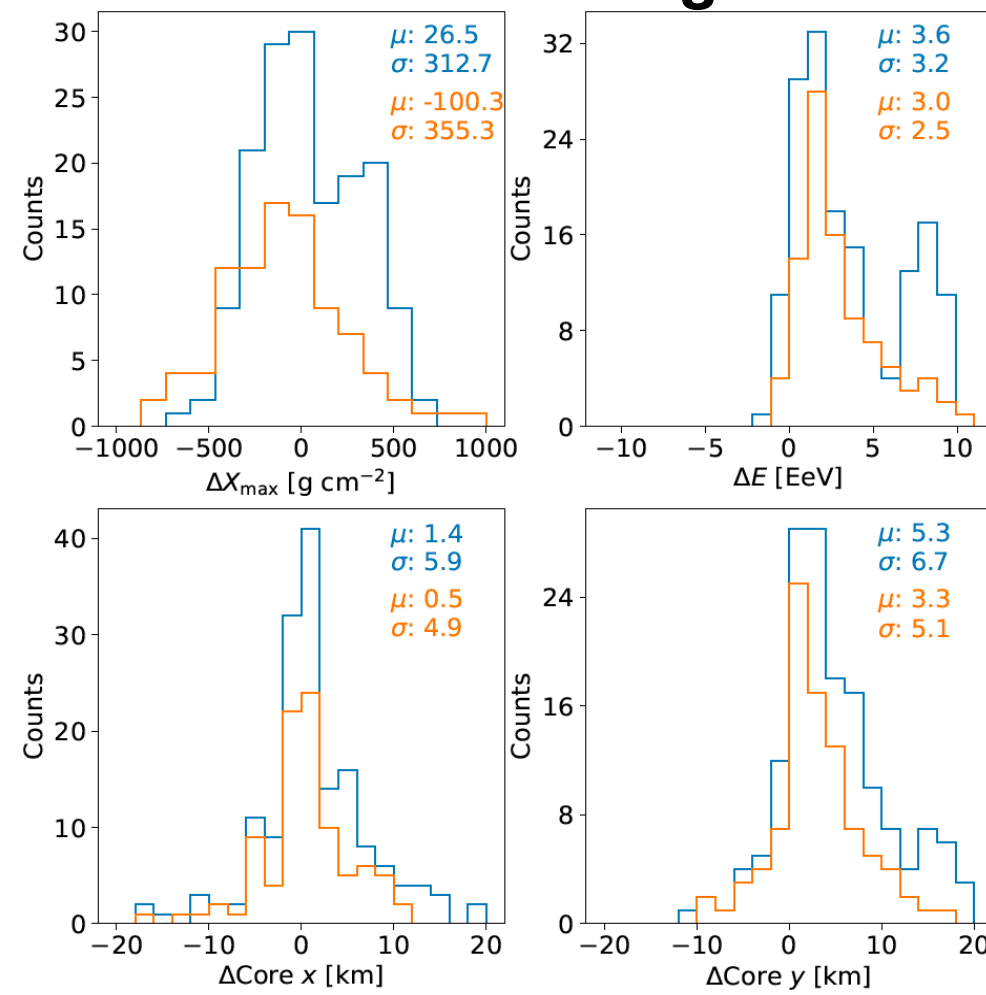
Blue = Temp. Method – TA / Auger

Orange = (Temp. Method + TDR) – TA / Auger

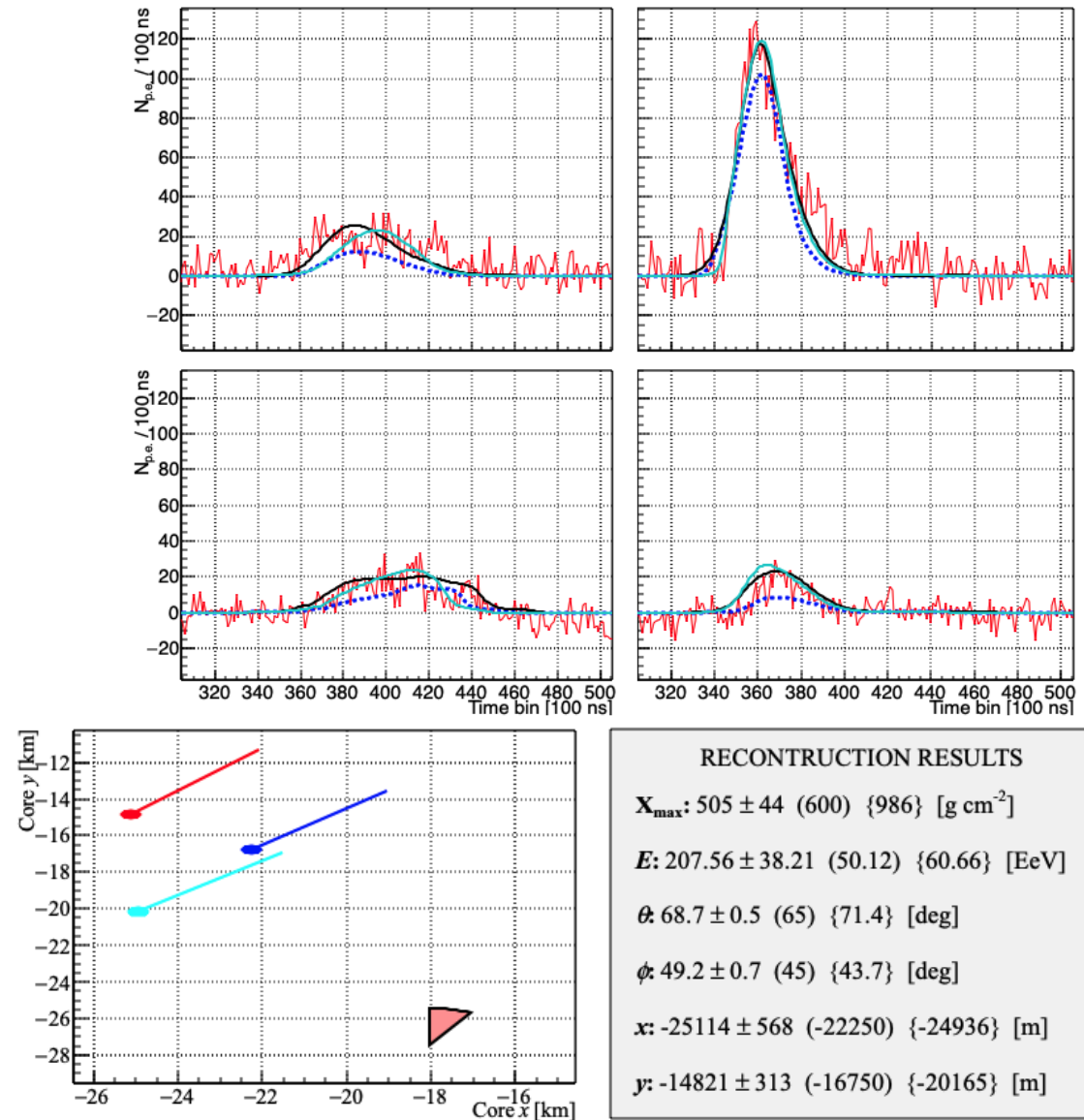
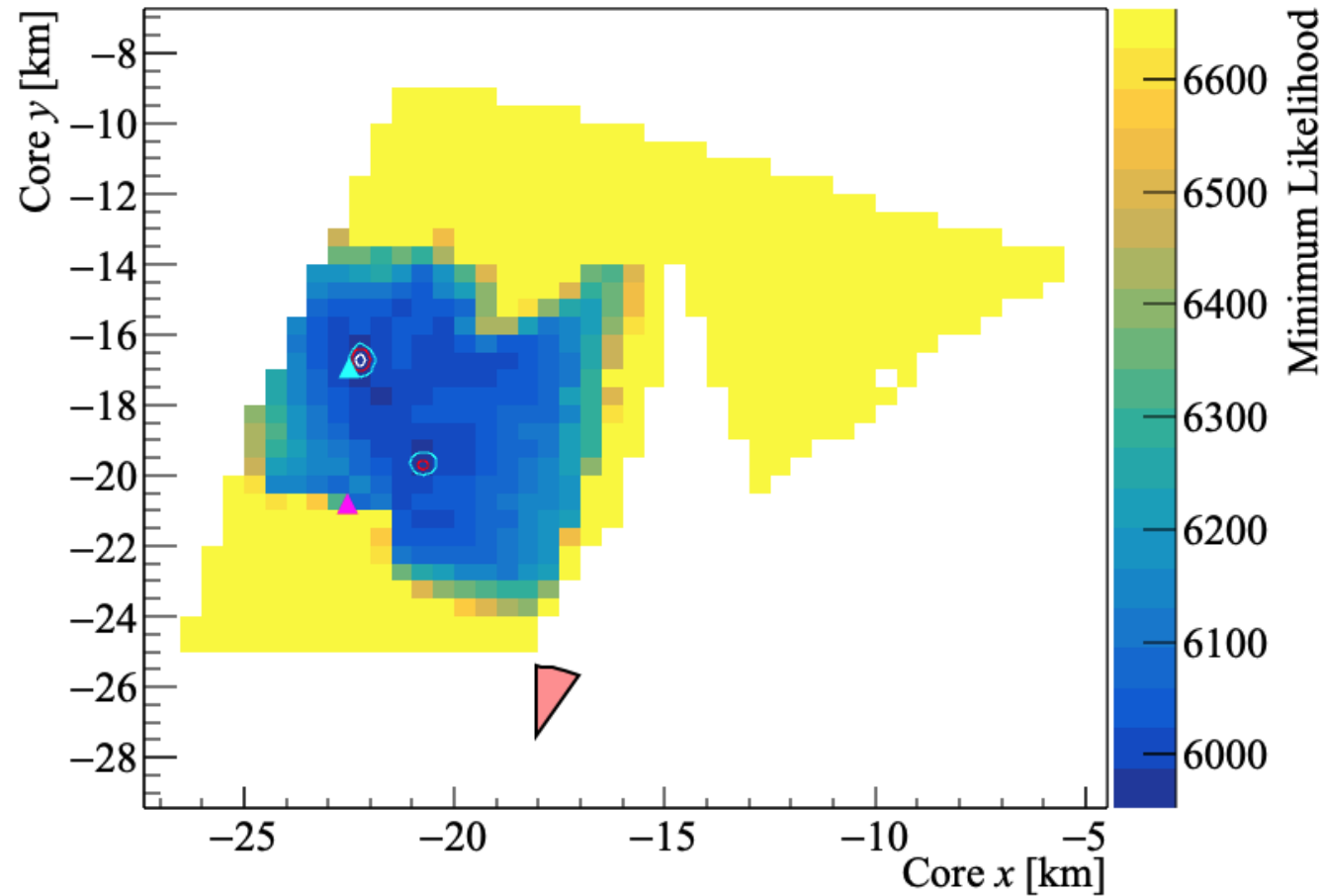
FAST@TA



