Bayesian Inference of the Reheating Temperature in Thermal Leptogenesis

Naturalness as typicality in the seesaw parameter space

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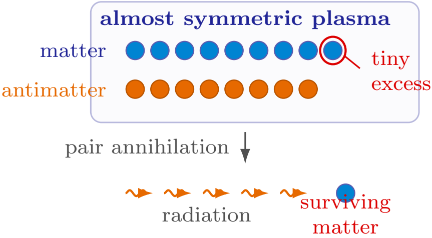
Matter–antimatter asymmetry

In the early Universe, particles and antiparticles were almost equally abundant.



- ✓ Pair annihilation leaves only a tiny excess.
- ✓ Observed value:

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 6.1 \times 10^{-10}.$$

- ✓ The Standard Model satisfies neither enough CP violation nor a strong enough departure from equilibrium.

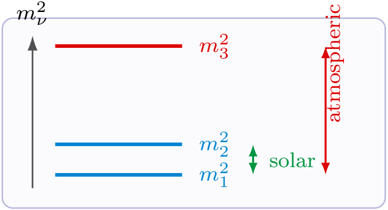
Question: Can the same new physics explain this number and neutrino masses?

Two facts pointing beyond the Standard Model

Neutrino oscillations

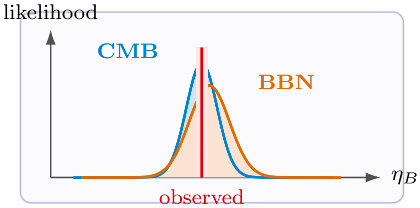
$$\Delta m^2_{21} \simeq 7.4 \times 10^{-5}\ \mathrm{eV}^2$$

$$|\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\ \mathrm{eV}^2.$$



Baryon asymmetry

$$\Omega_b h^2 \simeq 0.022, \qquad \eta_B \simeq 6.1\times10^{-10}.$$



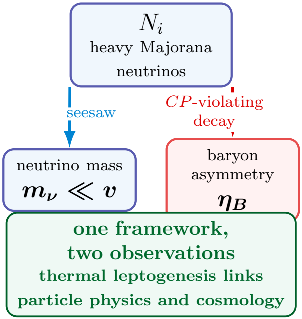
A minimal and economical extension is SM + heavy right-handed Majorana neutrinos.

Type-I seesaw: one extension, two targets

$$\mathcal{L} = \mathcal{L}_{\rm SM} + i\overline{N_i}\gamma^\mu\partial_\mu N_i - \ell_\alpha h_{\alpha i} N_i \widetilde{H} - \frac{1}{2} M_i \overline{N_i^c} N_i + \text{h.c.}$$

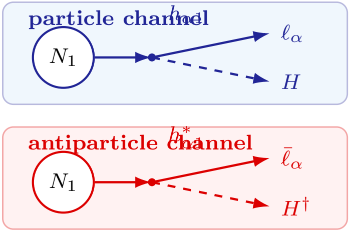
$$m_\nu^{\rm tree} = m_D M_R^{-1} m_D^T, \qquad m_D = vh.$$

- ✓ Small m_ν follows naturally for large M_R.
- ✓ Majorana mass violates lepton number.
- ✓ Complex Yukawa matrix gives new CP phases.



Thermal leptogenesis in one slide

CP-violating N_1 decays



$\boxed{\Gamma_{\ell H} \neq \Gamma_{\bar{\ell} H^\dagger}}$

Sphaleron redistribution

initial: $B = 0$, $L = -100$ → **sphalerons**: $\Delta(B+L) \neq 0$, $\Delta(B-L) = 0$ → **final**: $B = 35$, $L = -65$

$B-L$ stays constant conserved by the SM plasma

$$B = \frac{28}{79}(B{-}L), \qquad L = -\frac{51}{79}(B{-}L).$$

$$\varepsilon_1 = \frac{\Gamma(N_1 \to \ell H) - \Gamma(N_1 \to \bar{\ell} H^\dagger)}{\Gamma(N_1 \to \text{all})}$$

$$\eta_B \simeq 0.0096\, \varepsilon_1\, \kappa_f, \qquad 0 \leq \kappa_f \leq 1.$$

