TEST OF DARK MATTER SELF-INTERACTION AND ITS TEMPERATURE IN THE SUN

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base on 1408.5471(JCAP), 1505.03781, and 1508.05263
in collaboration with Guey-Lin Lin and Yen-Hsun Lin
We have long observed the gravitational pull of “DM” exerts on regular baryonic matter, no conclusive hint of the particle physics governing DM has so far shown up in laboratory exps.
search dark matter particle at our own galaxy

we are here!

arXiv:1110.4431

F. Iocco et al (nature physics2015)
DM is surrounding us of
\( q_{DM} \sim 0.3-0.4 \text{ GeV} / \text{cm}^3 \)
If DM interacts with nucleons, this can also happen

- remarks:
  - The indirect searches of DM in the Sun are tightly linked to direct detection searches, which are sensitive to the cross section for DM scattering off the nucleons of heavy nuclei.

The Sun has an escape velocity, from its surface, of about 618 km/s while the mean-squared velocity of the Galactic DM in the halo is about 270 km/s. Therefore, the gravitational effects of the Sun is significant.
current limits and future expectations

compilation of WIMP-nucleon spin-independent cross section limits
Capture and evaporation:

A DM can collide with nuclei and lose energy when it traverse the Sun. If the final velocity of the DM after collision is less than the local escape velocity $v_e(r)$, then it gets gravitationally trapped. However, the captured DM may scatter off energetic nuclei and be ejected, whenever the DM velocity after collision is larger than the local escape velocity.

Annihilation:

Once DM $\chi$ is captured by the Sun, the $\chi$ will come to thermal equilibrium and sink into the core. With time, the $\chi$ concentration will increase until the density is high enough for $\chi \chi$ annihilation to occur. A steady state will be achieved if the time to reach equilibrium is short compare to the age of the object.
Can DM interact with itself?

Collisionless cold dark matter?

- Inconsistencies between N-body simulations and observations

1. Cusp and core problem:


M.G. Walker and J. Penarrubia, 2011
2. Missing satellites

B. Moore, Astro. J. 1999
3. Too big to fail

**M. Boylan-Kolchin et al, 2012**

![Graph showing dark matter profile and inferred properties](image-url)

- **Initial dark matter profile**
- **$M_{\text{blow out}} = 2.2 \times 10^7 \, M_\odot$**
- **$M_{\text{blow out}} = 1.1 \times 10^8 \, M_\odot$**

**Graph legend**
- **bright MW dSphs (this work)**
- **Magellanic Clouds (lower limit)**
- **THINGS galaxies (Oh et al.)**

**Axes**
- $r$ [kpc]
- $V_{\text{circ}}$ [km s$^{-1}$]
- $M_*$ [$M_\odot$]
- $V_{\text{infall}}$ [km s$^{-1}$]
Remarks:

However, comparisons to cosmological models tend to be inconclusive for the simple reason: while most cosmological N-body simulations consider only dark matter particles, one observes only baryons.

Baryons complicate not only the measurement of a dark matter density profile but also its interpretation within the context of the CDM paradigm.

Measurement: the fact that any uncertainty (e.g., stellar mass-to-light ratios) in the baryonic mass profile propagates to the inferred dark matter profile, as the latter is merely the difference between dynamical and baryonic mass profiles.

Interpretation: the possibility that various poorly understood dynamical processes involving baryons might alter the original structure of a dark matter halo.
Dark Matter Self-interaction

If dark matter interacts with itself, it might solve these small scale problems.

**PHYSICAL REVIEW LETTERS**

**Observational Evidence for Self-Interacting Cold Dark Matter**

David N. Spergel and Paul J. Steinhardt

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(Received 20 September 1999)

Cosmological models with cold dark matter composed of weakly interacting particles predict overly dense cores in the centers of galaxies and clusters and an overly large number of halos within the Local Group compared to actual observations. We propose that the conflict can be resolved if the cold dark matter particles are self-interacting with a large scattering cross section but negligible annihilation or dissipation. In this scenario, astronomical observations may enable us to study dark matter properties that are inaccessible in the laboratory.
Constraints from Bullet Cluster matter distribution, halo shape, core densities, ........

- Analysis based on the kinematics of dwarf spheroidals, DMSI only alleviate small scale structure when

\[0.1 \text{ cm}^2/\text{g} < \sigma_{xx}/m_x < 1.0 \text{ cm}^2/\text{g}\]
The general DM evolution equation in the Sun is given by

\[
\frac{dN_X}{dt} = C_c + (C_s - C_e)N_X - (C_a + C_{se})N_X^2
\]

\[
N_X(t) = \frac{C_c \tanh(t/\tau_A)}{\tau_A^{-1} - (C_s - C_e) \tanh(t/\tau_A)/2}
\]

for \( N_X(0) = 0 \)

\[
\tau_A = \frac{1}{\sqrt{C_c(C_a + C_{se}) + (C_s - C_e)^2/4}}
\]

time scale for the DM in the Sun to reach the equilibrium in the equilibrium state, \( \tanh(t/\tau_A) \sim 1 \)