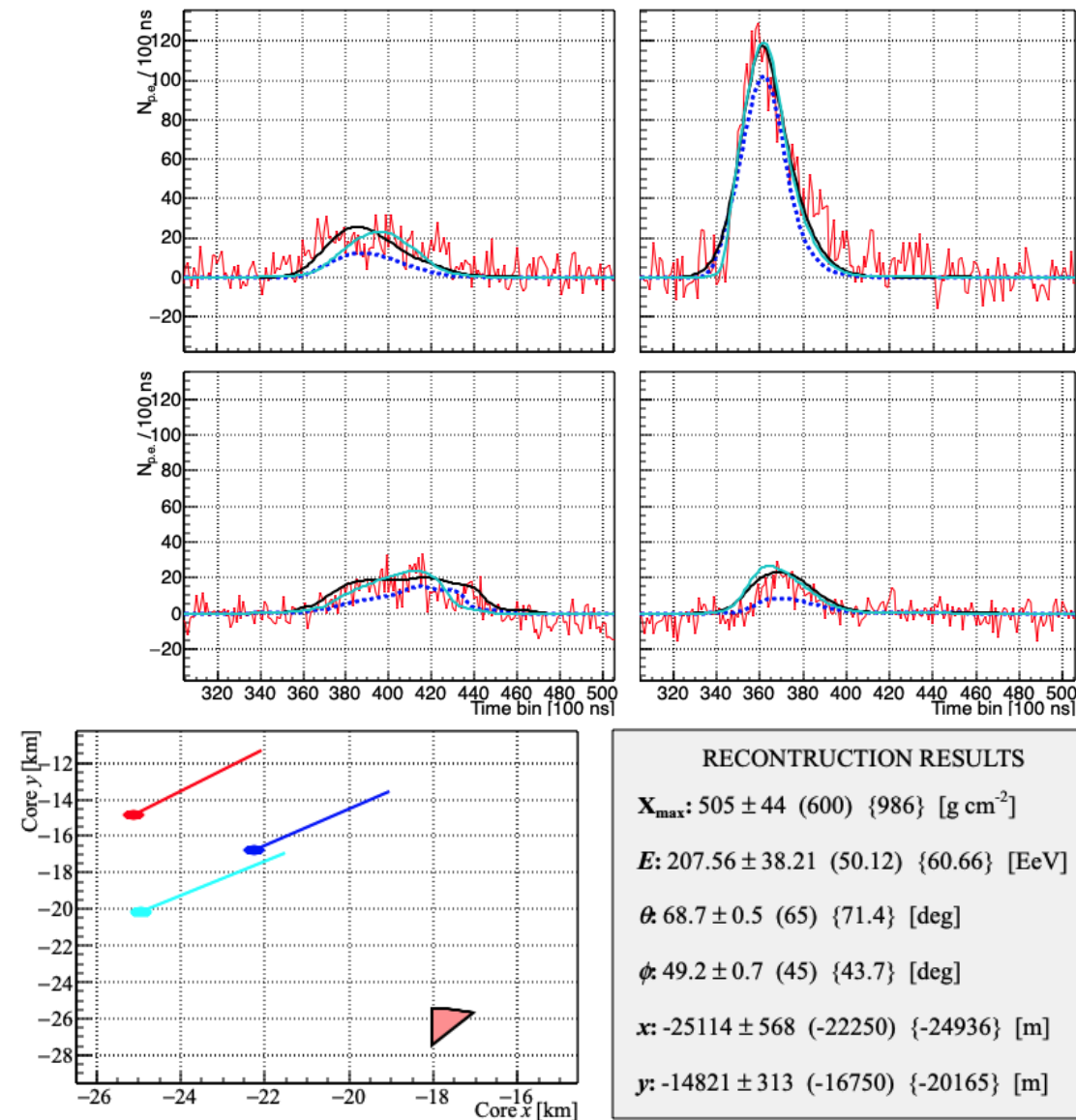
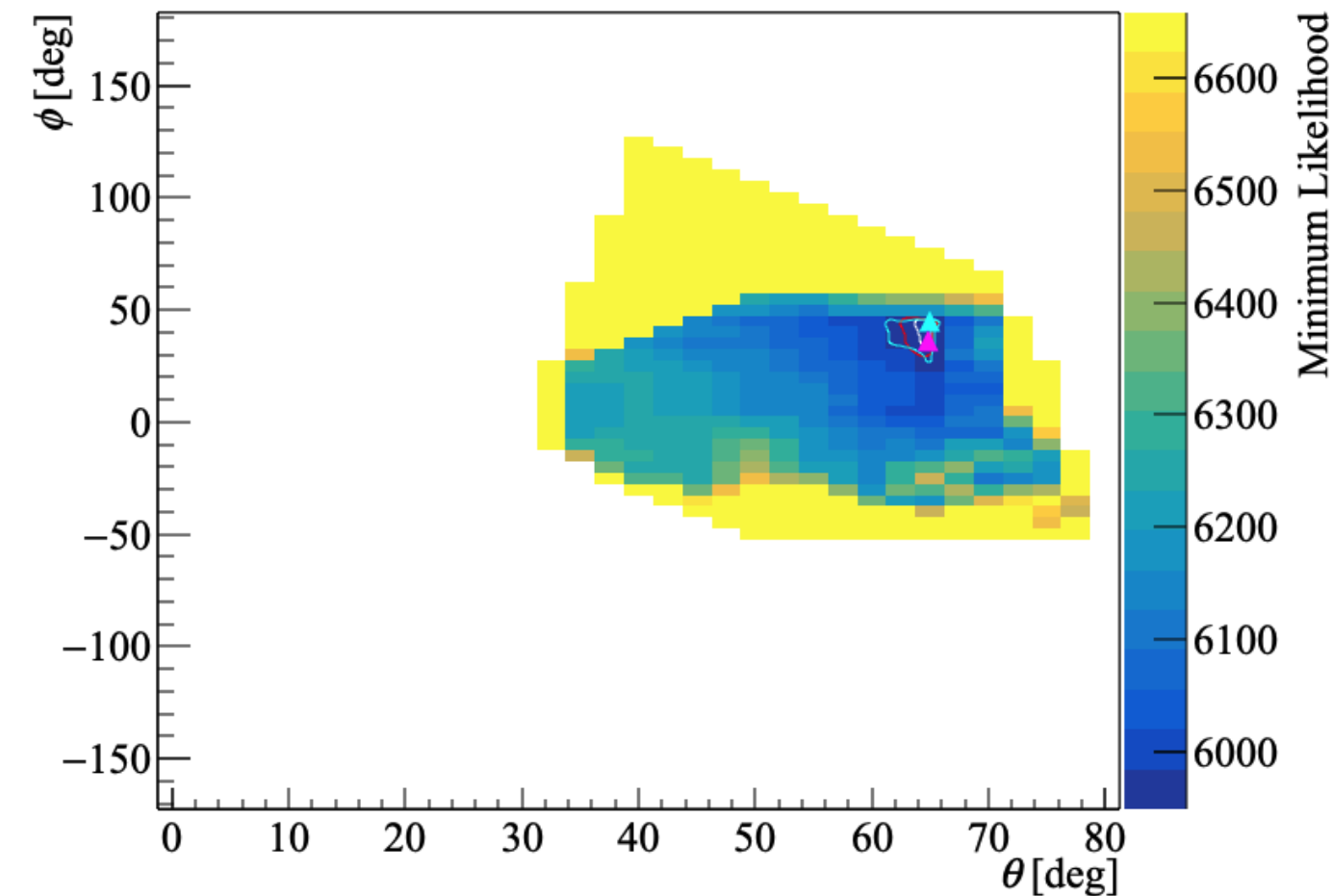
FAST@Auger



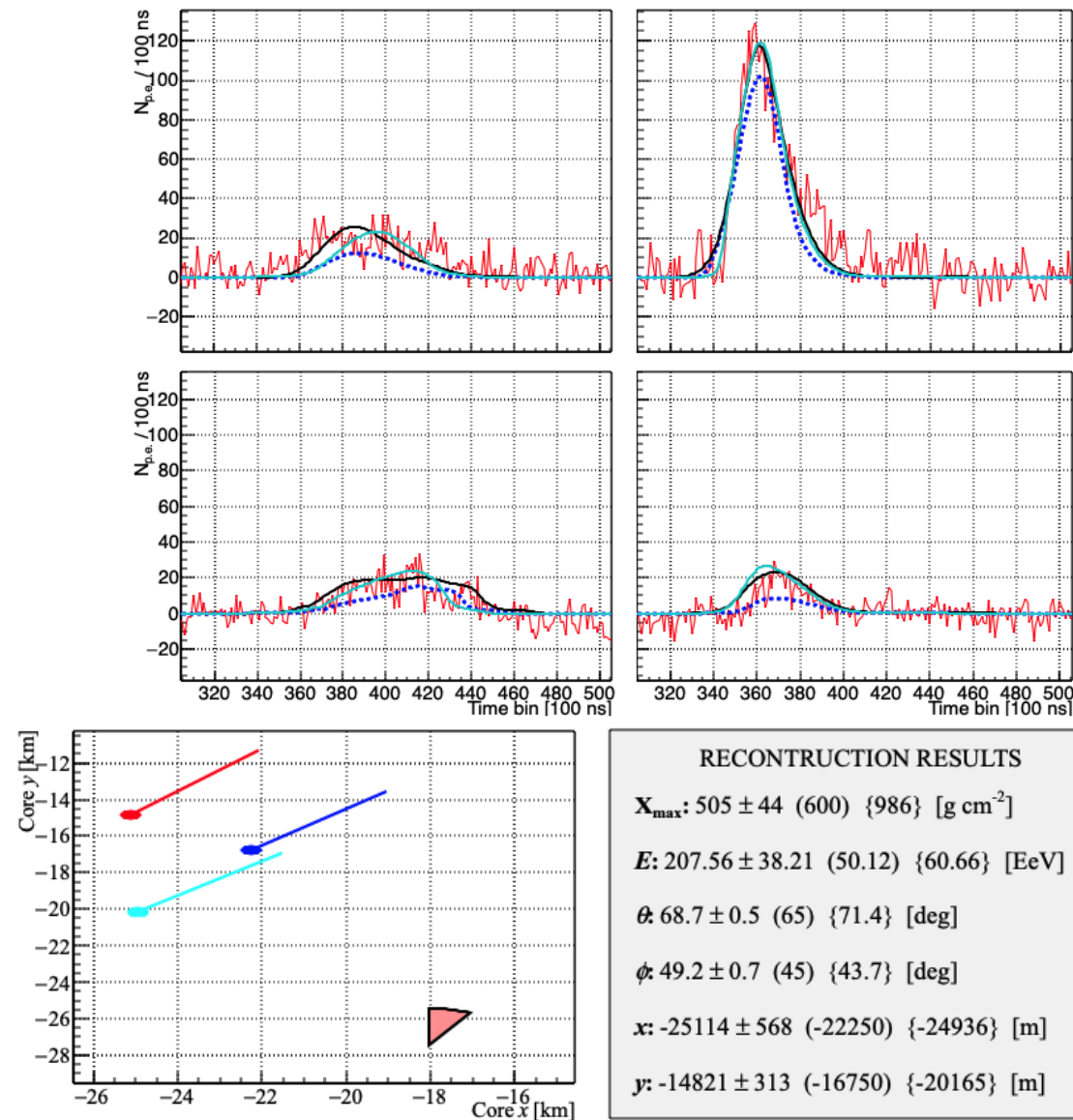
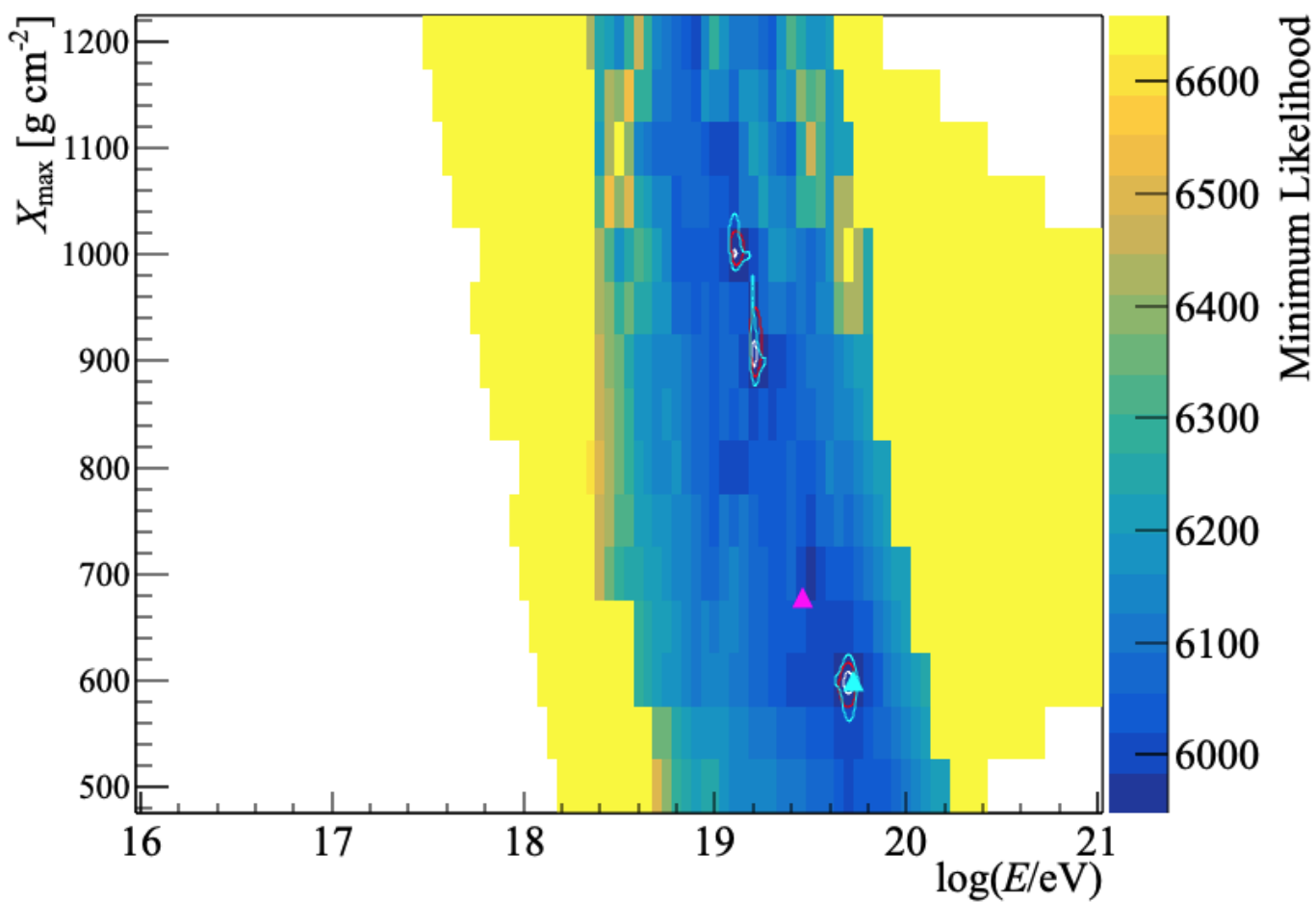
Template + TDR Examples



