E.Cartan's Supersymmetry and the Universe

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I. Introduction

- We want to understand the origin of universe based on particle physics and general relativity.

- The mass fraction of elements in the early universe, becomes a function of baryon density parameter and Hubble constants $\Omega_{m0}h^2$, ($h = H_0/(100\mathrm{km\,s^{-1}Mpc^{-1}})$ (Burles et al,2001)

- In our universe there are more particles than anti-particles. Cosmological observation indicates that the energy of dark matter is more than 5 times larger than that of our universe, and dominates about 27% of the whole energy.

- Dark energy is about three times of matter energy.

II. Cartan's supersymmetry and triality

- In Cartan's supersymmetric theory, fermions are defined by octonions, and interactions of vector current of fermions and gauge fields are studied. Its extension to interaction of axial current of fermions is straightforward, and gives differences in B_s decay and B_d decay, which the Cabibbo-Kobayashi-Maskawa CKM theory cannot explain.

- The Clifford algebra of $R_{3,0}$ is equipped with the basis

$$\{1, e_1, e_2, e_3, e_1e_2, e_2e_3, e_1e_3, e_1e_2e_3\},$$

which expresses Pauli algebra or quaternion algebra.

- Cartan considers fermion fields ψ

$$\begin{aligned}\psi &= \xi_1 i + \xi_2 j + \xi_3 k + \xi_4 = \begin{pmatrix} \xi_4 + i\xi_3 & i\xi_1 - \xi_2 \\ i\xi_1 + \xi_2 & \xi_4 - i\xi_3 \end{pmatrix} \\ C\psi &= -\xi_{234} i - \xi_{314} j - \xi_{124} k + \xi_{123} = \begin{pmatrix} \xi_{123} - i\xi_{124} & -i\xi_{234} + \xi_{314} \\ -i\xi_{234} - \xi_{314} & \xi_{123} + i\xi_{124} \end{pmatrix}\end{aligned}$$

and the spinor operator

$$\begin{aligned}\phi &= \xi_{14} i + \xi_{24} j + \xi_{34} k + \xi_0 = \begin{pmatrix} \xi_0 + i\xi_{34} & i\xi_{14} - \xi_{24} \\ i\xi_{14} + \xi_{24} & \xi_0 - i\xi_{34} \end{pmatrix} \\ C\phi &= -\xi_{23} i - \xi_{31} j - \xi_{12} k + \xi_{1234} = \begin{pmatrix} \xi_{1234} - i\xi_{12} & -i\xi_{23} + \xi_{31} \\ -i\xi_{23} - \xi_{31} & \xi_{1234} + i\xi_{12} \end{pmatrix},\end{aligned}$$

and vector field

$$\{x, x'\} = \{x_1, x_2, x_3, x_4, x'_1, x'_2, x'_3, x'_4\}.$$

- Cartan defines three quadratic forms

$$\begin{aligned} F &= x_1x_1' + x_2x_2' + x_3x_3' + x_4x_4' \\ \Psi &= -\xi_1\xi_{234} - \xi_2\xi_{314} - \xi_3\xi_{124} + \xi_4\xi_{123} \\ \Phi &= \xi_0\xi_{1234} - \xi_{23}\xi_{14} - \xi_{31}\xi_{24} - \xi_{12}\xi_{34} \end{aligned}$$

and obtained 5 Triality transformations

$$\{G_{23}, G_{12}, G_{13}, G_{132}, G_{123}\}$$

that leave F, Ψ and Φ invariant. The transformation table of the 5 transformations are given in [Furui,2012].

- Cartan studied interaction of even dimensional field x, x' and spinor fields $\psi, \mathcal{C}\psi$ and $\phi, \mathcal{C}\phi$ which are composed of quaternions, and each pair composes an octonion. Cartan showed that there is a group G which contains octonion transformation and leave ${}^t\phi C X\psi$, $F = x \cdot x'$, $\Phi = {}^t\phi C\phi$ and $\Psi = {}^t\psi C\psi$ invariant.

- The vertex of vector particles and spinors $\gamma_0 x^\mu \gamma_\mu$ can be extended to include weak interaction by replacing the vertex to $\gamma_0 x^\mu \gamma_\mu(1-\gamma_5)$ and choosing -1 or γ_5 such that the couplings are unified in the form

$$\sum_{i=1}^{4}(x_i \mathcal{C}\phi \mathcal{C}\psi + x_i' \mathcal{C}\phi\, \psi)$$

- We obtained the table of spinor vector field coupling via $\gamma_\mu\gamma_5$.