\[
N_{X,\text{eq}} = \frac{C_s - C_e}{2(C_a + C_{se})} + \sqrt{\frac{(C_s - C_e)^2}{4(C_a + C_{se})^2} + \frac{C_c}{C_a + C_{se}}}
\]
The capture rate can be categorized by the spin-dependent and spin-independent interactions

\[ C^\text{SD}_c \simeq 3.35 \times 10^{24} \text{ s}^{-1} \left( \frac{\rho_0}{0.3 \text{ GeV/cm}^3} \right) \left( \frac{270 \text{ km/s}}{\bar{v}} \right)^3 \left( \frac{\text{GeV}}{m_\chi} \right)^2 \left( \frac{\sigma^\text{SD}_H}{10^{-6} \text{ pb}} \right) \]

\[ \sigma^\text{SD}_i = A^2 \left( \frac{m_\chi + m_p}{m_\chi + m_A} \right)^2 \frac{4(J_i + 1)}{3J_i} |\langle S_{p,i} \rangle + \langle S_{p,i} \rangle|^2 \sigma^\text{SD}_{\chi p} \]

or

\[ C^\text{SI}_c \simeq 1.24 \times 10^{24} \text{ s}^{-1} \left( \frac{\rho_0}{0.3 \text{ GeV/cm}^3} \right) \left( \frac{270 \text{ km/s}}{\bar{v}} \right)^3 \left( \frac{\text{GeV}}{m_\chi} \right)^2 \left( \frac{2.6\sigma^\text{SI}_H + 0.175\sigma^\text{SI}_{\text{He}}}{10^{-6} \text{ pb}} \right) \]

\[ \sigma^\text{SI}_i = A^2 \left( \frac{m_A}{m_p} \right)^2 \left( \frac{m_\chi + m_p}{m_\chi + m_A} \right)^2 \sigma^\text{SI}_{\chi p} \]
The captured DM might collide with the nuclei inside the Sun and be kicked out from the Sun if the final state velocity is larger than the escape velocity. --- evaporation

\[ C_e \simeq \frac{8}{\pi^3} \sqrt{\frac{2m_\chi}{\pi T_\chi(\bar{r})}} \frac{v_{\text{esc}}(0)}{\bar{r}^3} \exp \left( -\frac{m_\chi v_{\text{esc}}^2(0)}{2T_\chi(\bar{r})} \right) \Sigma_{\text{evap}} \]

evaporation effect can be relevant only for low DM mass \( \sim O(1) \) GeV.

\[ \phi(r) = \int_0^r \frac{G_N M_\odot(r')}{r'^2} dr' \quad \text{gravitational potential} \]

\[ \frac{3}{2} kT(0) = m_\chi \phi(\bar{r}) \]

if take the solar core to have a constant density, we have

\[ \bar{r} \approx 0.13R_{\text{Sun}} \sqrt{\frac{m_p}{m_\chi}} \]
local gravitational potential:

\[ \phi(r) = \int_0^r \frac{G \, M_{\odot}(r')}{r'^2} \, dr' \]

DM number density is determined by solar gravitational potential and scales as

\[ n_\chi(r) = n_0 e^{-m_\chi \phi(r)/T_\chi} \]

The annihilation coefficient \( C_a \) is defined as

\[ C_a = \langle \sigma v \rangle_\odot \frac{\int_\odot n_\chi^2(r) \, d^3r}{\left[ \int_\odot n_\chi(r) \, d^3r \right]^2} \approx \frac{\langle \sigma v \rangle V_2}{V_1^2} \]

\[ V_j \approx 6.5 \times 10^{28} \, \text{cm}^3 \left( \frac{10 \, \text{GeV}}{j m_\chi} \right)^{3/2} \]
The DM self-capture rate

The DM scatters with the DM that have been captured inside the Sun.

\[
C_s \propto n_{\chi} \sigma_{\chi\chi} F(\bar{v}_\chi, v_{esc})
\]

\[
C_s = \sqrt{\frac{3}{2}} n_{\chi} \sigma_{\chi\chi} v_{esc}(R_\odot) \frac{v_{esc}(R_\odot)}{\bar{v}} \left\langle \phi_{\chi} \right\rangle \frac{\text{erf}(\eta)}{\eta}
\]

\[
\eta^2 = \frac{3(v_\odot/\bar{v})^2}{2}
\]

\[
\left\langle \phi_{\chi} \right\rangle \simeq 5.1
\]

dimensionless average solar potential experienced by the captured DM within the Sun
DM self-interaction induced evaporation $C_{se}$:

DM can interact among themselves, DM trapped in the solar core could scatter with other trapped DM and results in the evaporation. This process involves two DM particles just like annihilation. Both processes lead to the DM dissipation in the Sun. While $C_{se}$ does not produce neutrino flux as $C_{a}$ does.