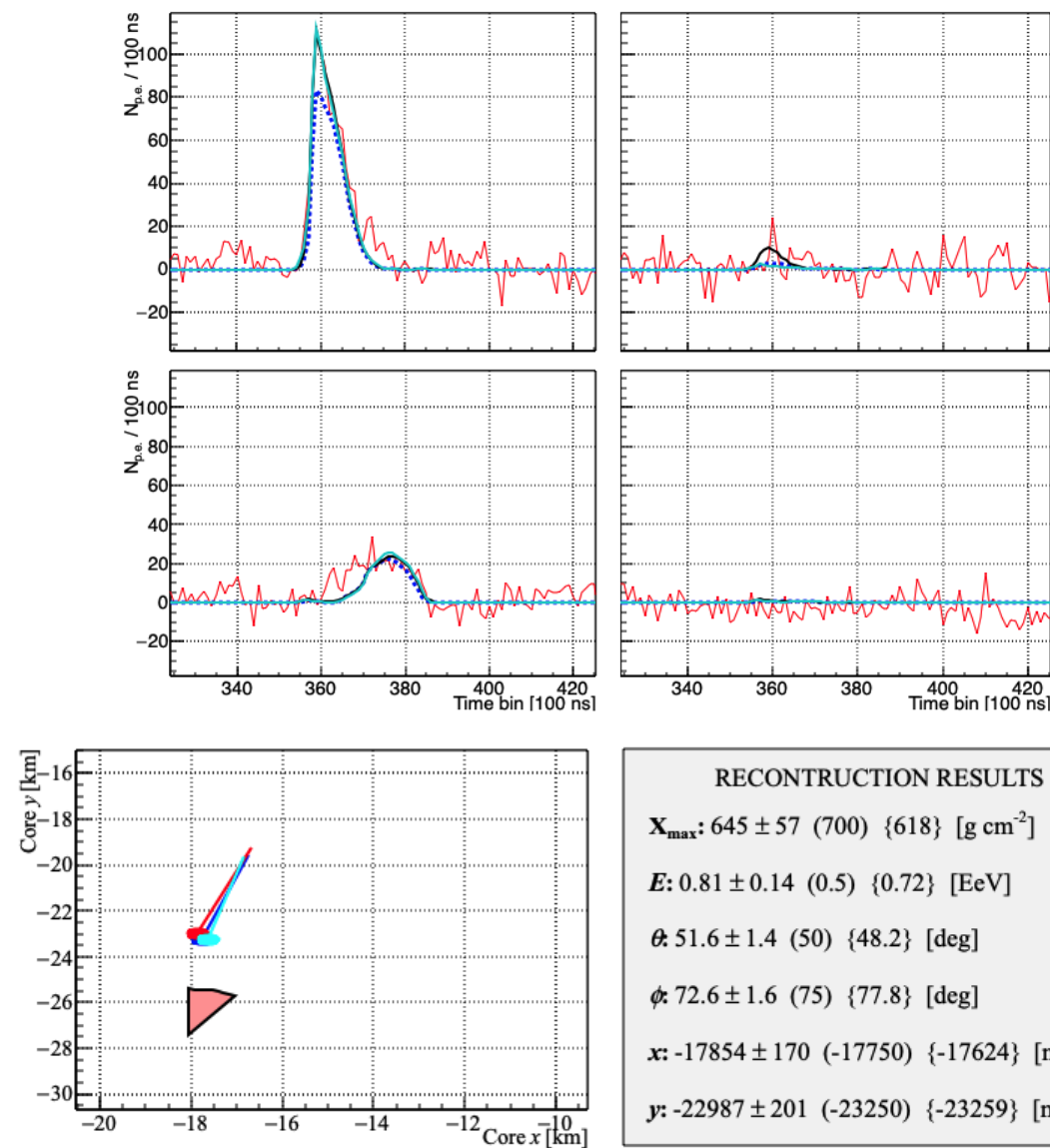
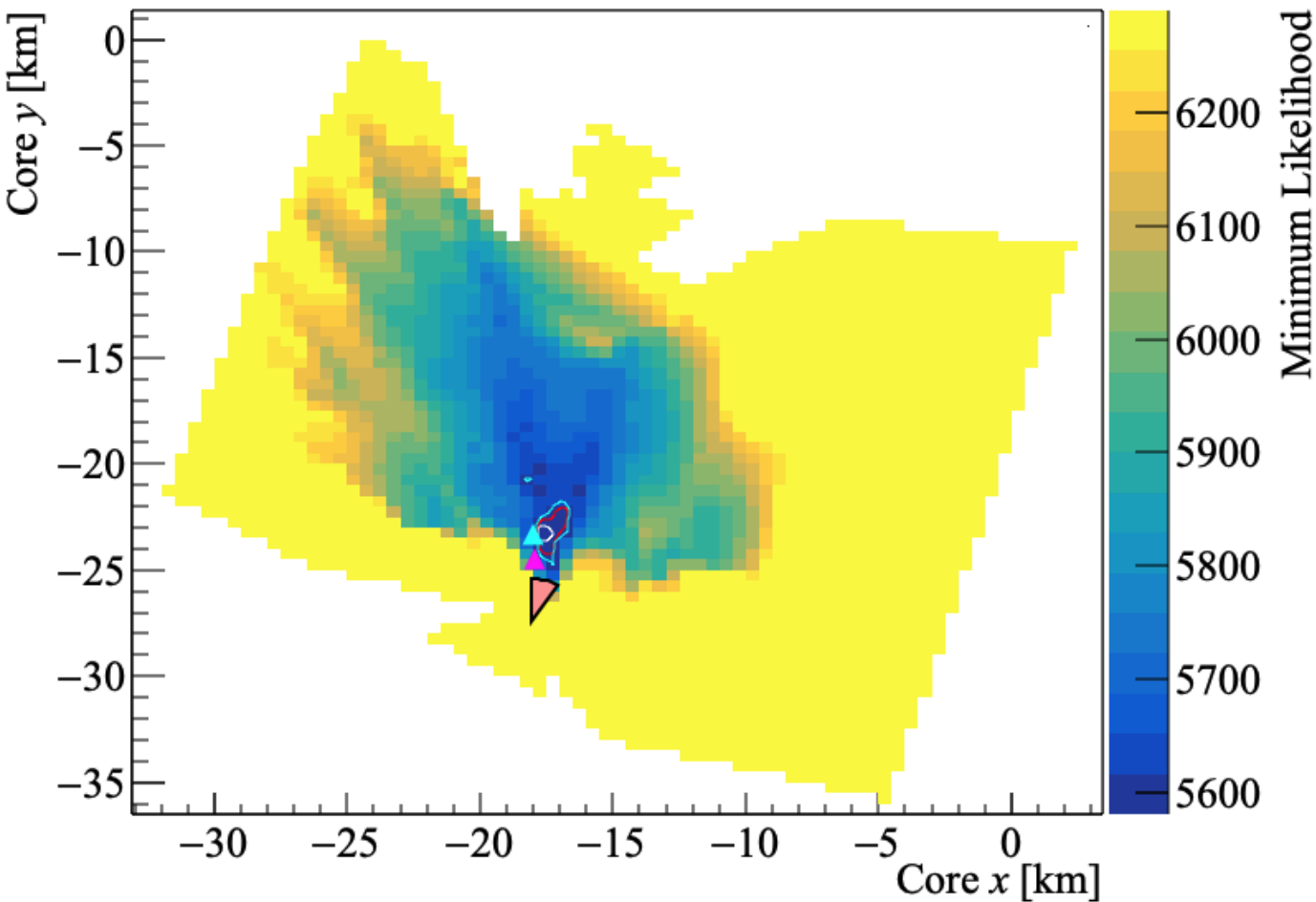
Template + TDR Examples



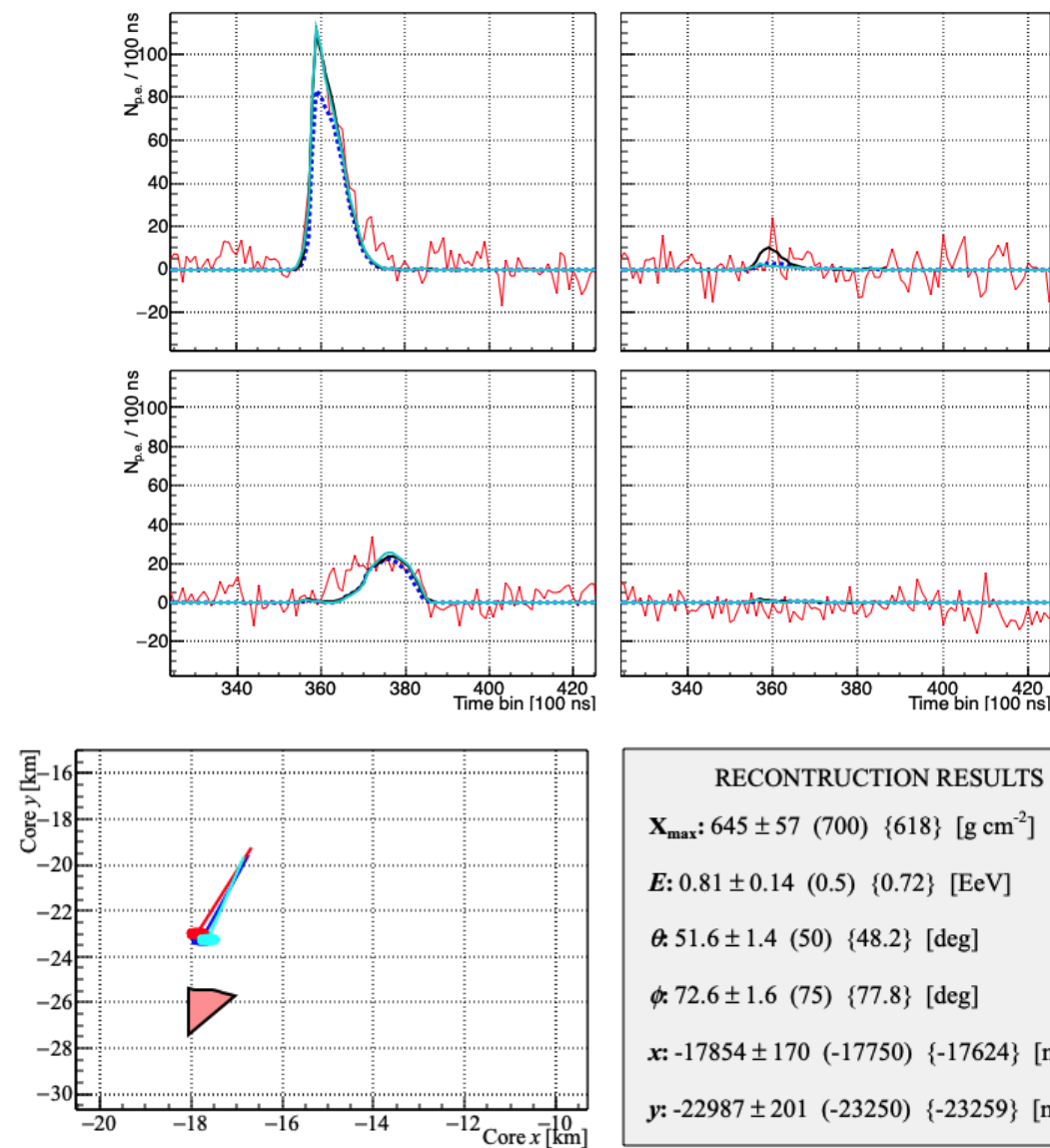
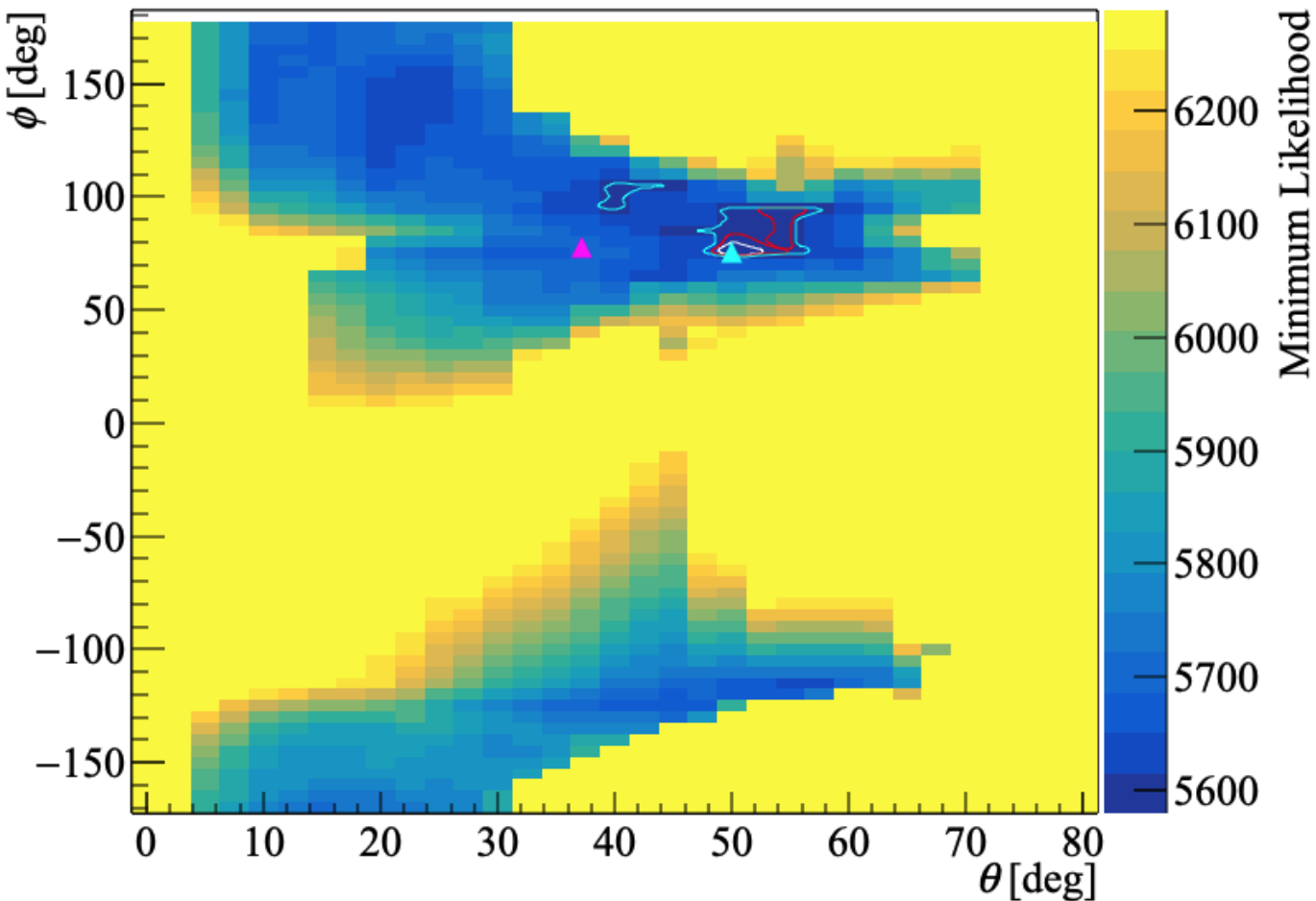
Template + TDR Examples



