The landscape of flavor composition of high-energy astrophysical neutrinos

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TeVPA 2015

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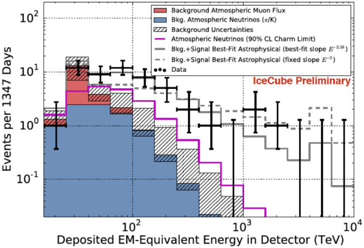
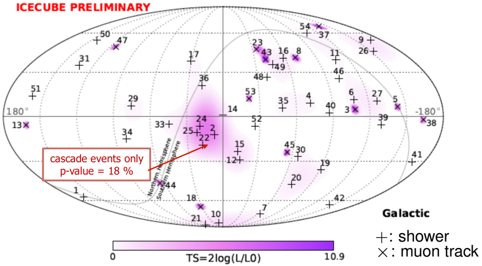
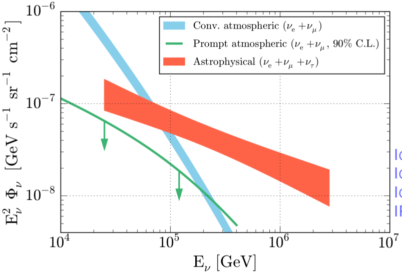
From high-energy neutrino astronomy . . .

IceCube has reported 54 events with 30 TeV – 2 PeV in 4 years





$\sim 7\sigma$ astrophysical excess



IceCube, *PRL* **111**, 021103 (2013)
IceCube, *Science* **342**, 1242856 (2013)
IceCube, *PRL* **113**, 101101 (2014)
IPA 2015, ICRC 2015

…to high-energy neutrino particle physics

After the sources, the second most important question to answer is:

What is the proportion of ν_e, ν_μ, ν_τ in the diffuse flux?

Knowing this can reveal two important pieces of information:

- the physical conditions at the neutrino sources; and
- whether there is new physics, and of what kind

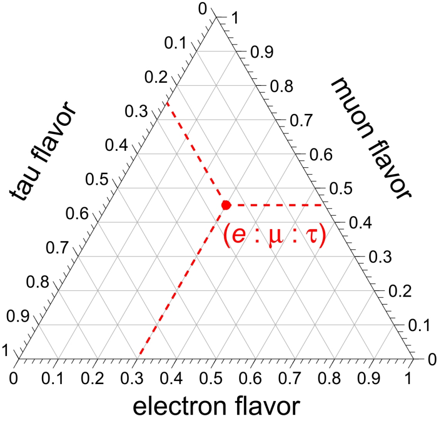
We need to explore what to expect from theory

[Barenboim, Quigg, *PRD* **67**, 073024 (2003)]
[Winter, *PRD* **88**, 083007 (2013)]
[Mena, Palomares, Vincent, *PRL* **113**, 091103 (2014)]
[Palomares, Vincent, Mena, *PRD* **91**, 103008 (2015)]
[Palladino, Pagliaroli, Villante, Vissani, *PRL* **114**, 171101 (2015)]

“Flavor triangle” or Dalitz/Mandelstam plot

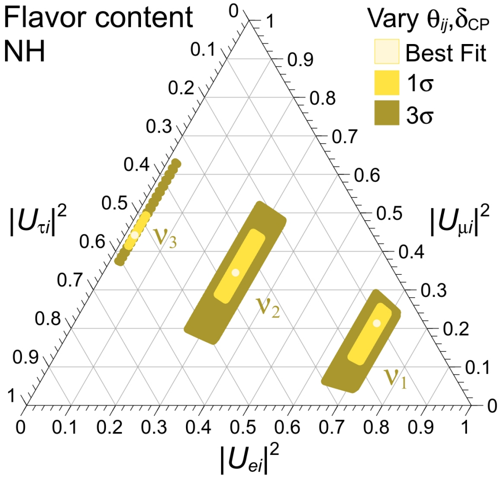
Assumes underlying unitarity: sum of projections on each axis is 1

How to read it: follow the tilt of the tick marks, *e.g.*,



Flavor content of the mass eigenstates ν_1, ν_2, ν_3

Show the *e*, μ, and τ content of the ν_i via ternary plots:



[MB, BEACOM, WINTER, *PRL* **115**, 161302 (2015)]

Flavor ratios

- Neutrino production at the source via pion decay:

$$p\gamma \to \Delta^+(1232) \to \pi^+ n \qquad \pi^+ \to \mu^+ \nu_\mu \to e^+ \nu_e \bar{\nu}_\mu \nu_\mu$$

- Flavor ratios at the source: $(f_e : f_\mu : f_\tau)_S \approx (1/3 : 2/3 : 0)$
- At Earth, due to flavor mixing:

$$f_{\alpha,\oplus} = \sum_\beta P_{\nu_\beta \to \nu_\alpha} f_{\beta,S} = \sum_\beta \left(\sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2 \right) f_{\beta,S}$$

$$(1/3 : 2/3 : 0)_S \xrightarrow{\text{flavor mixing, NH, best-fit}} (0.36 : 0.32 : 0.32)_\oplus$$

- Other compositions at the source:

$$\begin{aligned} (0 : 1 : 0)_S &\longrightarrow (0.26 : 0.36 : 0.38)_\oplus \quad \text{(“muon damped”)} \\ (1 : 0 : 0)_S &\longrightarrow (0.55 : 0.26 : 0.19)_\oplus \quad \text{(“neutron decay”)} \\ (1/2 : 1/2 : 0)_S &\longrightarrow (0.40 : 0.31 : 0.29)_\oplus \quad \text{(“charmed decays”)} \end{aligned}$$

How does IceCube see flavor?