Why a reheating-temperature bound appears

η_B in the vanilla picture:

$$\eta_B \simeq 0.0096\, \varepsilon_1\, \kappa_f$$

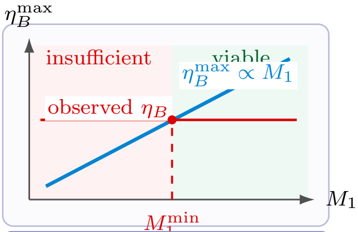
For hierarchical heavy neutrinos,

$$|\varepsilon_1| \leq \frac{3M_1}{16\pi v^2}(m_3 - m_1).$$

Davidson–Ibarra bound

Therefore, the maximal baryon asymmetry scales linearly with M_1:

$$\eta_B^{\max} = \text{Constant} \times M_1.$$



$T_{\rm RH} \gtrsim M_1$: the plasma must thermally produce N_1

Conventional estimates give a scale around

$$M_1, T_{\rm RH} \gtrsim 10^{9\text{–}10}\ \text{GeV},$$

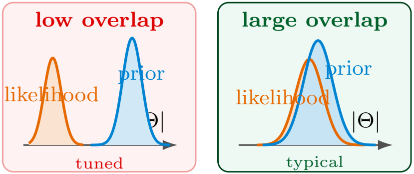
depending on hierarchy, flavor, and degeneracy.

Our question: typicality, not only existence

A low T_{RH} point may exist after tuning parameters.

Question:

How typical is successful leptogenesis at each (M_0, T_{RH})?



Naturalness = overlap of likelihood and prior.

- ✓ Prior expresses “anarchy-like” flavor ignorance.
- ✓ Likelihood imposes oscillation data and η_B.
- ✓ Evidence measures overlap between prior volume and data.

Bayesian setup

$$\pi(Y, M) = \frac{8}{\pi^{15} Y_0^{18} M_0^{12}} \exp\left[-\frac{\mathrm{tr}(Y^\dagger Y)}{Y_0^2} - \frac{\mathrm{tr}(M^\dagger M)}{M_0^2}\right].$$

$$L(x) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{(x_i - x_i^{\mathrm{obs}})^2}{2\sigma_i^2}\right].$$

$$x_i = \{\sin^2\theta_{ij}, \delta_{\mathrm{CP}}, \Delta m_{31}^2, \Delta m_{21}^2/\Delta m_{31}^2, \eta_B\}.$$

Without leptogenesis

$$Z_{Y_0, M_0} = \int dY\, dM\, L_\nu(Y, M)\pi(Y, M),$$

With thermal leptogenesis

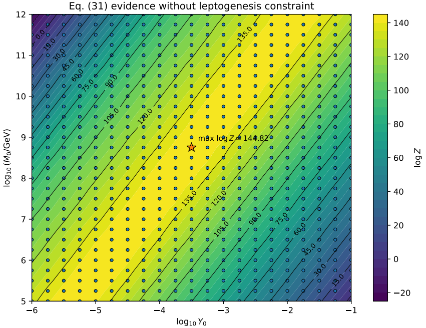
$$Z_{M_0, T} = \left\langle \exp\left[-\frac{(\eta_B^{\mathrm{th}} - \eta_B^{\mathrm{obs}})^2}{2(\Delta\eta_B)^2}\right]\right\rangle_0.$$

The average is over the analytically reduced seesaw measure after imposing low-energy neutrino data.

Result I: natural seesaw scale

- ✓ First, ignore the η_B constraint.
- ✓ Evidence tests whether neutrino data prefer a particular seesaw scale.
- ✓ In the scale-invariant limit, the analytic calculation predicts no sharp M_0 preference.

$$\widetilde{Z}(M_0) = \text{const.}$$



The numerical contour is consistent with no typical seesaw mass scale from oscillation data alone.

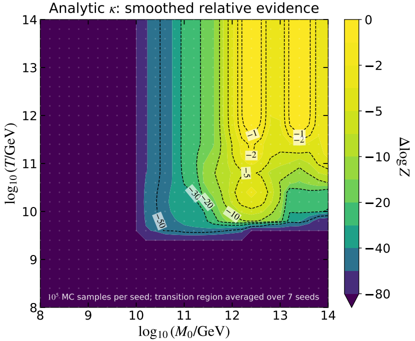
Result II: natural leptogenesis scale

Evidence to be evaluated on

$$10^8\,\text{GeV} < M_0 < 10^{14}\,\text{GeV},$$

$$10^8\,\text{GeV} < T_{\text{RH}} < 10^{14}\,\text{GeV}.$$

- ✓ Draw Monte Carlo points once from the reduced measure.
- ✓ Reuse them for each (M_0, T_{RH}) and average the η_B likelihood.
- ✓ Perturbativity condition: $|Y_{\alpha i}| < \sqrt{4\pi}$.



Take-home message

Low-T_{RH} solutions may exist, but they are not typical in the seesaw parameter space.