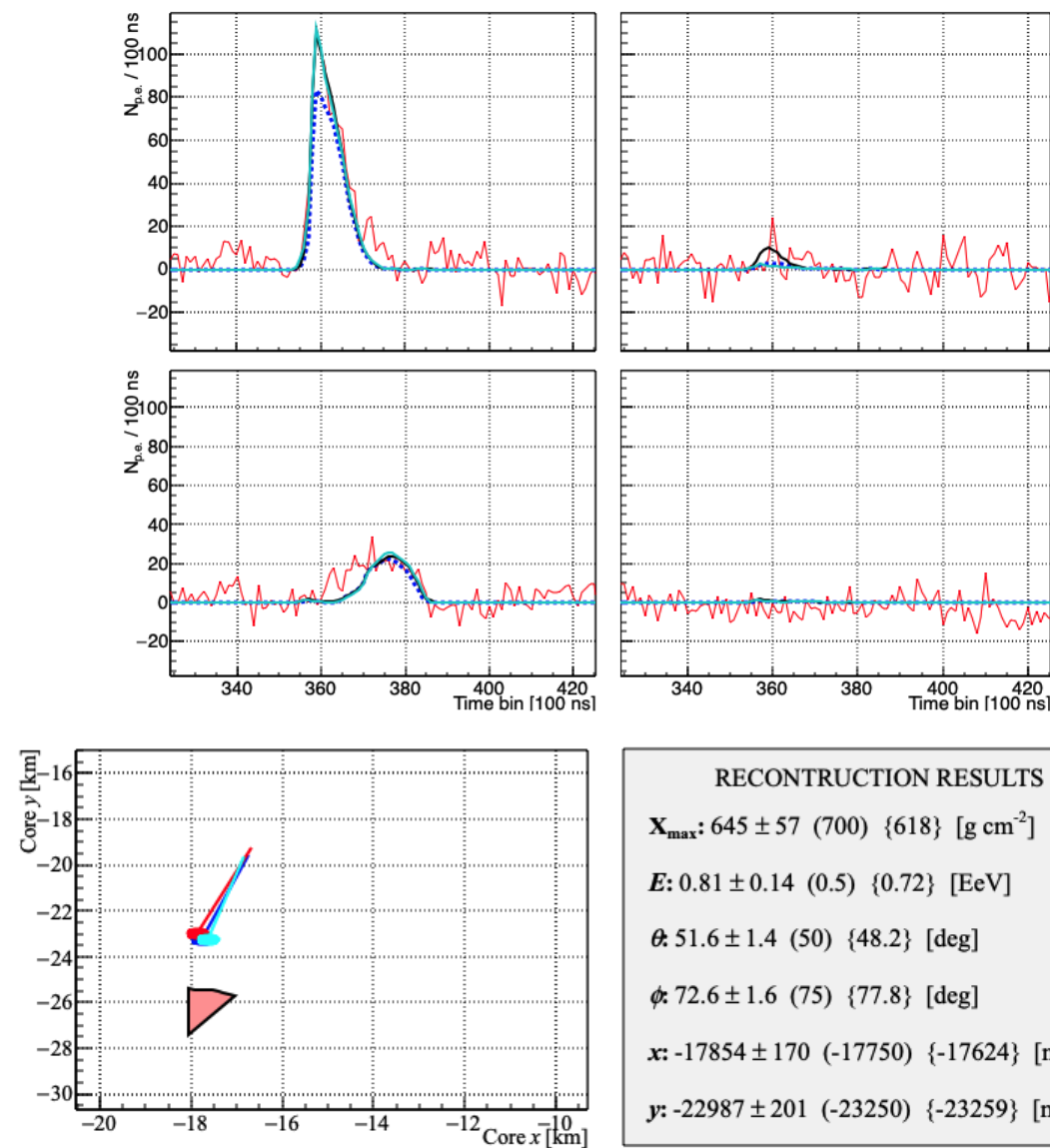
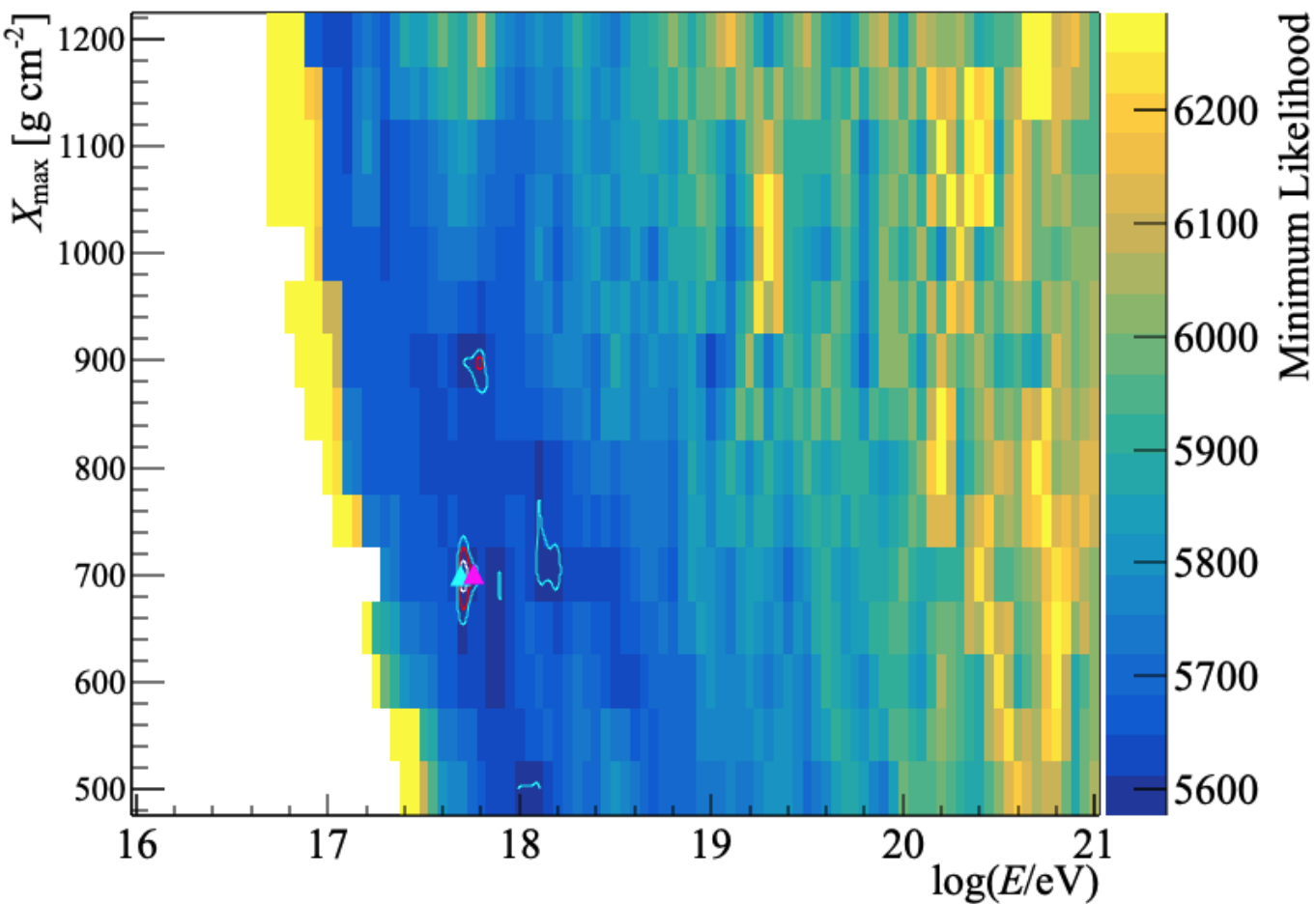
Template + TDR Examples



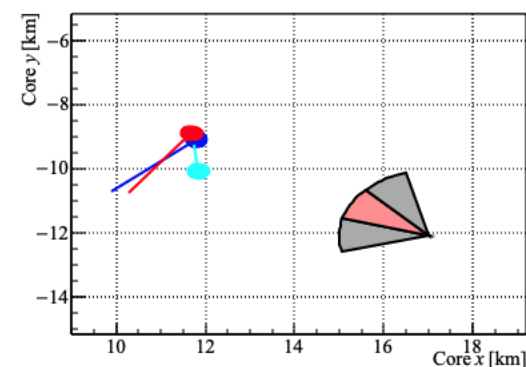
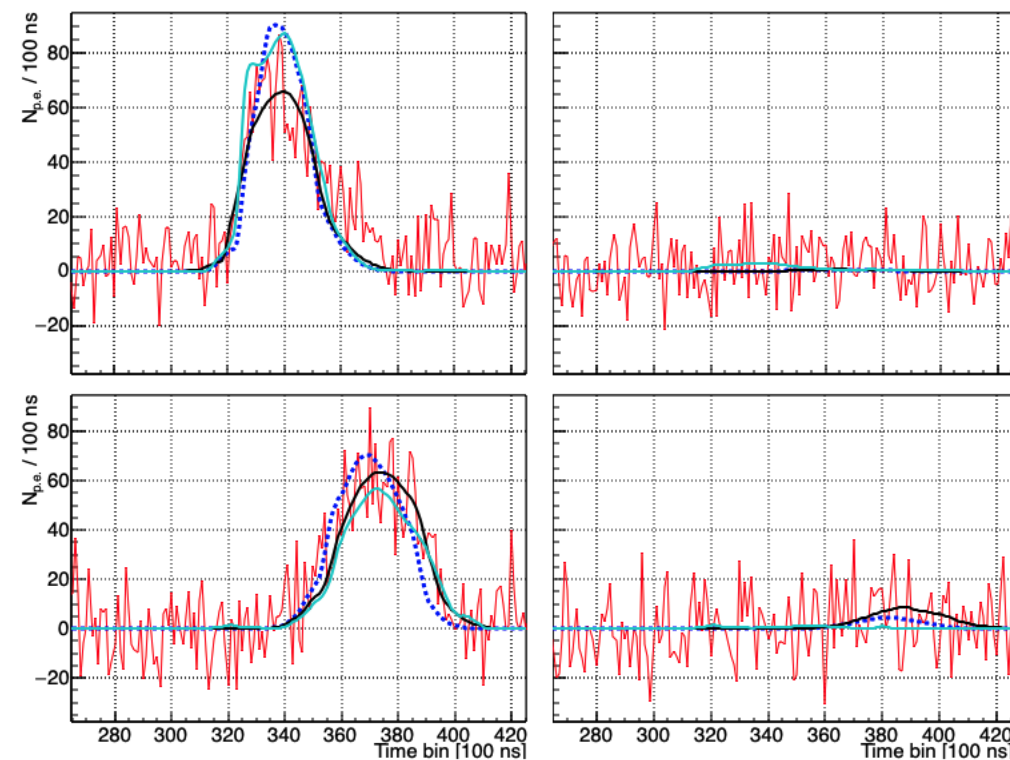
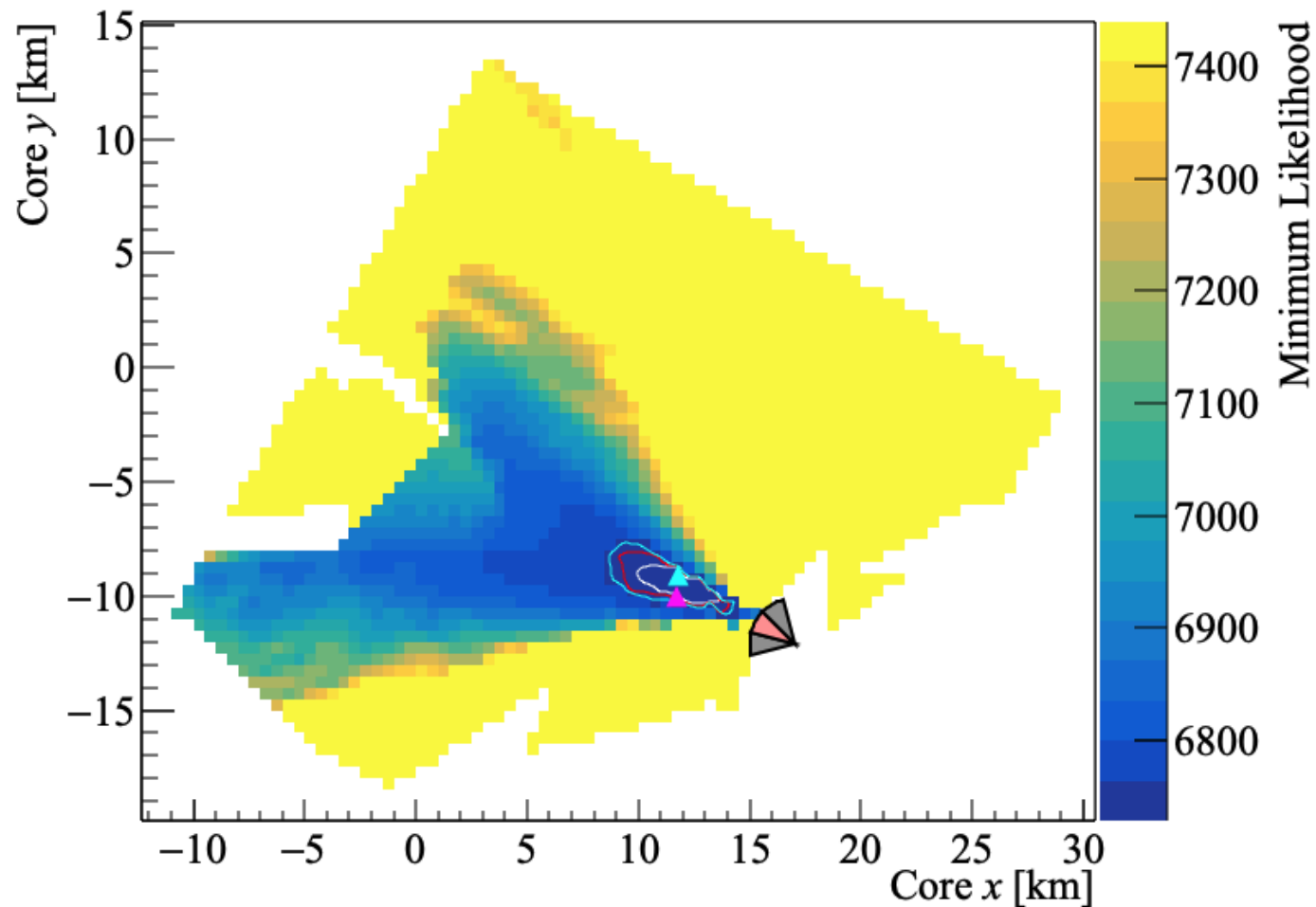
Template + TDR Examples