Below $E_\nu \sim 5$ PeV, there are two event topologies:

- Showers: generated by CC ν_e or ν_τ; or by NC ν_x
- Muon tracks: generated by CC ν_μ

(Some muon tracks can be mis-reconstructed as showers)

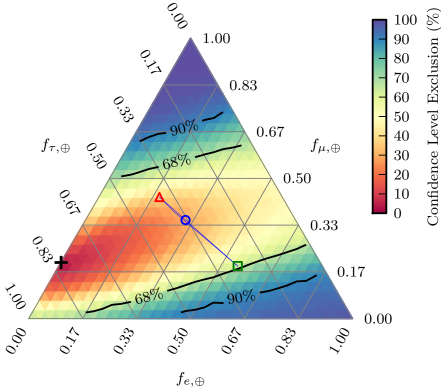
At $\gtrsim 5$ PeV (no events so far), all of the above, plus:

- Glashow resonance: CC $\bar{\nu}_e e$ interactions at 6.3 PeV
- Double bangs: CC $\nu_\tau \to \tau \to \nu_\tau$

Flavor ratios must be inferred from the number of showers and tracks

Using contained events only

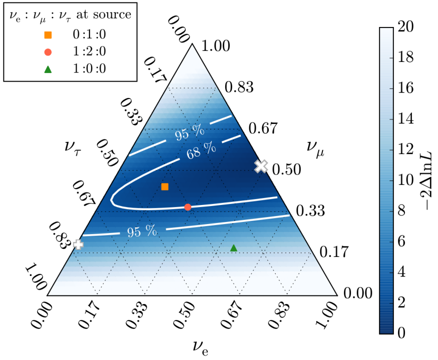
[IceCube Coll., *PRL* **114**, 171102 (2015)]



Best fit: $(0 : 0.2 : 0.8)_\oplus$

Using contained events + throughgoing muons

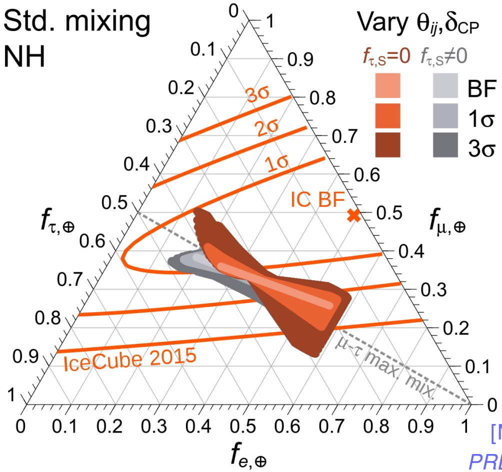
[IceCube Coll., *ApJ* **809**, 98 (2015)]



Best fit: $(0.49 : 0.51 : 0)_\oplus$

- Compatible with standard source compositions
- Bounds are weak – need more data and better flavor-tagging

Assume unconstrained flavor composition at source (with and w/o ν_τ):



[MB, Beacom, Winter, *PRL* **115**, 161302 (2015)]

Std. mixing can access *only* $\sim$ 10% of the possible combinations

Flavor combinations at Earth from std. mixing

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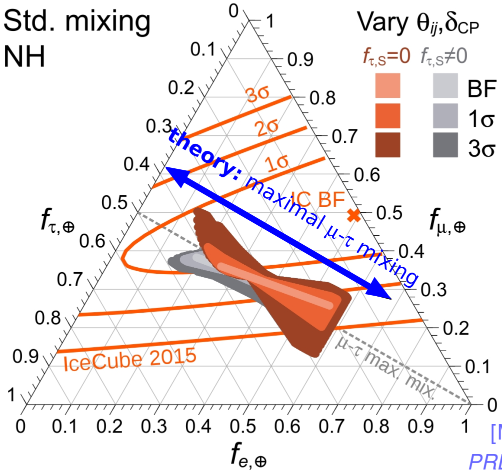

[MB, Beacom, Winter, *PRL* **115**, 161302 (2015)]