The derivation of this term is similar to the nucleon induced evaporation with the parameter replacements

$$m_N \rightarrow m_\chi, \ T_N \rightarrow T_\chi$$

DM satisfies the Maxwell-Boltzmann distribution

$$f_\odot(w) = \frac{4}{\sqrt{\pi}} \left( \frac{m_\chi}{2T_\chi} \right)^{3/2} n_\chi w^2 \exp \left( - \frac{m_\chi w^2}{2T_\chi} \right)$$
Take the final state velocity $v$ such that $v > w$. The DM-DM differential scattering rate with the velocity transition $w \to v$ is

$$R^+(w \to v) dv = \frac{2}{\sqrt{\pi}} n_{\chi} \sigma_{\chi \chi} \frac{v}{w} e^{-\kappa^2(v^2 - w^2)} \chi(\beta_-, \beta_+) dv$$

with $\beta_\pm = \pm \kappa w$ with $\kappa = \sqrt{\frac{m_{\chi}}{2T_{\chi}}}$ and $\chi(a, b) \equiv \int_a^b du e^{-u^2} = \frac{\sqrt{\pi}}{2} [\text{erf}(b) - \text{erf}(a)]$

we integrate the scattering rate with $v$ greater than the escape velocity

$$\Omega^+_{v_{\text{esc}}} (w) = \int_{v_{\text{esc}}}^{\infty} R^+(w \to v') dv' = \frac{2}{\sqrt{\pi}} n_{\chi} \sigma_{\chi \chi} \frac{T_{\chi}}{m_{\chi}} \exp \left[ \frac{m_{\chi}(v_{\text{esc}}^2 - w^2)}{2T_{\chi}} \right] \chi(\beta_-, \beta_+)$$

the evaporation rate per unit volume at position $r$, and we sum up all possible states of the incident DM

$$\frac{dC_{se}}{dV} = \int_0^{v_{\text{esc}}} f_{\odot}(w) \Omega^+_{v_{\text{esc}}} (w) dw$$
The escape velocity where

\[ n_x(r) = n_0 \exp \left( -\frac{m_x \phi(r)}{T_x} \right) \]

\[
\frac{dC_{se}}{dV} = \frac{4}{\sqrt{\pi}} \sqrt{\frac{m_x n_0^2 \sigma_{xx}}{2T_x}} m_x \exp \left[ -\frac{2m_x \phi(r)}{T_x} \right] \exp \left[ -\frac{E_{esc}(r)}{T_x} \right] \tilde{K}(m_x)
\]

with

\[ \tilde{K}(m_x) = \sqrt{\frac{E_{esc}(r) T_x}{\pi}} \exp \left[ -\frac{E_{esc}(r)}{T_x} \right] + \left( E_{esc}(r) - \frac{T_x}{2} \right) \text{erf} \left( \sqrt{\frac{E_{esc}(r)}{T_x}} \right) \]

\[ E_{esc}(r) = \frac{1}{2} m_x v_{esc}^2(r) \]

\[ C_{se} = \frac{\int_{\odot} \frac{dC_{se}}{dV} d^3r}{\left( \int_{\odot} n_x(r) d^3r \right)^2} \]
The general DM evolution equation in the Sun is given by

\[
\frac{dN_X}{dt} = C_c + (C_s - C_e)N_X - (C_a + C_{se})N_X^2
\]

\[
N_X(t) = \frac{C_c \tanh(t/\tau_A)}{\tau_A^{-1} - (C_s - C_e)\tanh(t/\tau_A)/2}
\]

for \(N_X(0) = 0\)

\[
\tau_A = \frac{1}{\sqrt{C_c(C_a + C_{se}) + (C_s - C_e)^2/4}}
\]

\[\text{time scale for the DM in the Sun to reach the equilibrium}\]

in the equilibrium state, \(\tanh(t/\tau_A) \sim 1\)

\[
N_{X,eq} = \frac{C_s - C_e}{2(C_a + C_{se})} + \sqrt{\frac{(C_s - C_e)^2}{4(C_a + C_{se})^2} + \frac{C_c}{C_a + C_{se}}}.
\]
Figure 1. The values of \( \tanh(\tau/\tau_A) \) over the plane at the present day, \( t = t_\odot \). The red-circled area is the non-equilibrium region for \( N \).

Figure 2. The values of \( \tanh(\tau/\tau_A) \) over the plane at the present day, \( t = t_\odot \). The blue-circled area is the non-equilibrium region for \( N \). The vertical line at the right panel indicates the LUX bound, \( SI_P \lesssim 10^{45} \text{cm}^2 \), for \( m = 20 \text{ GeV} \).

The DM annihilation rate in the Sun's core is given by

\[
A = C a^2 N^2.
\]  

(2.13)

By setting \( C_s = C_{se} = 0 \), we can recover the results in refs. \([5, 8–11]\) for the absence of DM self-interaction. By setting \( C_e = C_{se} = 0 \), we recover the result in ref. \([26]\), which includes the DM self-interaction while neglects the DM evaporation.

2.2 Numerical results

The coefficients \( C_{c,e,s} \) have been worked out in refs. \([5, 8, 26]\), which we adopt for our numerical studies. The numerical result for \( C_{se} \) is based on the analytic expression we have given in

\(\cdot\)
Figure 1. The values of $\tanh(t/\tau_A)$ over SD plane at the present day, $t = t_\odot$. The red-circled area is the non-equilibrium region for $N$. The vertical line at the right panel indicates the LUX bound, $\sigma^{SI} < 10^{-45}$ cm$^2$, for $m = 20$ GeV.