Template + TDR Examples



Template + TDR Examples



RECONSTRUCTION RESULTS

X_{max} : 868 ± 32 (700) {531} [g cm^{-2}]

E : 5.32 ± 0.34 (5.35) {6.2} [EeV]

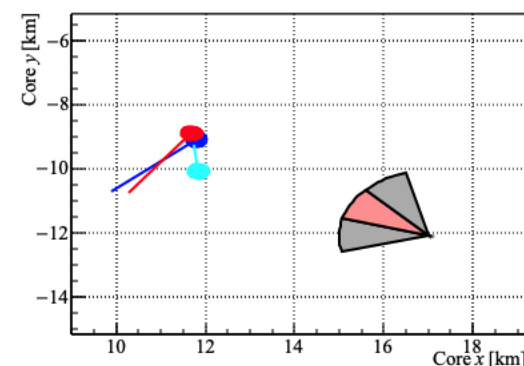
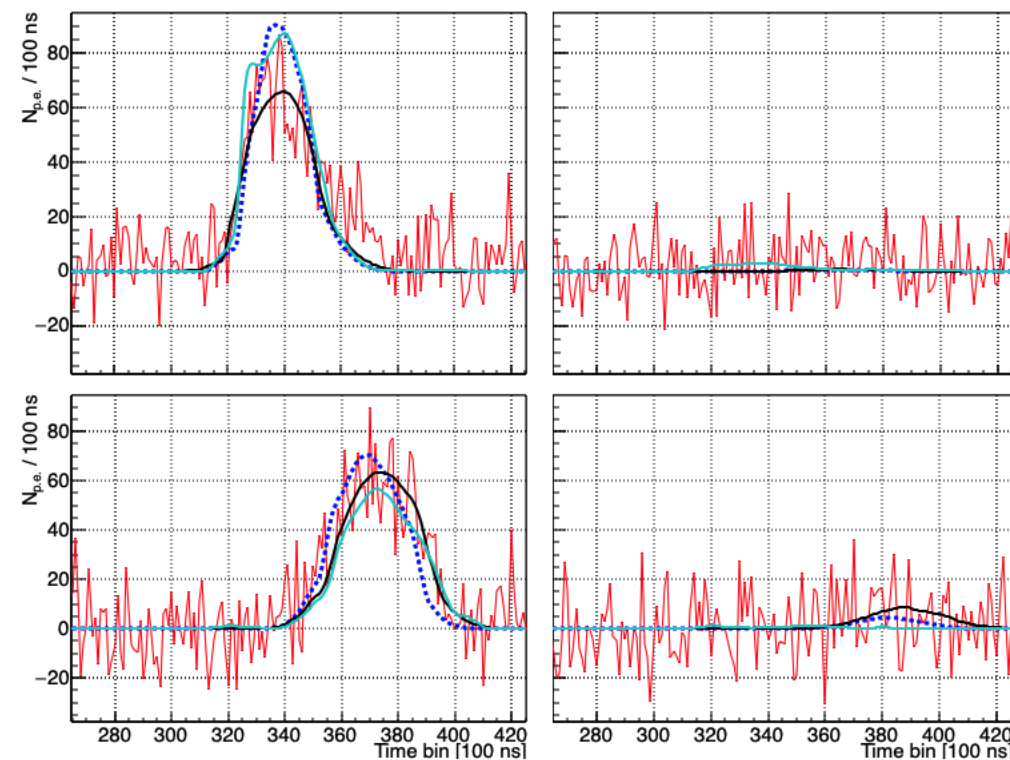
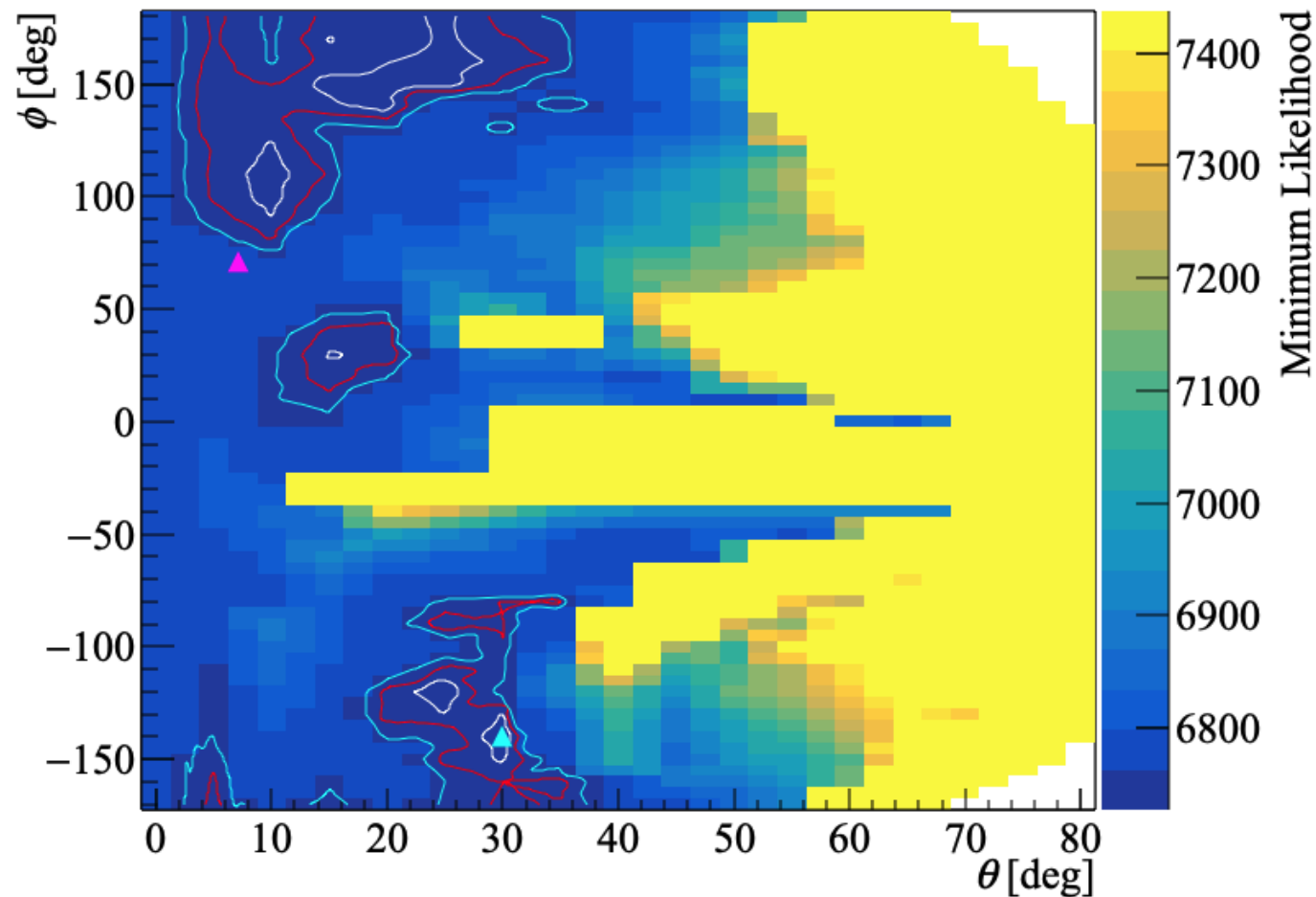
θ : 27.7 ± 1 (30) {9.5} [deg]

ϕ : -127.3 ± 4 (-140) {98} [deg]

x : 11692 ± 95 (11803.8) {11845} [m]

y : -8907 ± 67 (-9100) {-10082} [m]

Template + TDR Examples



RECONSTRUCTION RESULTS

$X_{\text{max}}: 868 \pm 32 \text{ (700) } \{531\} \text{ [g cm}^{-2}\text{]}$

$E: 5.32 \pm 0.34 \text{ (5.35) } \{6.2\} \text{ [EeV]}$

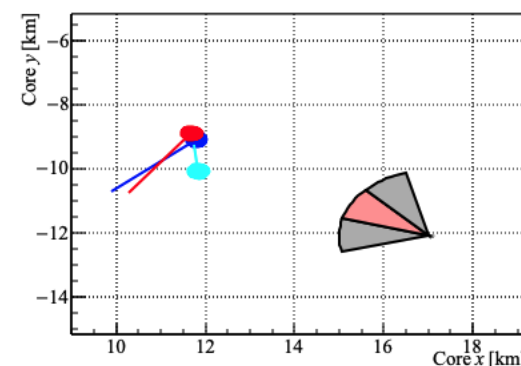
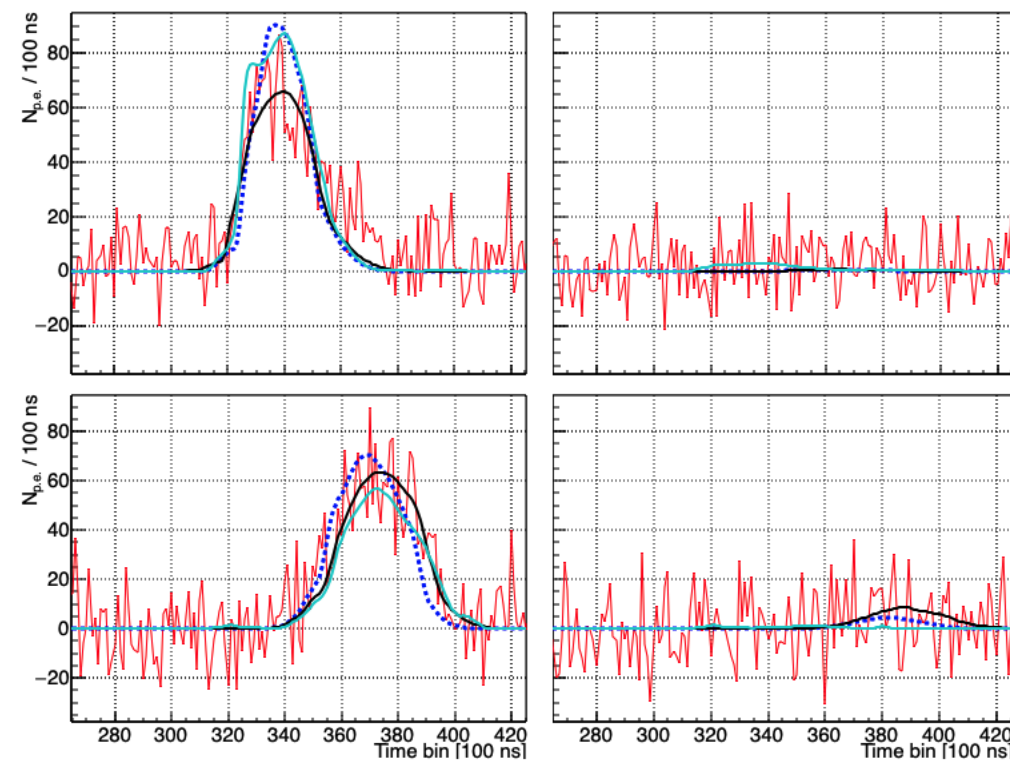
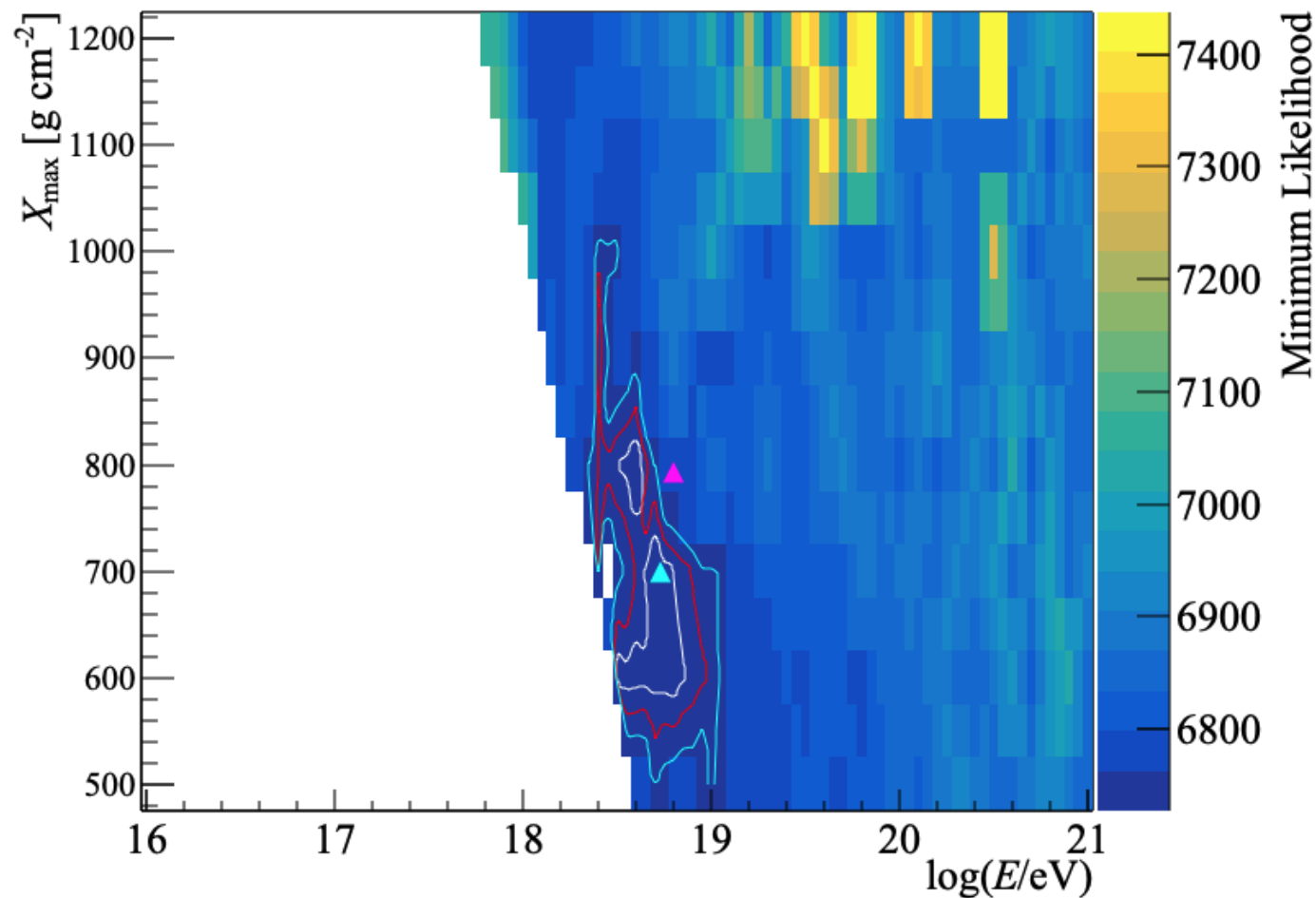
$\theta: 27.7 \pm 1 \text{ (30) } \{9.5\} \text{ [deg]}$