In the SM, B meson decay into $\ell\bar{\ell}$ in the lowest order is given by

$$(\frac{g}{\sqrt{2}})^2 \sum_{\mu=1}^{4} \bar{\ell}(1-\gamma_5)\gamma_\mu \ell \frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{m_Z^2}}{q^2 - m_Z^2} \bar{\psi}(1-\gamma_5)\gamma_\mu \psi$$

Fig.1 Typical diagrams of $B_s(B_d) \to \ell\bar{\ell}$ decay in SM.
They contain two $1-\gamma_5$ operators.

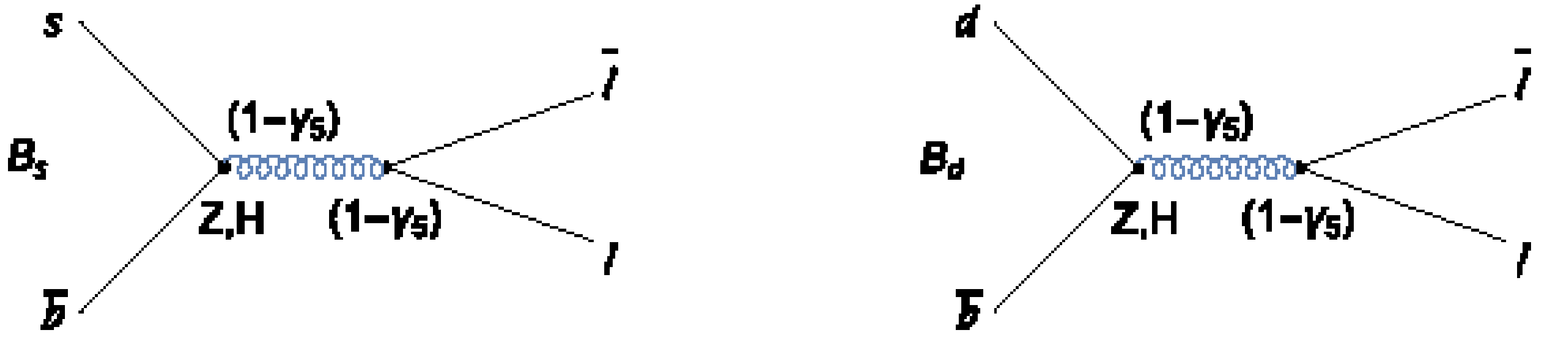


Fig.2a Typical diagrams of $B \to \ell\bar{\ell}$ decay which contain one γ_5 operator. $B \to \ell\bar{\ell}$ (left) contains one loop, and $B \to \ell\bar{\ell}$ (right) contains two loops.