The DM annihilation rate in the Sun's core is given by

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2.2 Numerical results

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\hfill –5–
General speaking, the inclusion of DM self-interaction will increase the capture DM number

\[ \sigma_{\text{SD}} = 10^{-41} \text{ cm}^2 \]

\[ \sigma_{\text{SI}} = 10^{-44} \text{ cm}^2 \]

\[ \sigma_{\text{SD}} = 10^{-43} \text{ cm}^2 \]

\[ \sigma_{\text{SI}} = 10^{-45} \text{ cm}^2 \]
we define the dimensionless quantity

\[ N_{\chi, eq} = \sqrt{\frac{C_c}{C_a + C_{se}}} \left( \pm \sqrt{\frac{R}{4}} + \sqrt{\frac{R}{4} + 1} \right) \]

\[ R = \frac{(C_s - C_e)^2}{C_c(C_a + C_{se})} \]

\[ \Gamma_A = \frac{1}{2} \frac{C_c C_a}{C_a + C_{se}} \left( \pm \sqrt{\frac{R}{4}} + \sqrt{\frac{R}{4} + 1} \right)^2 \]

\[ \Gamma_A = \frac{C_a}{2} N_{\chi}^2 \]
Figure 5. Ratio $R$ over the $p$ plane. The upper panel is for SI interaction and the lower panel is for SD interaction. The red-circled region is for $R > 1$. It has been seen that DM self-interaction not only enhances $N$ significantly for $m < 10$ GeV but also affects the evaporation mass scale. Therefore in the next session we shall explore the possibility of probing DM self-interaction for $m < 10$ GeV by IceCube-PINGU detector.

3 Probing DM self-interaction at IceCube-PINGU

The annihilation rate of the captured DM in the Sun is given by eq. (2.13). It is worth mentioning that in the absence of both evaporation (the one due to $C_e$) and self-interaction, the annihilation rate $A$ with an equilibrium $N$ is

$$A = \frac{1}{2} C_a \times C_c C_a = C_c^2,$$

(3.1)

which only depends on the capture rate $C_c$. However, with the presence of either $C_e$ or self-interaction, $A$ depends on other coefficients as well even $N$ has reached to the equilibrium. We plot $A$ as a function of $m$ with and without self-interaction in figure 6.
IceCube-PINGU is a proposed low-energy infill extension to the IceCube Observatory. PINGU will feature the world’s largest effective volume for neutrinos at an energy threshold of a few GeV.

The DM annihilation rate in the Sun’s core is given by

\[ \Gamma_A = \frac{C_a}{2} N_{\chi,\text{eq}}^2. \]

The primary DM annihilation spectrum is model dependent, here we consider $\chi\chi$ to $\tau^+\tau^-$ and $\nu\nu$, for neutrino final state productions since the low mass region is our interest.
If no DM self-interaction and evaporation, the annihilation rate with an equilibrium $N_\chi$ is

$$\Gamma_A = \frac{1}{2} C_a \times \frac{C_c}{C_a} = \frac{C_c}{2}$$

depends only on the capture rate.
The neutrino differential flux is

\[
\frac{d\Phi_{\nu_i}}{dE_{\nu_i}} = P_{\nu_i \rightarrow j}(E_\nu) \frac{\Gamma_A}{4\pi R^2} \sum_f B_f \left( \frac{dN_{\nu_j}}{dE_{\nu_j}} \right)_f
\]

The neutrino event rate in the detector from the Sun DM is given by

\[
N_\nu = \int_{E_{\text{th}}}^{m_x} \frac{d\Phi_\nu}{d\Omega dE_\nu} A_\nu(E_\nu) dE_\nu d\Omega
\]

the detector effective area

\[
A^\nu_{\text{eff}}(E_\nu) = \rho_{\text{ice}} V_{\text{eff}} \frac{N_A}{M_{\text{ice}}} (n_p \sigma_{\nu p}(E_\nu) + n_n \sigma_{\nu n}(E_\nu))
\]

mass of ice per mole

\[
\begin{align*}
\frac{\sigma_{\nu N}(E_\nu)}{E_\nu} &= 6.66 \times 10^{-3} \text{ pb} \cdot \text{GeV}^{-1} \\
\frac{\sigma_{\bar{\nu} N}(E_\bar{\nu})}{E_{\bar{\nu}}} &= 3.25 \times 10^{-3} \text{ pb} \cdot \text{GeV}^{-1}
\end{align*}
\]

for 1 GeV \( \leq E_\nu \leq 10 \text{ GeV} \)
In our calculation, the atmospheric neutrino flux has been replaced by the atmospheric neutrino flux. Hence, we encounter high-density medium in the Sun, the vacuum in space, and the Earth medium. For 1 GeV

\[ \frac{N}{\Delta E} \sim 10^{-3} \text{ cm}^{-1} \equiv \chi \text{ cm}^{-1} \]