$\phi: -127.3 \pm 4 \text{ (-140) } \{98\} \text{ [deg]}$

$x: 11692 \pm 95 \text{ (11803.8) } \{11845\} \text{ [m]}$

$y: -8907 \pm 67 \text{ (-9100) } \{-10082\} \text{ [m]}$

Template + TDR Examples



RECONSTRUCTION RESULTS

X_{max} : 868 ± 32 (700) {531} [g cm^{-2}]

E : 5.32 ± 0.34 (5.35) {6.2} [EeV]

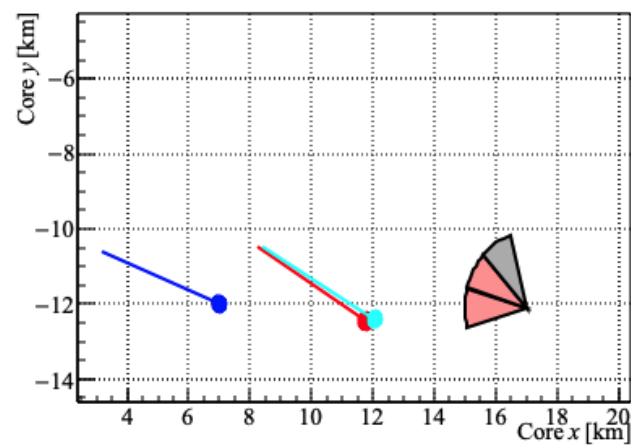
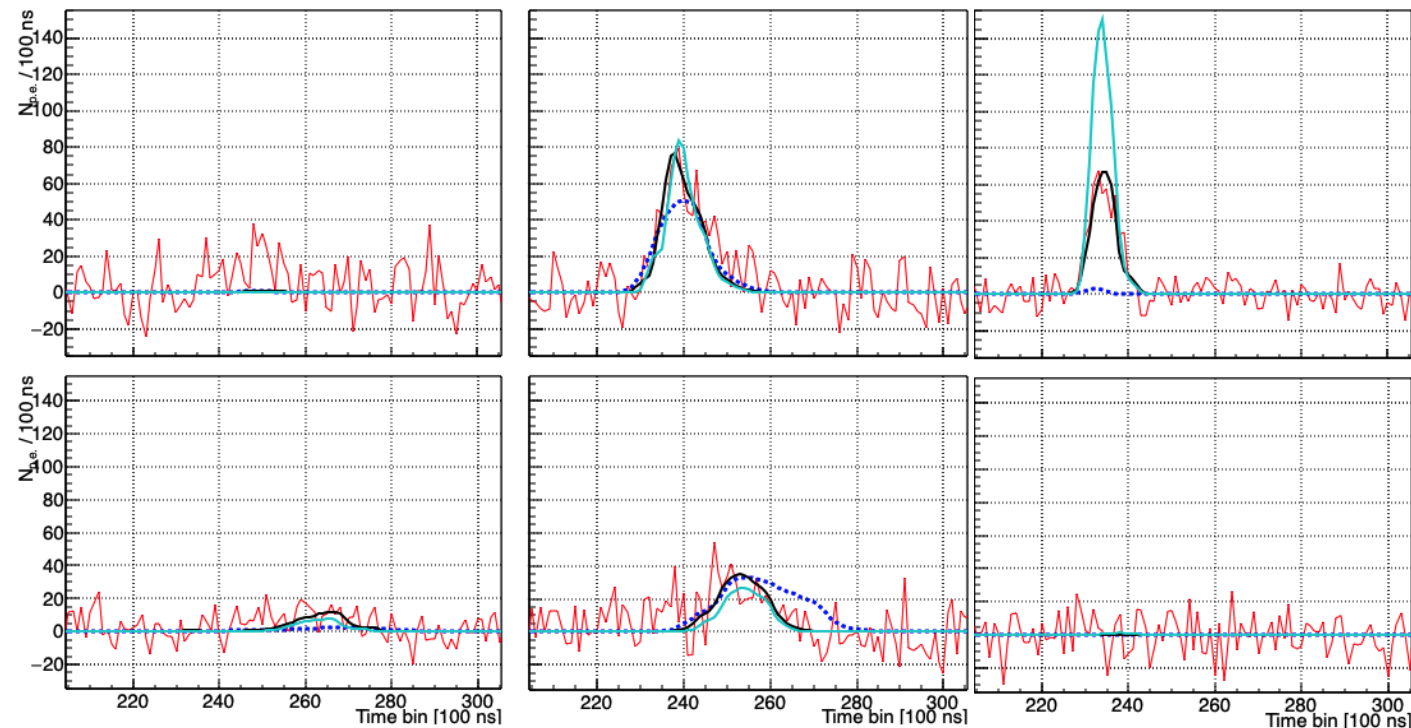
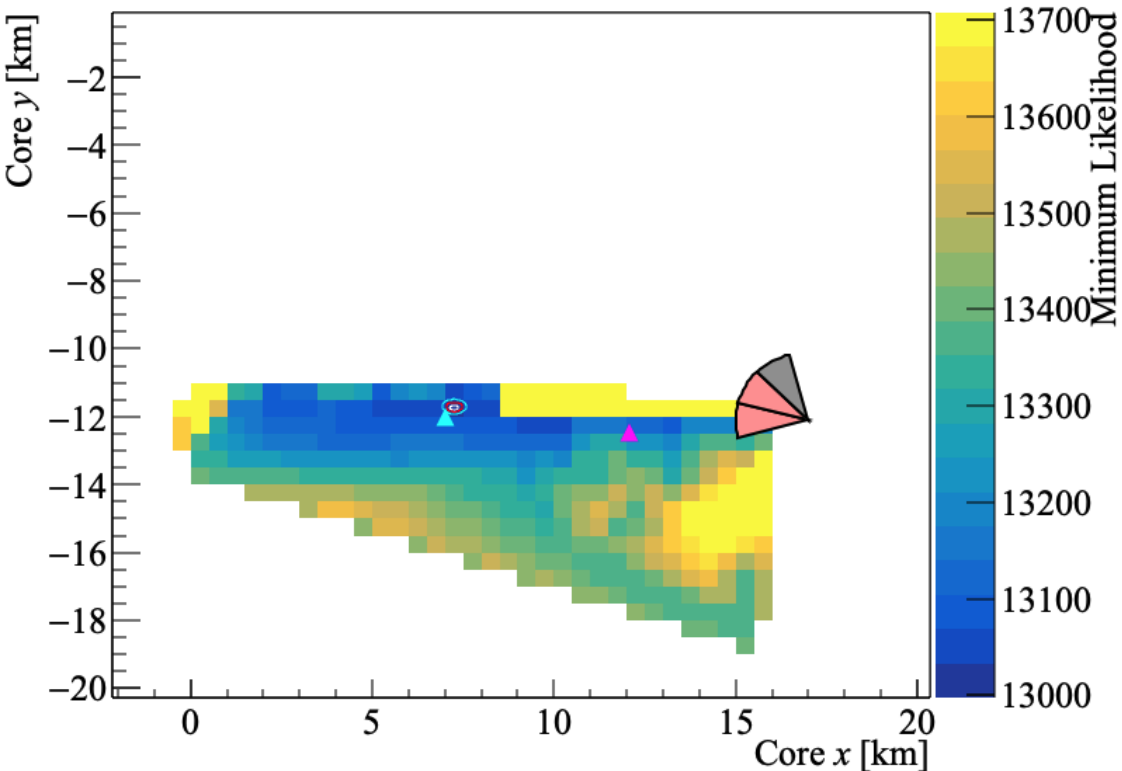
θ : 27.7 ± 1 (30) {9.5} [deg]

ϕ : -127.3 ± 4 (-140) {98} [deg]

x : 11692 ± 95 (11803.8) {11845} [m]

y : -8907 ± 67 (-9100) {-10082} [m]

Template + TDR Examples



RECONSTRUCTION RESULTS

X_{max} : 846 ± 59 (700) {665} [g cm^{-2}]

E : 1.67 ± 0.22 (6.31) {1.68} [EeV]