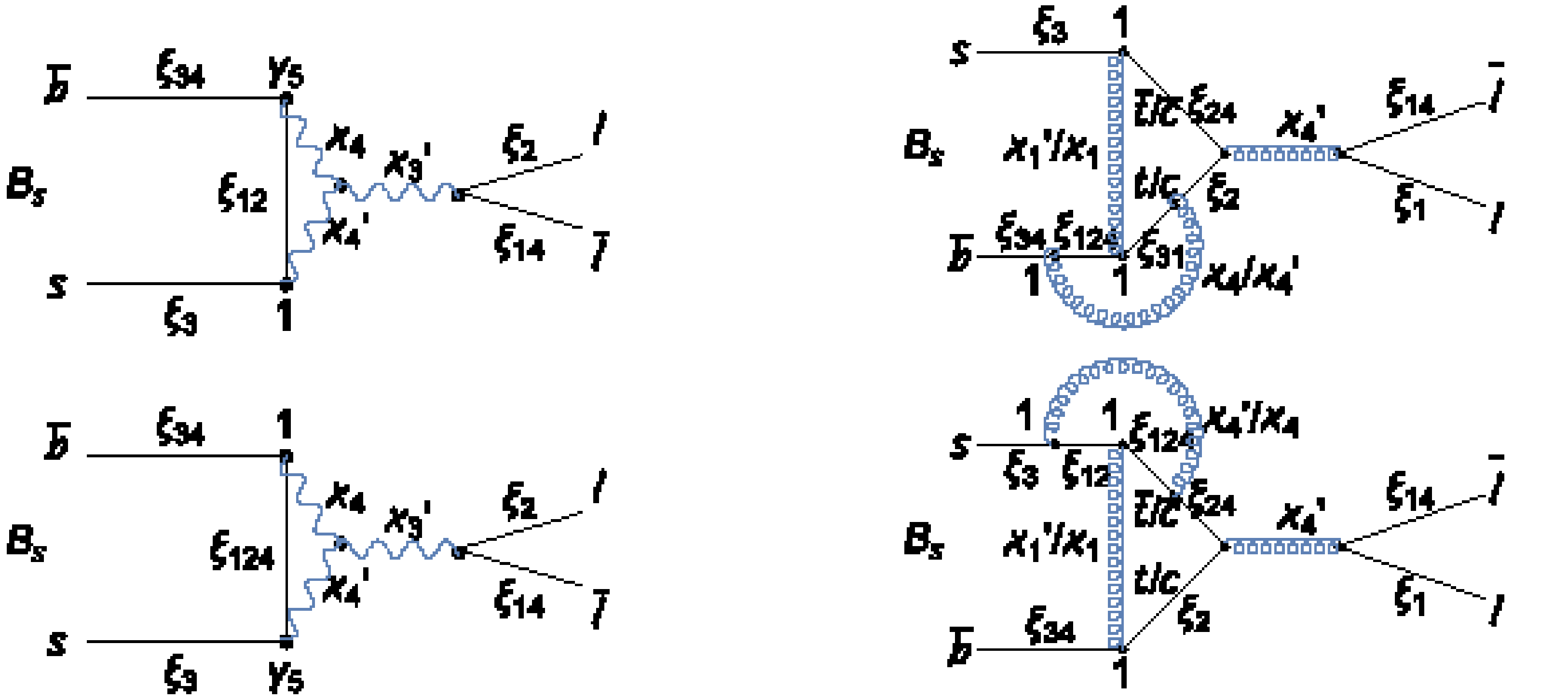
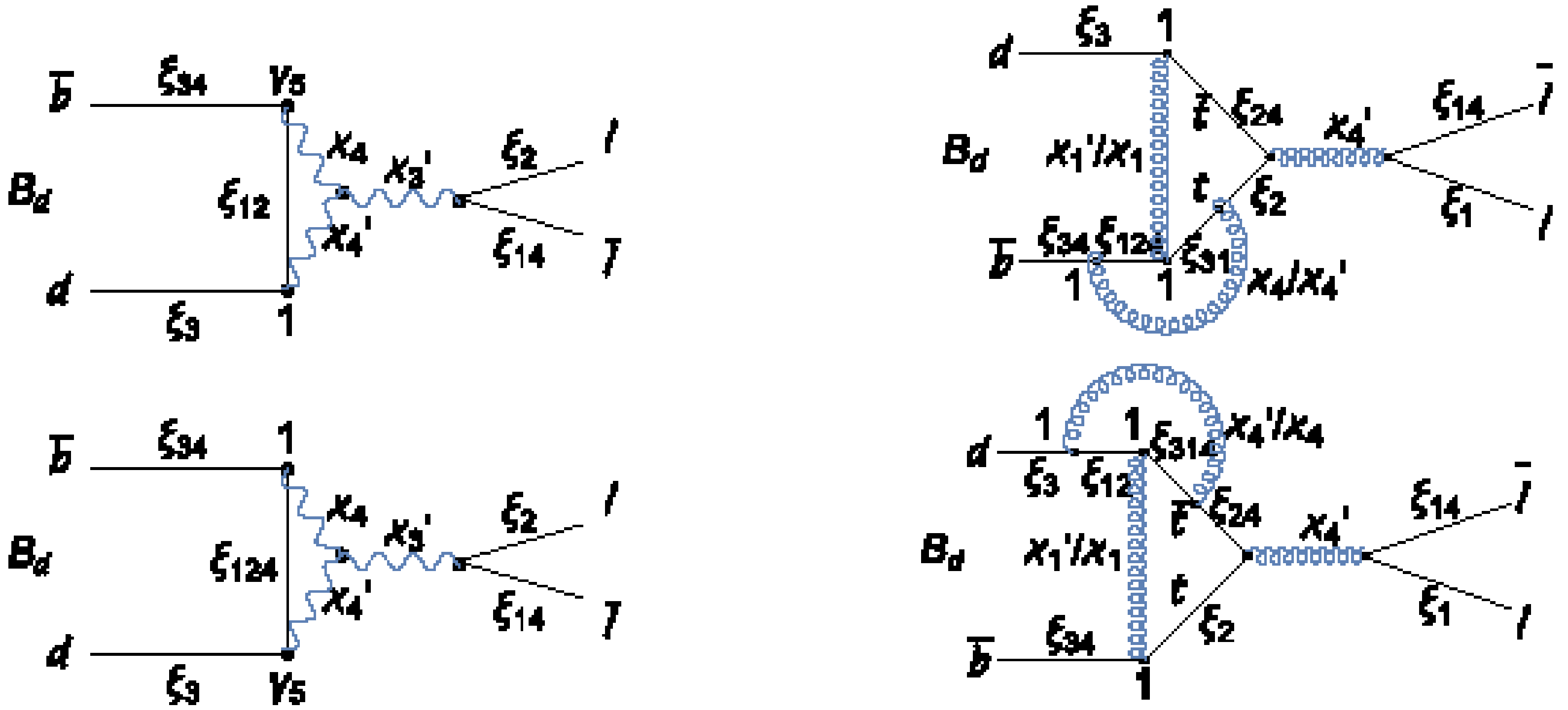