The DM-nucleus interaction inside the Sun is assumed to be dominated by SD interaction. Halo shapes excl. and Bullet Cluster excl. The IceCube-PINGU sensitivities to DM self-interaction cross section \( \sigma_{\chi\chi} \rightarrow \nu \bar{\nu} \) have been considered in order to compare with the DM signal. The testability of \( \sigma_{\chi\chi} \) for spin-dependent, \( xx \rightarrow vv \) is taken from ref. [1].

\[ \langle \sigma v \rangle = \begin{cases} 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} & (3.5a) \\ 3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1} & (3.5b) \end{cases} \]

\[ \chi \chi \rightarrow \nu \bar{\nu}, \sigma^{\text{SD}}_{\chi p} = 10^{-41} \text{ cm}^2 \]

\[ \chi \chi \rightarrow \nu \bar{\nu}, \sigma^{\text{SD}}_{\chi p} = 10^{-43} \text{ cm}^2 \]
testability of $\sigma_{\chi\chi}$ for spin-dependent $\chi\chi$ to $\tau\tau$
interaction than track events do in all cases. One can also see that the sensitivity to
for

is below the LUX bound for

IceCube bound

and 10

freeze-out.

the relic density, since DM annihilation inside the Sun occurs much later than the period of

are used for our studies. We note that the latter value for

of thermal average cross section,

weak to resolve the core/cusp problem of the structure formation. Two benchmark values
Cluster and halo shape analyses. Below the black solid line, the DM self-interaction is too

the surrounding the Sun with

Hence we consider neutrino events arriving from the solid angle range

Figure 8

\( \chi \chi \rightarrow \nu \bar{\nu}, \sigma_{\chi p}^{\text{SI}} = 10^{-44} \text{ cm}^2 \)

\( \chi \chi \rightarrow \nu \bar{\nu}, \sigma_{\chi p}^{\text{SI}} = 10^{-45} \text{ cm}^2 \)

\[ \langle \sigma v \rangle = 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} \]

\[ \langle \sigma v \rangle = 3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1} \]

Bullet Cluster excl. (Randall et al.)

Halo shapes excl. (Peter et al.)

Track

Cascade

\[ m_{\chi} \text{ [GeV]} \]

\[ \sigma_{\chi \chi} \text{ [cm}^2] \]

\[ \sigma_{\chi \chi} \text{ [cm}^2] \]
testability of $\sigma_{\chi\chi}$ for spin-independent, $\chi\chi$ to $\tau\tau$

$\chi\chi \rightarrow \tau^+\tau^-$, $\sigma_{\chi p}^{\text{SI}} = 10^{-44}$ cm$^2$

$\chi\chi \rightarrow \tau^+\tau^-$, $\sigma_{\chi p}^{\text{SI}} = 10^{-45}$ cm$^2$

sensitivity to $\sigma_{\chi\chi}$ becomes better for smaller annihilation cross section $<\sigma v>$. 
The accumulation of DM depends on these three processes:

- DM self-interactions
- DM annihilation
- Direct detection

We can show that if DM self-interaction exists, the total captured DM can be less relevant to DM-nuclei cross-section.
Framework of SIDM

\[
\sigma_{xx} \approx 7.6 \times 10^{-24} \text{ cm}^2 \left( \frac{\alpha_x}{0.01} \right)^2 \left( \frac{m_\phi}{30 \text{ MeV}} \right)^{-4} \left( \frac{m_x}{\text{GeV}} \right)^2
\]

H.B. Yu (PRL2010)

velocity dependent cross section, dark U(1) force

\[
L_{\text{mixing/vector}} = \left( \epsilon_\gamma e J^\mu_{\text{em}} + \epsilon_Z g_2 \frac{g_2}{c_W} J^\mu_{\text{NC}} \right) \phi_\mu
\]

\[
L_{\text{mixing, } U(1)} = \frac{\epsilon_\gamma}{2} \phi_{\mu\nu} F^{\mu\nu} + \epsilon_Z m_Z^2 \phi_\mu Z^\mu
\]
SIDM cross section

\[ \sigma_{\chi \tilde{\chi}} \approx 4\pi \alpha_{\chi}^2 \frac{m_{\chi}^2}{m_{\phi}^4} \]

\[ \alpha_{\chi} \equiv \frac{e_D^2}{(4\pi)} \]

DM-nucleus scattering

\[ \sigma_{\chi A} \approx \frac{16\pi \alpha_{\chi} \alpha_{em}}{m_{\phi}^4} \left[ \varepsilon_p Z + \varepsilon_n (A - Z) \right]^2 \mu_{\chi A}^2 = \frac{16\pi \alpha_{\chi} \alpha_{em}}{m_{\phi}^4} \varepsilon_p^2 [Z + \eta (A - Z)]^2 \mu_{\chi A}^2 \]

\[ \mu_{\chi A} \equiv m_{\chi} m_A / (m_{\chi} + m_A) \]
isospin symmetry does not necessarily satisfy & spin-independent cross section dominated