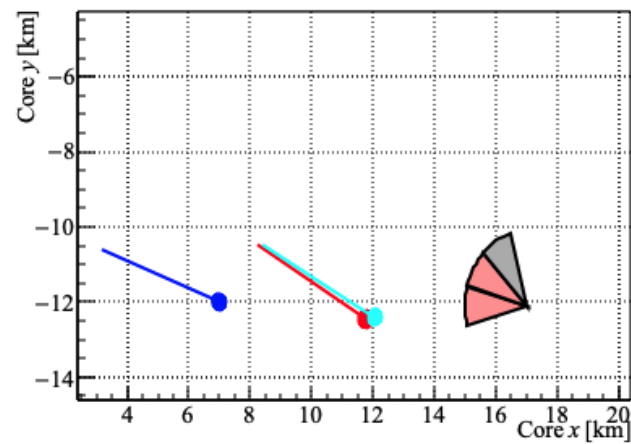
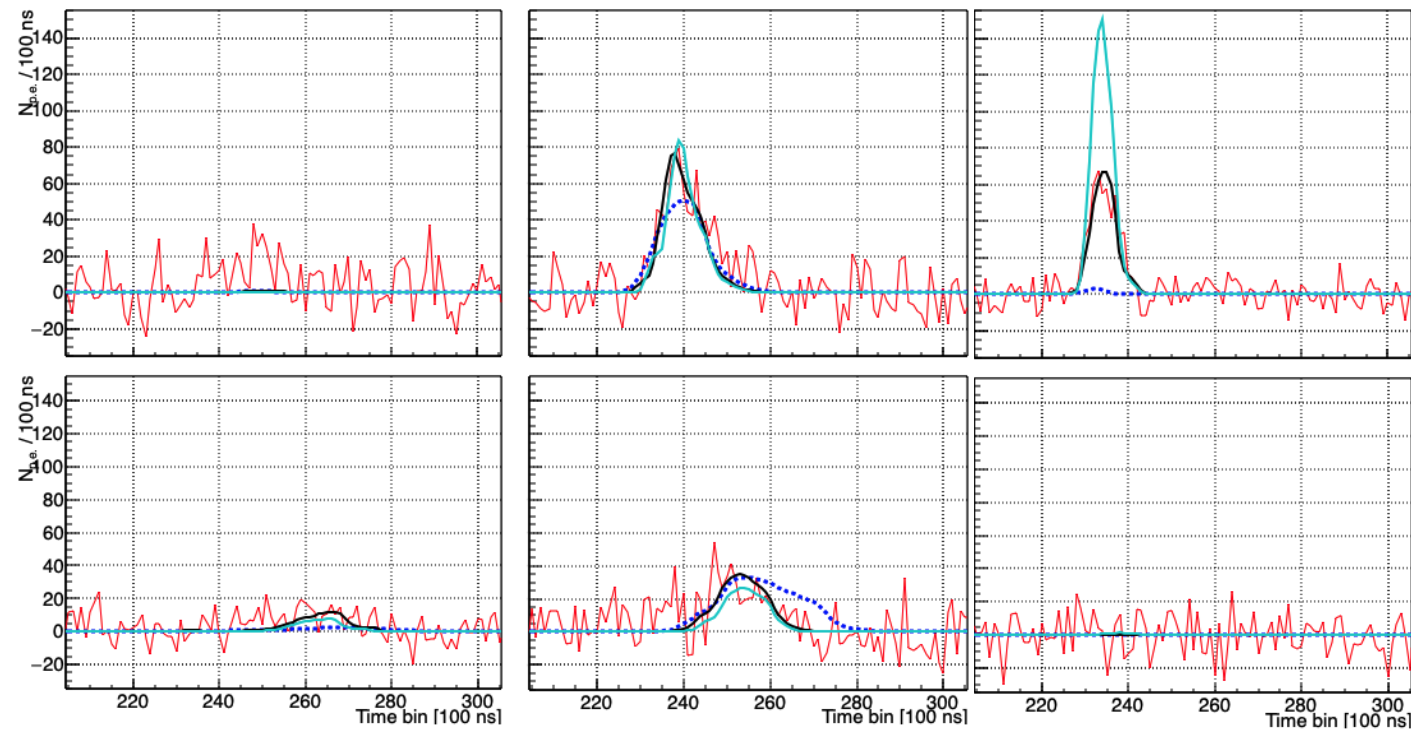
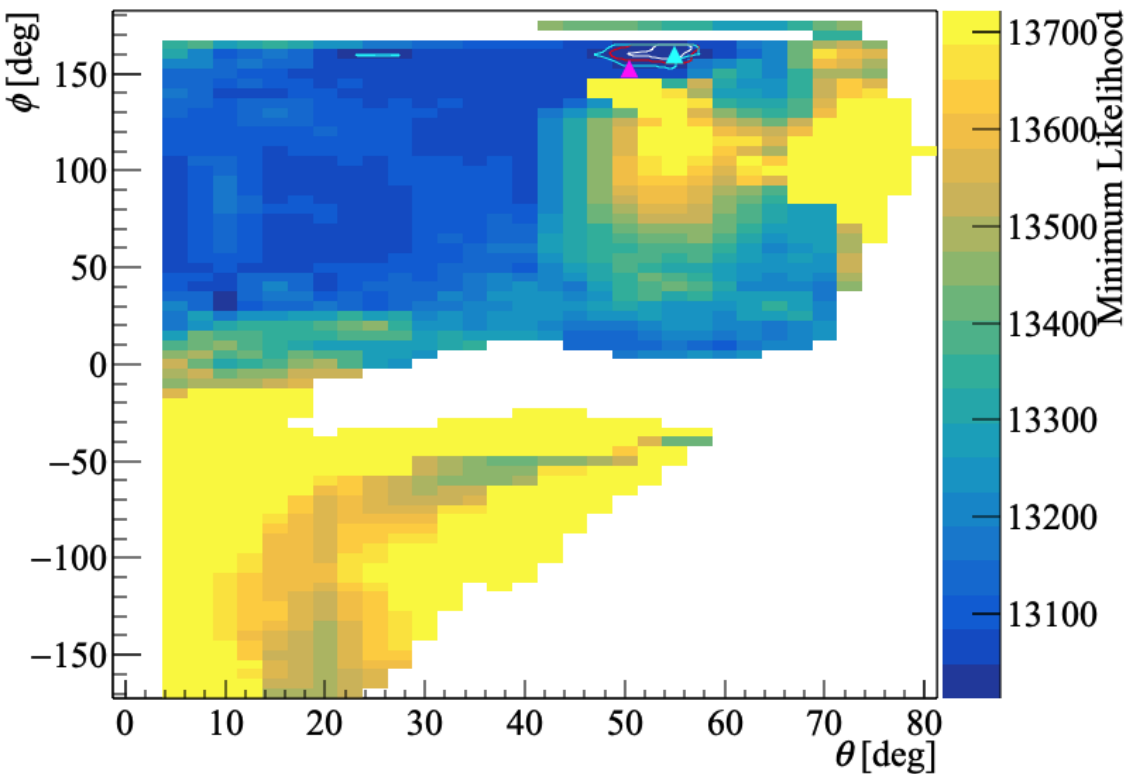
θ : 54.2 ± 0.8 (55) {55.7} [deg]

ϕ : 150.7 ± 0.6 (160) {152.4} [deg]

x : 11774 ± 194 (7000) {12066} [m]

y : -12450 ± 81 (-12000) {-12398} [m]

Template + TDR Examples



RECONSTRUCTION RESULTS

X_{max} : 846 ± 59 (700) {665} [g cm^{-2}]

E : 1.67 ± 0.22 (6.31) {1.68} [EeV]

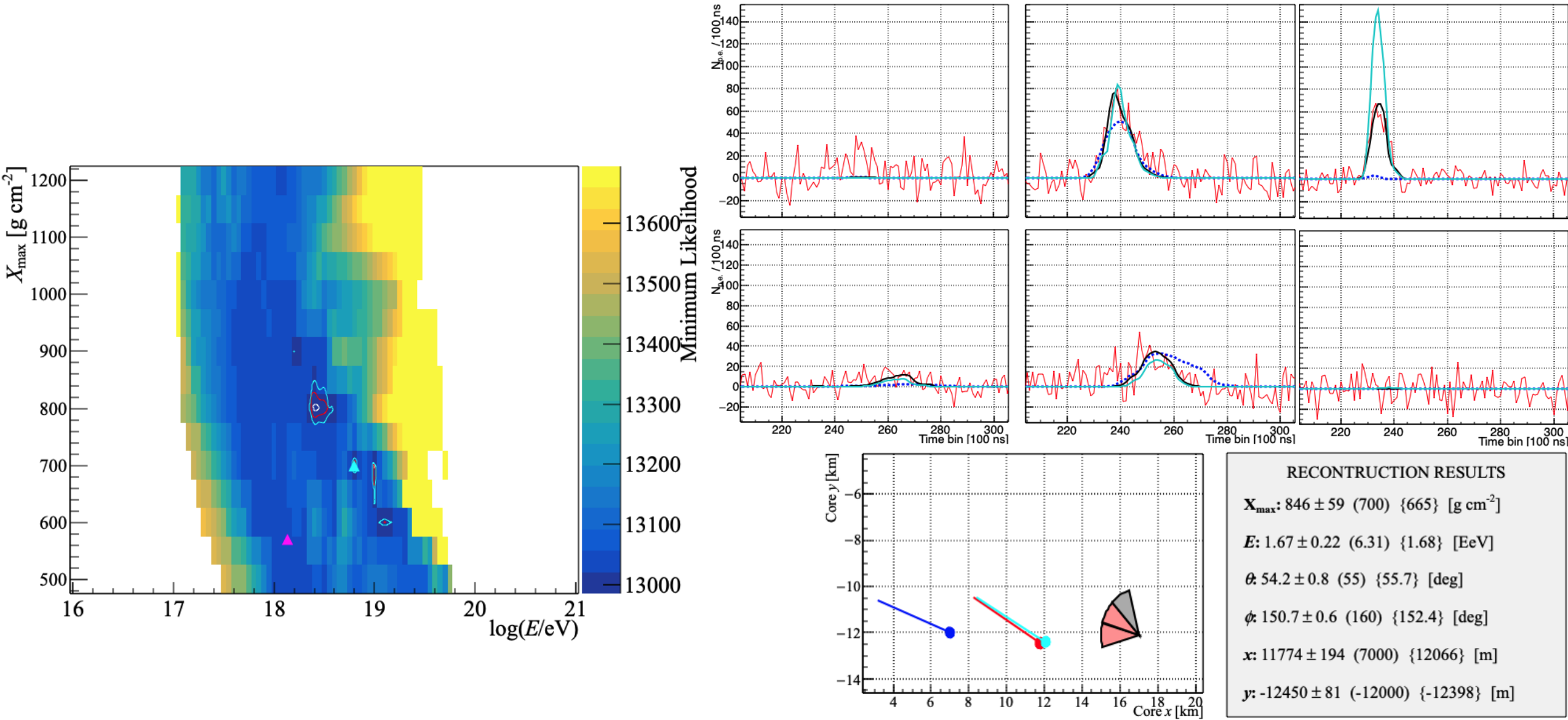
θ : 54.2 ± 0.8 (55) {55.7} [deg]

ϕ : 150.7 ± 0.6 (160) {152.4} [deg]

x : 11774 ± 194 (7000) {12066} [m]

y : -12450 ± 81 (-12000) {-12398} [m]

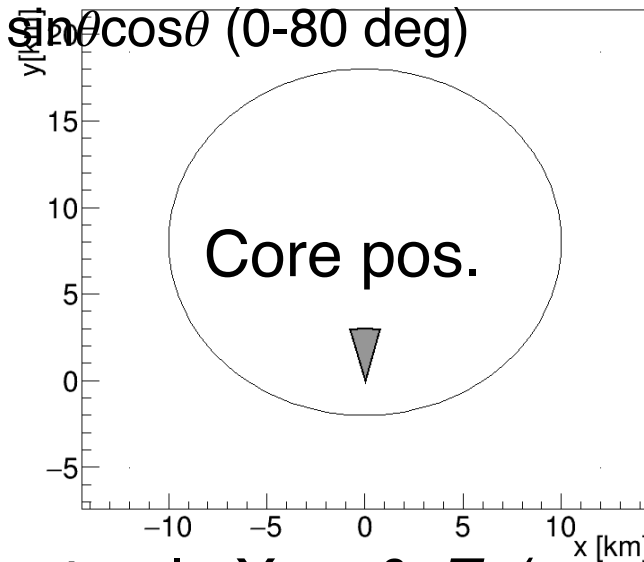
Template + TDR Examples



Construct $X_{\max}$ rails for FAST

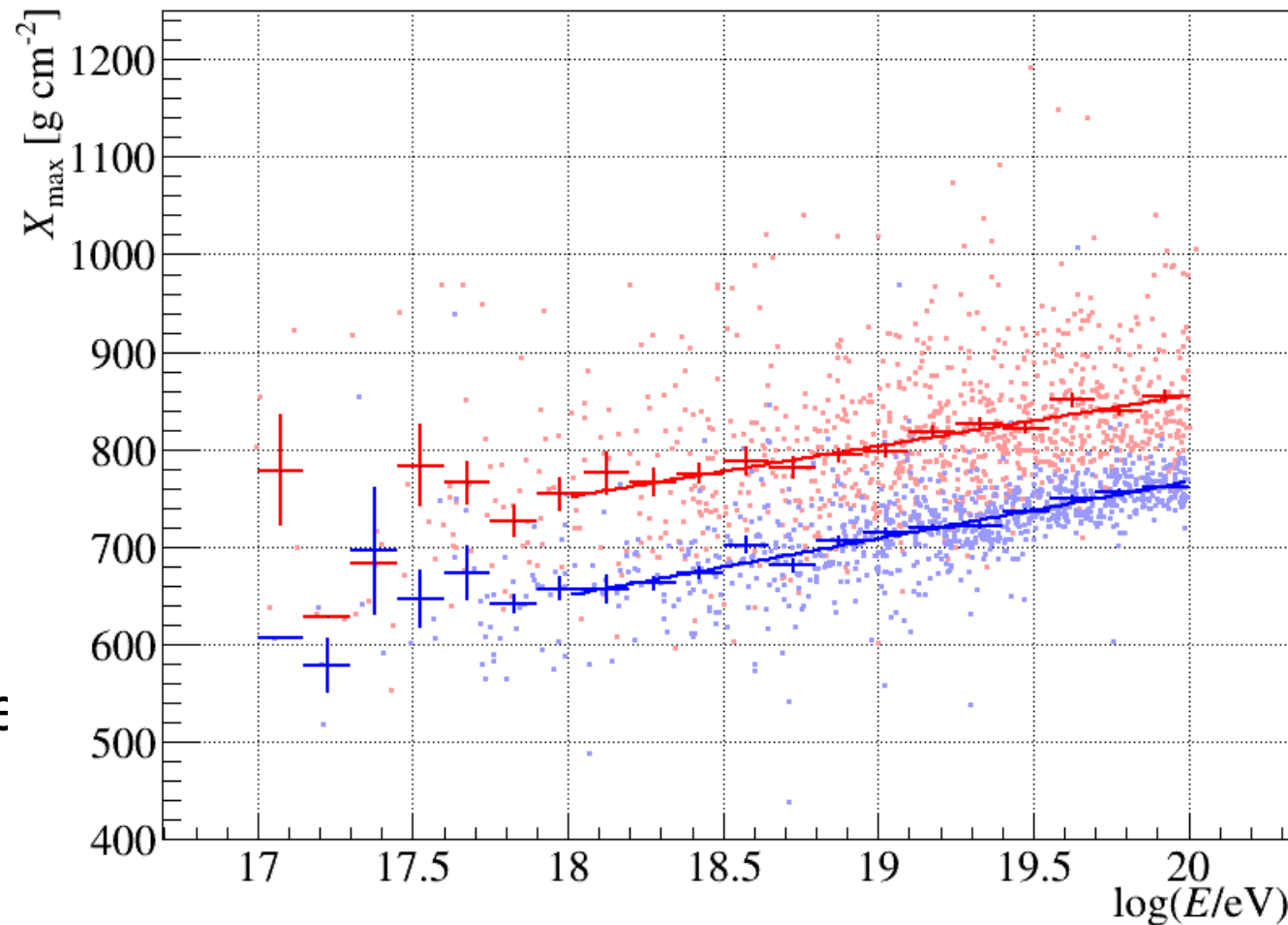
Simulation conditions:

- $X_{\max}$ dist. : EPOS (500-1200 gcm⁻²)
- Energy dist. : E^{-1} (10¹⁸-10²⁰ eV)
- θ dist. : $\sin\theta \cos\theta$ (0-80 deg)