Fig.2b Typical diagrams of $B_d \to \ell\bar{\ell}$ decay.

In the right diagrams $\xi_{124}x_1\xi_{31}$ and $\xi_{12}\gamma_5 x_1'\xi_{314}$ couplings and x_1/x_1' mixing allow the decay.



- The CKM model was successful since Higgs meson separated t quark from other quarks, and $m_u \simeq m_d$. The problem of B meson decay branching ratios indicates importance of the difference of $m_s < m_c < m_H < m_t$ and $m_K < m_c < m_B$, and supersymetric interactions.

- Presence of c meson in the same sector as that of s meson is not crucial in the decay of K mesons since $m_c > m_K$, but crucial in that of B_s since $m_c < m_B$. In the case of Dirac spinors, our model contains the triality symmetry, which can be interpreted as the color degrees of freedom (r, g, b). The neutrinos in a definite flavor sector are

$$\nu_e, [\nu_u, \nu_d]_{r,g,b}, \quad \nu_\mu, [\nu_c, \nu_s]_{r,g,b}, \quad \nu_\tau, [\nu_t, \nu_b]_{r,g,b}$$

all left-handed, and there are seven right-handed neutrinos.

- We expect that our electromagnetic detector can detect electromagnetic fields transformed by G_{23}, but cannot detect electromagnetic fields transformed by G_{12}, G_{13}, G_{123} and G_{132}. There are six lepton sectors

$$|e, \nu_e)^*, |\mu, \nu_\mu)^*, |\tau, \nu_\tau)^*, |e, \nu_e)^{**}, |\mu, \nu_\mu)^{**}, |\tau, \nu_\tau)^{**}$$

  whose massive neutrino components cannot be detected by our detectors, and two additional right-handed neutrinos.

- The number of massive neutrinos becomes $21 + 6 = 27$, and the number of right-handed neutrinos becomes $7 + 2 = 9$ in this model. It is possible to construct a model satisfying the Z_3 symmetry using Dirac lepton neutrinos and Majorana quark neutrinos.

- If in the univese there are world which are transformed by G_{ij} and G_{ijk}, and the uncertainty principle applys not only in our world but also in whole universe, we can understand the presence of dark matter.

- Using Clifford algebra, one can assign vectors of $(\psi, \mathcal{C}\psi)$ and $(\phi, \mathcal{C}\phi)$ where ϕ corresponds to the charge conjugate state of ψ. There is no preference of $(\psi, \mathcal{C}\psi)$ and $(\phi, \mathcal{C}\phi)$, in general, there are no unique bivector $\mathrm{F} \in \wedge^2 R^8$ which contains octonions, whose field has characteristic 2 (Lounesto,2001).

- But combining with the commutation relations of quaternions ψ and ϕ, or $\mathcal{C}\psi$ and $\mathcal{C}\phi$ we can define unique bivectors which contains octonions satisfying Cartan's supersymmetry.