\[
\sigma_{\chi p}^{\text{SI}} \approx 1.5 \times 10^{-24} \text{ cm}^2 \; \varepsilon_\gamma^2 \left( \frac{\alpha_X}{0.01} \right) \left( \frac{m_\phi}{30 \text{ MeV}} \right)^{-4}
\]

\[
\sigma_{\chi n}^{\text{SI}} \approx 5 \times 10^{-25} \text{ cm}^2 \; \varepsilon_Z^2 \left( \frac{\alpha_X}{0.01} \right) \left( \frac{m_\phi}{30 \text{ MeV}} \right)^{-4}
\]

\[
\varepsilon_p = \varepsilon_\gamma + \frac{\varepsilon_Z}{4s_W c_W} (1 - 4s_W^2) \approx \varepsilon_\gamma + 0.05 \varepsilon_Z
\]

\[
\varepsilon_n = -\frac{\varepsilon_Z}{4s_W c_W} \approx -0.6 \varepsilon_Z
\]
For a light mediator $\phi$ at MeV range, $\sigma_{\chi A}$ is sensitive to the momentum transfer

$$\sigma_{\chi A}(q^2) = \frac{m_\phi^4}{(m_\phi^2 + q^2)^2} \sigma_{\chi A}^0$$

$$q^2 = 2m_A E_R$$

$$C_c \propto \left( \frac{\rho_\chi}{0.15 \text{ GeV/cm}^3} \right) \left( \frac{\text{GeV}}{m_\chi} \right) \left( \frac{270 \text{ km/s}}{v_\chi} \right) \sum_A F_A(m_\chi, \eta) \sigma_{\chi A}^0 \frac{m_\phi^4}{(m_\phi^2 + q_A^2)^2}$$

$$C_s(q^2) \propto \left( \frac{\rho_\chi}{0.15 \text{ GeV/cm}^3} \right) \left( \frac{\text{GeV}}{m_\chi} \right) \left( \frac{270 \text{ km/s}}{v_\chi} \right) \sigma_{\chi \chi}^0 \left( \frac{\text{erf}(\eta)}{\eta} \right) \frac{m_\phi^4}{(m_\phi^2 + q^2)^2}$$

\( \eta = 1, \sigma_{\chi p} = 10^{-45} \text{ cm}^2 \)

\( \eta = -0.7, \sigma_{\chi p} = 10^{-45} \text{ cm}^2 \)
complementary test of SIDM model in IceCube-PINGU

\[ \chi \bar{\chi} \rightarrow \phi \phi \rightarrow 4\nu \]

\[ \text{BR}(\phi \rightarrow \nu \bar{\nu}) \approx 75\%, 39\%, 48\%, \text{and 67\% for } \eta = 1, -0.3, -0.5, \text{and } -0.7, \text{respectively.} \]

\[ \tau_\phi \lesssim \mathcal{O}(1) \text{ s} \quad \text{BBN constraint} \]

<table>
<thead>
<tr>
<th>( E_{\text{max}} ) [GeV]</th>
<th>Track ( N_{\nu}^{\text{atm}} )</th>
<th>Track ( N_{\nu}^{\text{DM}} )</th>
<th>Cascade ( N_{\nu}^{\text{atm}} )</th>
<th>Cascade ( N_{\nu}^{\text{DM}} )</th>
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annual signal and background event numbers for reaching 2\( \sigma \) detection significance in 5 years
IceCube-PINGU sensitivities to $m_\phi$ for different values of $\eta$
If the DM-nuclei cross-section is small enough, the heat exchange between DM sphere and the Sun is small. As a result, DM can be treated as an adiabatic system (isolated from the Sun).

- The correct temperature evolution provides modifications to DM signals
DM thermal transport in the Sun

DM thermal system is governed by

1. DM number evolution equation

\[
\frac{dN_x}{dt} = C_c + C_s N_x - C_a N_x^2
\]

2. the energy transport equation

\[
\frac{d(N_x E_x(t))}{dt} = J_c + (J_x + J_s) N_x - J_a N_x^2
\]
suppose $u$ is the DM velocity in the halo which is in falling to the spherical shall (with radius $r$) of the Sun

$w$ is the velocity of infall DM at the shell and the local escape speed on the shell is $v_{esc}(r)$

$$w = \sqrt{u^2 + v_{esc}^2(r)}$$

To be captured, the DM must loses its energy in a fraction in between

$$\frac{u^2}{w^2} \leq \frac{\Delta E}{E} \leq \frac{4m Xm}{(mX + m)^2}$$

$m$ is the target mass, it can be the nucleus mass or the DM mass
Assume the energy distribution after the collision is equipartition, the average DM kinetic energy being captured after collision is

$$\bar{E} = \frac{m_\chi}{4} \left( \frac{m_\chi - m_A}{m_\chi + m_A} \right)^2 u^2 + \frac{m_\chi}{2} \frac{(m_\chi^2 + m_A^2)}{(m_\chi + m_A)^2} v_{\text{esc}}^2(r)$$