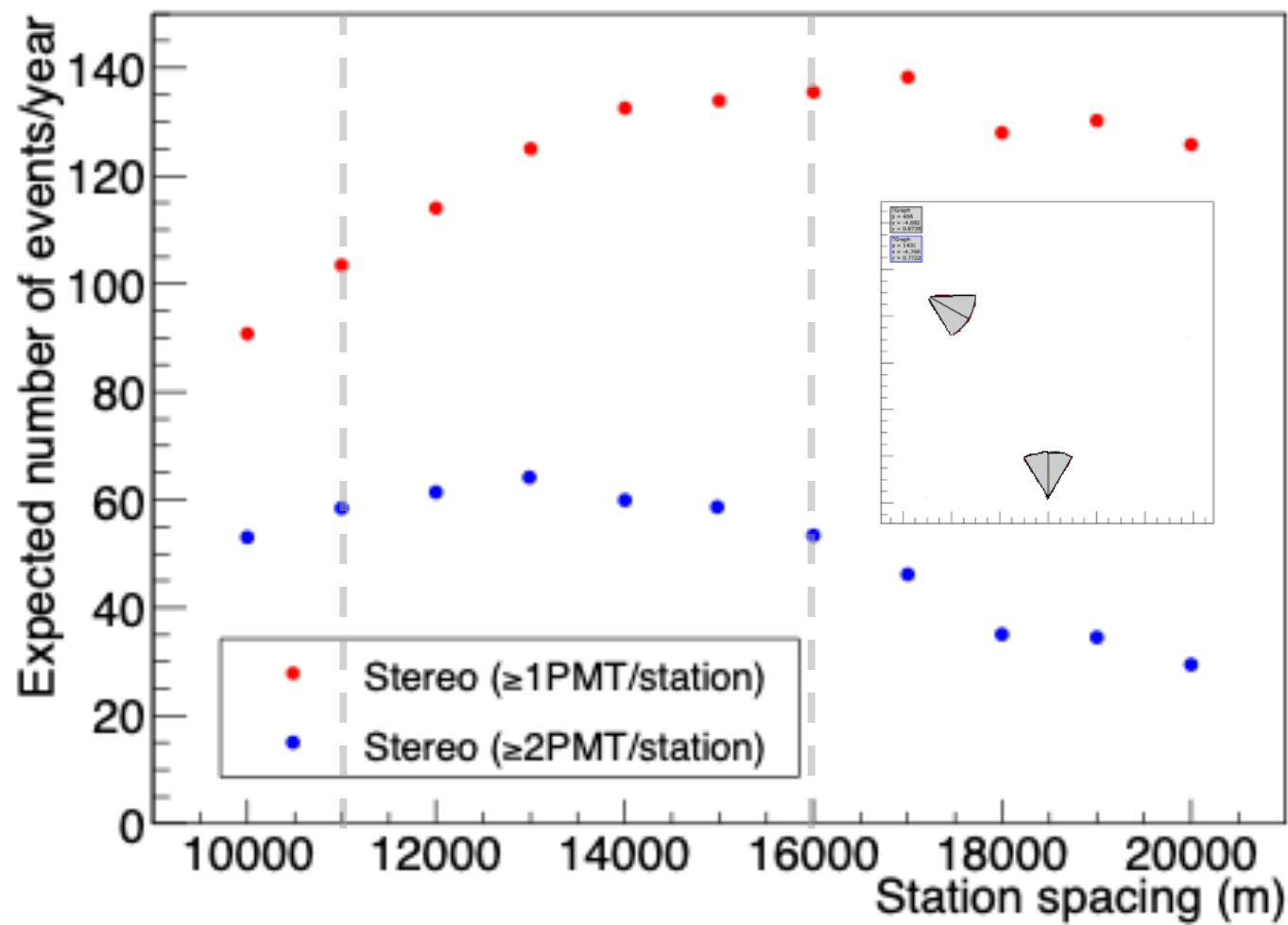
Reconstruct only $X_{\max}$ & E (geometry fixe

- Initial $X_{\max}$ and E varied by 30g/cm² and 10%
- Cuts:
 - One PMT with SNR>6
 - Successful minimization
 - Relative uncertainty in E & X both ≤ 0.5



FAST mini-array

- Future planning, what should spacing be?

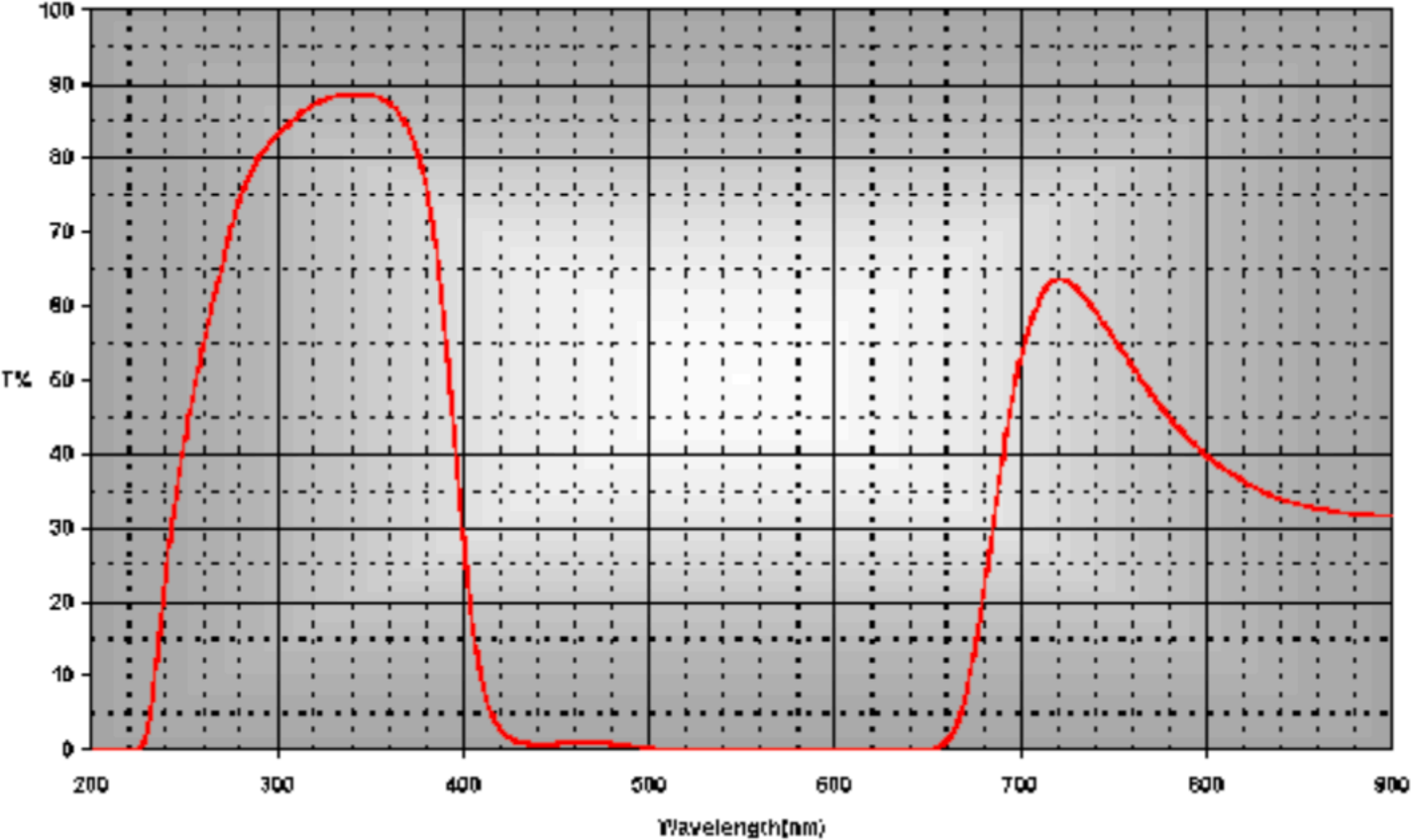


Number of showers observed by FAST in one night

Above say $10^{18.5}\text{eV}$

- 1 particle / km² / year (24/7)
 $\times 60,000 \text{ km}^2 = 60,000 \text{ particles per year} = 164 \text{ a day}$
- Only observe at night so ~ 50 events per day
 - (Ignoring non observation nights e.g. moon out)
- Want to reconstruct all detected events within 24 hr – can be achieved if reconstruction take under 30 min per event
 - TDR can already take 10+mins
 - First guess should really be quick a few minutes at most.
- Will need to calculate expected rate of showers passing pre reconstruction cuts to determine real necessary rate.

UV Filter Wavelength Response



Numbers

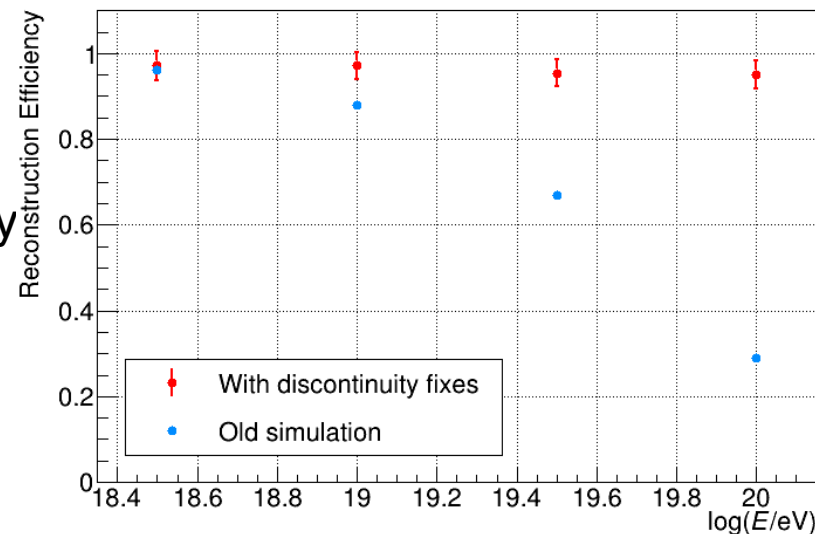
- Gyro radius of 10^{20} proton $\rightarrow$

Summary

- Want to find out the sources of UHECRs
 - FAST is a next generation cosmic ray experiment aiming to do so using giant array of low-cost fluorescence telescopes
 - **Need to ensure an accurate and precise reconstruction procedure!**

Results:

- **Fixed recon. efficiency issue** (30% $\rightarrow$ 95% @ 10^{20} eV)
 - Removing sources of discontinuities in simulations
 - For similar top-down approaches to air shower reconstruction, critical to **ensure smooth changes in signal in every bin**
- Introduced $X_{\max}$ in FOV cut
 - Important to remove degeneracy between reconstructed $X_{\max}$ and energy

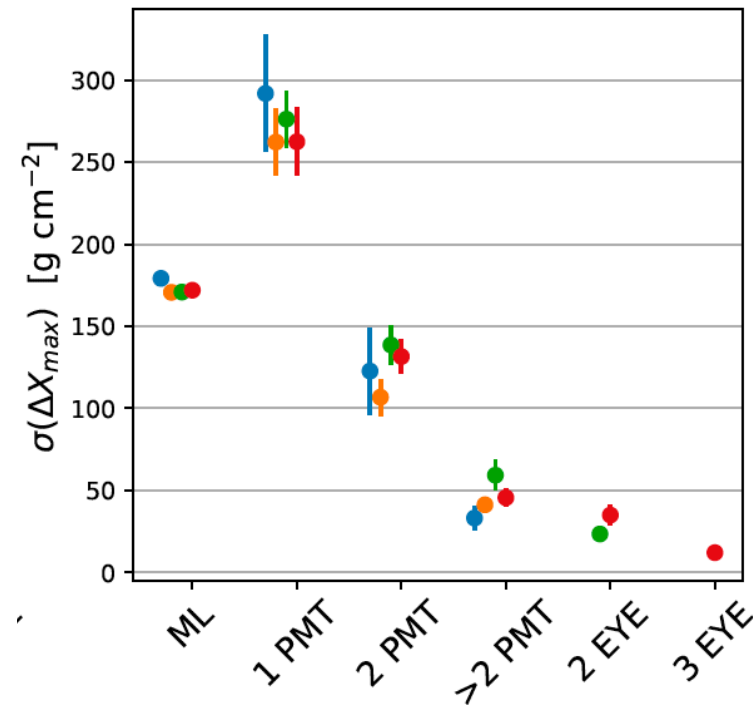


Summary

- Want to find out the sources of UHECRs
 - FAST is a next generation cosmic ray experiment aiming to do so using giant array of low-cost fluorescence telescopes
 - **Need to ensure an accurate and precise reconstruction procedure!**

Results:

- Developed two first guess methods, TSFEL DNN and Template Method
 - Resolutions improve for more pixels / more eyes
- Full reconstruction chain with TSFEL DNN shows promising results for FAST-Mini Array
 - **Resolutions in $X_{\max} \sim 30 \text{ g cm}^{-2}$ and energy $\sim 10\%$ seem achievable**
 - Need to ensure simulations match prototypes as closely as possible!



Summary

- Want to find out the sources of UHECRs
 - FAST is a next generation cosmic ray experiment aiming to do so using giant array of low-cost fluorescence telescopes
 - **Need to ensure an accurate and precise reconstruction procedure!**

Results:

- Reconstructed coincidence events
 - Ok agreement between expectations and data
 - Decreasing trend in $\ln(E_{\text{FAST}} / E_{\text{TA}}) \rightarrow$ calibration factor?
 - First guess methods favor large energy / far core position
 - **Stereo observation is important for FAST**
- **First measurements** of energy spectrum and mass composition with FAST
 - Need more statistics + careful consideration of uncertainties