- The Dirac fermion is defined by $\psi, \mathcal{C}\psi$ and $\phi, \mathcal{C}\phi$ both in 8 dimensional space, and together with the vector fields x, x', they follow the supersymmetry that has the triality symmetry.

III. Nature of $G_{23}, G_{12}, G_{123}, G_{13}$ and G_{132} Transformations

- The G_{23} transforms matters to anti-matters and has the commutation relation

$$\frac{1}{2}(\xi_{ij}\xi_{ij4} - \xi_{ij4}\xi_{ij}) = 1, \quad (ij = 12, 23, 31).$$

By G_{12}, one obtains the commutation relation of spinors and vectors

$$\frac{1}{2}(\xi_{12}x^3 - x^3\xi_{12}) = 1, \quad \frac{1}{2}(\xi_{23}x^1 - x^1\xi_{23}) = 1, \quad \frac{1}{2}(\xi_{31}x^2 - x^2\xi_{31}) = 1$$

and anti-commutation relations of time component of spinors and vectors

$$\frac{1}{2}(\xi_{i4}x^{i'} + x^{i'}\xi_{i4}) = -1, \quad (i = 1, 2, 3).$$

By G_{13}, one obtains the commutation relation of vectors and spinors

$$\frac{1}{2}(\xi_i x^{i'} - x^{i'}\xi_i) = 1, \quad (i = 1, 2, 3)$$

$$\frac{1}{2}(x^3\xi_{124} - \xi_{124}x^3) = 1, \quad \frac{1}{2}(x^2\xi_{314} - \xi_{314}x^2) = 1, \quad \frac{1}{2}(x^1\xi_{234} - \xi_{234}x^1) = 1.$$

By G_{123}, one obtains the commutation relation between vectors and spinors

$$\frac{1}{2}(x^3\xi_{12} - \xi_{124}x^3) = 1, \quad \frac{1}{2}(x^2\xi_{31} - \xi_{314}x^2) = 1, \quad \frac{1}{2}(x^1\xi_{23} - \xi_{234}x^1) = 1.$$

By G_{132}, one obtains the commutation relatin between vectors and spinors

$$\frac{1}{2}(\xi_{12}x^3 - x^3\xi_{124}) = 1, \quad \frac{1}{2}(\xi_{31}x^2 - x^2\xi_{314}) = 1, \quad \frac{1}{2}(\xi_{23}x^1 - x^1\xi_{234}) = 1$$

The main part of the universe transformed by $G_{23}, G_{12}, G_{13}, G_{123}$ and G_{132} are difficult to detect by our electromagnetic detecters, and they appear as dark matter.

IV. Gravitation and the Universe

- Description of Dirac fermions is not unique. The sub algebra $R^+_{3,1}$ with a basis
$$\{1, e_1e_2, e_1e_3, e_2e_3, e_1e_4, e_2e_4, e_3e_4, e_1e_2e_3e_4\}$$
where e_i satisfy $e_1^2 = e_2^2 = e_3^2 = 1, e_4^2 = -1$ is equivalent to Pauli algebra, defined by $e_1e_4 = \sigma_1, e_2e_4 = \sigma_2, e_3e_4 = \sigma_3$ and
$$e_1e_2e_3e_4 = \sigma_1\sigma_2\sigma_3 = i.$$
This non-commutative algebra can be calculated on PC using Mathematica[Aragon et al.,2008].

- In the n dimensional linear space V over a field F and exterior algebra $\wedge V$, octonion appears by defining $e_1e_2e_3 = \ell$, and choosing
$$\{1, e_1, e_2, e_3, -e_3\ell, -e_2\ell, -e_1\ell, \ell\}$$
as the basis of the field[Dray and Manogue, 1998]. The commutation relations of e_i are not same as those of Cartan's ξ_i.

- In the theory of general relativity, using time τ of freely falling observer, the Newton's equation becomes

$$\ddot{r}(\tau) = -\frac{GM}{r^2}$$

and when $r > r_s = 2GM/c^2$(Schwarzshild radius), using the watch of the observer, the equation becomes

$$\frac{dv}{dt} = -(1 - \frac{r_s}{r})\frac{GM}{r^2}.$$

In the gravitational field of a star of mass M, the light that has the path with closest distance of r_0 is bent with angle

$$\Delta\phi \sim \frac{4GM}{r_0 c^2}.$$

These relations are independent of the color of quarks that compose matter.