Base on equipartition, the probability that an individual scattering leads to capture is

$$P_{\text{cap}} = \frac{v_{\text{esc}}^2(r)}{w^2} \left[ 1 - \frac{u^2}{v_{\text{esc}}^2(r)} \frac{(M_x - m)^2}{4M_x m} \right]$$
Therefore, the energy flow per shell volume due to DM-nucleus scattering is

\[
\frac{dJ_c}{dV} = \int n_A \sigma v_{esc}(r) \frac{f(u)}{u} \times \left[ 1 - \frac{(m_X - m_A)^2}{4m_X m_A} \frac{u^2}{v_{esc}(r)^2} \right] \tilde{E} du
\]

The DM velocity distribution in the halo, \( f(u) \), is assumed to be Maxwell-Boltzmannian

\[
f(x) = \sqrt{\frac{6}{\pi}} \frac{\rho_o}{m_X \bar{v}} x^2 e^{-x^2} e^{-\eta^2} \frac{\sinh(2x\eta)}{x\eta}
\]

\[
x^2 = 3(u/\bar{v})^2 / 2 \quad , \quad \eta^2 = 3(v_\odot/\bar{v})^2 / 2
\]

\( \bar{v} \sim 270 \text{ kms}^{-1} \) is the DM dispersion velocity in the halo

\( v_\odot = 220 \text{ kms}^{-1} \) is the relative velocity between the Sun and the MW
The leading contribution of the total energy flow due to gravitational capture is

\[ J_c = \xi \sum_A b_A \frac{(m^2_{\chi} + m^2_A)}{(m_{\chi} + m_A)^2} \left( \frac{\sigma_{\chi A}}{\text{pb}} \right) \langle \phi_A^2 \rangle \]

\( b_A \) is the number fraction of nucleus, \( \langle \phi_A^2 \rangle \) is the average gravitational potential square as a result of nucleus A

\[ \xi \equiv \sqrt{\frac{3}{8}} N_\odot \rho_0 \frac{v_{\text{esc}(R_\odot)}}{\bar{v}} v_{\text{esc}(R_\odot)} \frac{\text{erf}(\eta)}{\eta} \]

\[ \approx 1.2 \times 10^{23} \text{ GeV s}^{-1} \left( \frac{\rho_0}{0.3 \text{ GeV/cm}^3} \right) \left( \frac{270 \text{ km/s}}{\bar{v}} \right) \]
Similarly, the energy flow due to self-capture $J_S$ can be derived by setting $m_A \rightarrow m_\chi$ and $n_A \rightarrow n_\chi$ ($n_\chi$ is the DM number density in the Sun)

$$J_s \approx \sqrt{\frac{3}{32}} \rho_0 \sigma_{xx} \frac{\text{erf}(\eta)}{\eta} \frac{v_{\text{esc}}(R_\odot)}{\bar{v}} v_{\text{esc}}^3(R_\odot) \langle \phi_\chi \rangle^2$$

Here I use $\langle \phi_\chi^2 \rangle \approx \langle \phi_\chi \rangle^2$ since DM is concentrated about less than 0.1 solar radius and $\langle \phi_\chi \rangle = 5.1$ is the average gravitational potential of the DM
The energy of captured DM could be dissipated due to annihilation. The energy flow due to this process is

\[ J_a = \frac{\int 4\pi r^2 n_X^2(r) E_X(t)dr}{(\int 4\pi r^2 n_X(r)dr)^2} \langle \sigma v \rangle \]

\[ \langle \sigma v \rangle \] is the thermal-average DM annihilation cross section.

\[ J_a(t) \approx 7.5 \times 10^{-65} \text{ GeV s}^{-1} \left( \frac{m_X}{10 \text{ GeV}} \right)^{3/2} \left( \frac{E_X(t)}{\text{GeV}} \right)^{-1/2} \]
Lastly, the captured DMs will continuously exchange energy with the solar nuclei.

\[
J_{\chi} = 8 \sqrt{\frac{2}{\pi}} \rho_c m_{\chi} \frac{k_B(T_{\odot} - T_{\chi})}{(m_{\chi} + m_A)^2} \\
\times \sum_A f_A \sigma_{\chi A} \left( \frac{m_A k_B T_{\chi} + m_{\chi} k_B T_{\odot}}{m_{\chi} m_A} \right)^{1/2}
\]

\( \rho_c \approx 110 \text{ g/cm}^3 \) is the core density of the Sun, \( f_A \) is the mass fraction of nuclei A.
Thermal equilibrium conditions

In order to study the temperature evolution of the trapped DM, let’s compare the mean collision time between a pair of trapped DMs and that between a trapped DM and nucleus in the Sun.

\[
\tau_{xx}(t) \simeq \frac{V_\odot}{N_x(t)\sigma_{xx}\bar{v}} \quad \text{and} \quad \tau_{\chi\odot} \simeq \frac{V_\odot}{\sum_i N_i\sigma_{\chi A_i}^{SI}\bar{v}}
\]

The time scale \(\tau_{\chi}^{eq}\) for DMs in the Sun to reach thermal equilibrium can be estimated by the condition

\[
\tau_{\chi}^{eq} \simeq \tau_{xx}(\tau_{\chi}^{eq})
\]
Consider at early stage and $N_\chi(\tau_\chi^{eq})$ is still far from the maximal value. In this case, 

$$\dot{N}_\chi(\tau_\chi^{eq}) = C_c \tau_\chi^{eq} \quad \text{for} \quad C_s^2 \gg 4C_c C_a,$$

we obtain

$$\tau_\chi^{eq} = \sqrt{V_\odot/C_c \sigma_{\chi\chi} \bar{v}}.$$
we are interested in the region that

\[ r < 1 \quad \text{and} \quad C_s^2 / 4C_cC_a \equiv 1.9 \times 10^3 (\sigma_{\chi\chi} / \sigma_{\chi p}^{\text{SI}}) (\sigma_{\chi\chi} / \text{cm}^2) \gtrsim 1 \]

\[(\sigma_{\chi p}^{\text{SI}}, \sigma_{\chi\chi}) = (10^{-45} \text{cm}^2, 10^{-23} \text{cm}^2) \] (typical values for current constraints)

gives \( \tau_{\chi}^{\text{eq}} = 4.5 \times 10^{13} \text{ s} \) which is much shorter than the age of the Sun \( 10^{17} \text{ s} \)

we justify the thermal equilibrium state of DM

we are able to write \( E_{\chi}(t) = s\kappa_B T_{\chi}(t) / 2 \)

and put in the equations derived above for \( t > 10^{13} \text{ s} \)
$N_X$ AND $T_X$ EVOLUTIONS

- stronger $\sigma_{xp}$
weaker $\sigma_{\chi p}$
some remarks:

- $\bar{E}$ is the DM average kinetic energy before the thermalization, and it is taken as the initial condition for $E_\chi(t)$ at $t = 10^{13}$ s.

$$\bar{E} = \frac{m_\chi}{4} \left( \frac{m_\chi - m_A}{m_\chi + m_A} \right)^2 u^2 + \frac{m_\chi}{2} \frac{m_\chi^2 + m_A^2}{(m_\chi + m_A)^2} v_{\text{esc}}(r)$$

- When DMs reach to the thermal equilibrium, they are populated more closely to the solar core. Hence one expects $E_\chi(t_0) > \bar{E}$.

- $J_a$ does not affect the DM temperature. $J_c$ and $C_c$ are constants, as $N_\chi$ accumulates they become negligible. We have

$$\frac{dE_\chi(t)}{dt} \approx J_\chi + J_s - C_s E_\chi(t)$$

This eq. approaches zero when the system is balanced. Hence the final $T_\chi$ depends on $m_\chi$, $\sigma_{\chi p}$ and $\sigma_{\chi\chi}$, does not depend on the initial condition.
annihilation rate with temperature correction
CONCLUSION

- We solve the general DM evolution equation with $C_s$ and $C_e$ inside the Sun.

- DM self-interaction is significant in $m_\chi \sim $ GeV scales.

- DM self-interaction will enhance the trapped DM number density and lower the critical mass.

- DM self-interaction help the reach of equilibrium state quicker.

- DM self-interaction is testable in IceCube-PINGU.

- Complementary (direct and indirect detections) test of SIDM models is studied, in particular, for the isospin violation regions and low DM mass regions.

- Temperature evolution of DM is resolved and it corrected annihilation rate is given. The derivations are quit general, one can apply to any halo systems and celestial objects if the distributions are known.
Thank you for your attention!
effective volume for both track and cascade signals

baseline 40 string configuration efficiencies for MultiNest reconstruction of neutrinos

<table>
<thead>
<tr>
<th>Flavor (Interaction)</th>
<th>$N_{\text{reco}}/N_{\text{total}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_e$(CC)</td>
<td>90.1 ± 0.5%</td>
</tr>
<tr>
<td>$\nu_\mu$(CC)</td>
<td>93.1 ± 0.6%</td>
</tr>
<tr>
<td>$\nu_\tau$(CC)</td>
<td>99.0 ± 1.0%</td>
</tr>
<tr>
<td>$\nu$(NC)</td>
<td>87.2 ± 1.7%</td>
</tr>
</tbody>
</table>
we consider $2\sigma$ detection significance in 5 years

$$\frac{N_\nu}{\sqrt{N_\nu + N_{\text{atm}}}} = 2.0$$

the atmospheric neutrino backgrounds

$$N_{\text{atm}} = \int_{E_{\text{th}}}^{\infty} \frac{d\Phi_{\nu, \text{atm}}}{dE_\nu, d\Omega} A^{\nu}_{\text{eff}}(E_\nu) dE_\nu d\Omega$$

ATM fluxes, Honda et al.

$0.3 \leq \cos \theta_z \leq 0.4$
The Sun is roughly 23° above the horizon, $\cos \theta_z \sim 0.39$, in Antarctica during the daylight, that is why we take the atmospheric neutrino flux within $0.3 < \cos \theta_z < 0.4$

It is across about 6 ~ 7 degrees which roughly fits the angular resolution of IceCube-PINGU in the relevant range.

The IceCube-PINGU detector at $E_\nu = 5$ GeV is roughly $10^0$. We consider neutrino events arriving from the solid angle range surrounding the Sun with 10 degrees.

arXiv:1401.2046