- In the Hubble's expansion theorie, the matter density $\rho_m(r,t)$ and the velocity field $v(r,t)$ satisfy continuity equation and Euler's equation

$$\partial_t \rho_m + div(\rho_m v) = 0$$
$$\partial_t v + v \cdot \nabla v = -\frac{1}{\rho_m} \nabla p_m + f \qquad (1)$$

where f contains the gravitational force $g = -\nabla \Phi$

$$\Delta \Phi - div\, g = 4\pi G \rho_m$$

- There is an additional force

$$f_\Lambda = \frac{\Lambda c^2}{3} r, \quad f(t) = \frac{\Lambda c^2}{3} - \frac{4\pi G}{3} \rho_m$$

and the corresponding energy is called dark energy.

- The energy of ordinary matter is about 5% of the total energy of the universe and about 1% is luminary.

- Since there are expected to exist other universe transformed by 5 transformations, the baryonic energy/total energy of the whole universe is expected to be

$$\Omega_m|_{theory} = 0.04 \times 6 \sim 0.24,$$

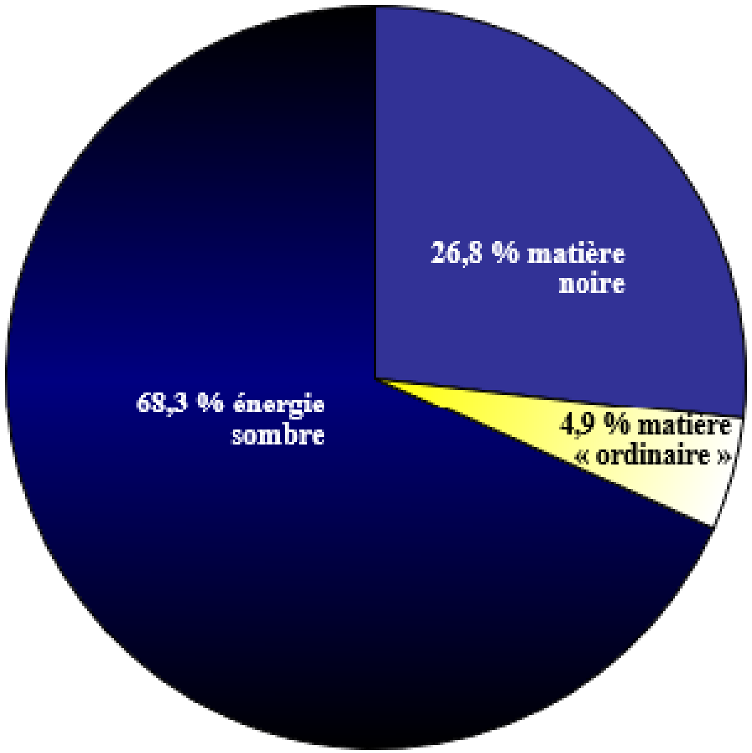
which is not far from the dark matter parameter of the universe 0.268(Wikimedia).

- Since gravitational energy is blind to the color of electro-weak and strong interactions, 1/3 of the dark energy parameter of the universe, i.e. 0.683/3=0.228 should be close to the baryonic energy of the universe $\Omega_m|_{theory}$.

- Rebhan(2012) derived as the sum of dark matter parameter and lightning matter parameter $\Omega_{m0} = 0.28$, and the dark energy parameter

$$\Omega_{\Lambda 0} = 1 - \Omega_{m0} = 0.72.$$

Our $\Omega_m|_{theory}$ agrees with 1/3 of $\Omega_{\Lambda 0}$.



(Wikimedia, Matière noire)

Bitbol(2010) wrote, paraphrasing Kant, that les faits sans formes sont certes aveugles, mais les formes sans faits sont vides*(facts without forms are surely blind, but forms without facts are empty)*. One may be able to say that a recognition not related to experiences from detectors but based on mathematical forms is possible.

- The gravitational interaction between our universe and the universe transformed by $G_{23}, G_{12}, G_{13}, G_{123}$ and G_{132} is to be investigated.

- Neutrino interaction between our universe and the universe transformed by $G_{23}, G_{12}, G_{13}, G_{123}$ and G_{132} will give useful information for establishing Cartan's supersymmetry of electro weak interactions.

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