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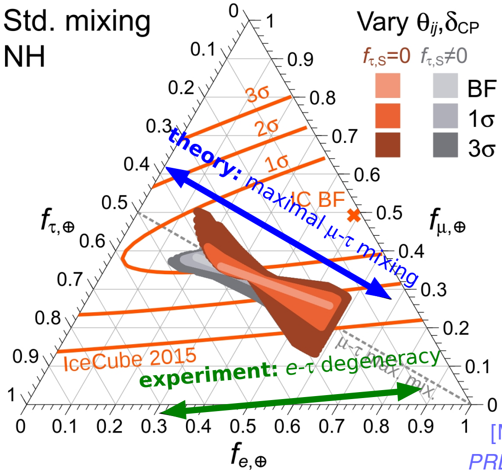
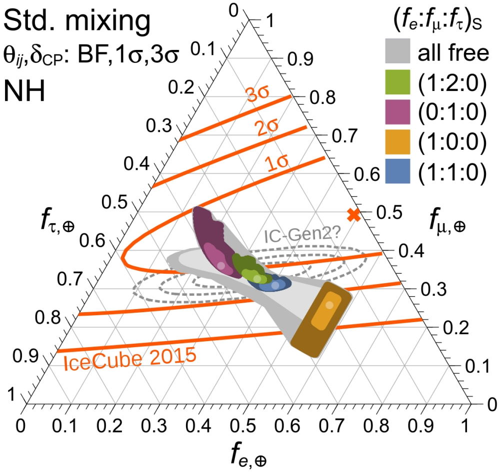


[MB, Beacom, Winter, *PRL* **115**, 161302 (2015)]

Std. mixing can access *only* $\sim$ 10% of the possible combinations

Selected source compositions

We can look at results for particular choices of ratios at the source:



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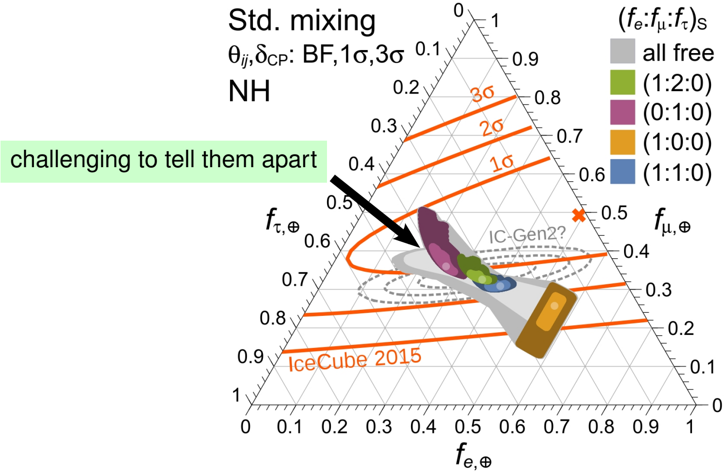

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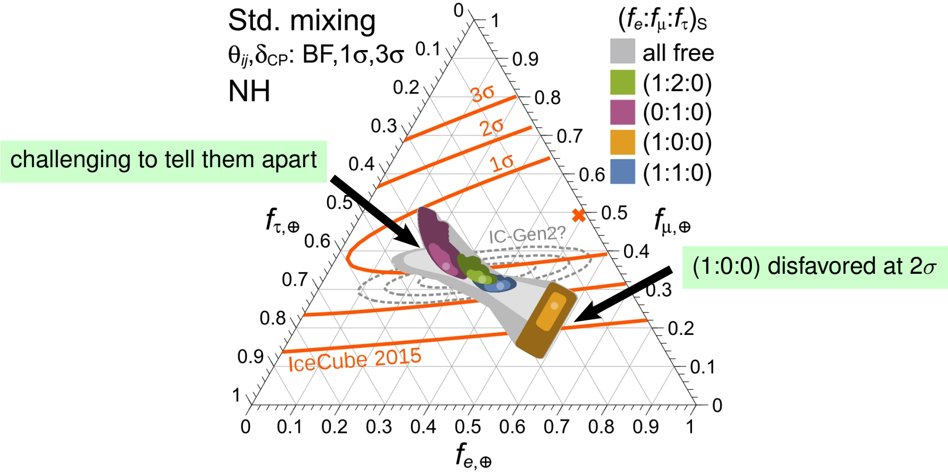
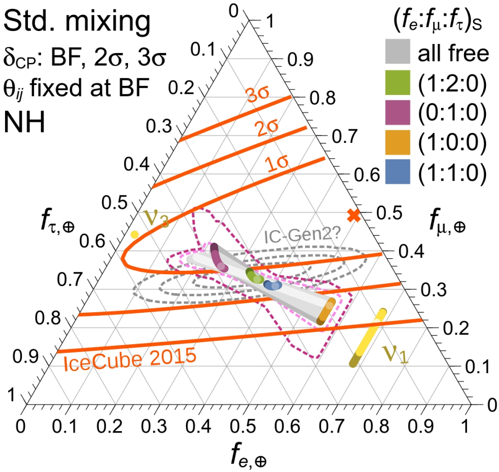


[MB, Beacom, Winter, *PRL* **115**, 161302 (2015)]

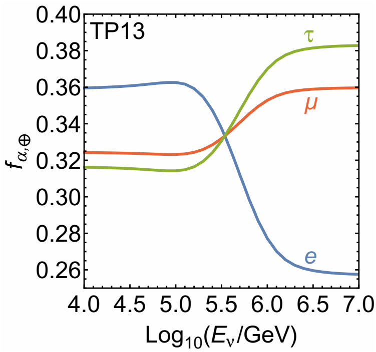
In a few years, we might know all the mixing parameters except δ_{CP}:

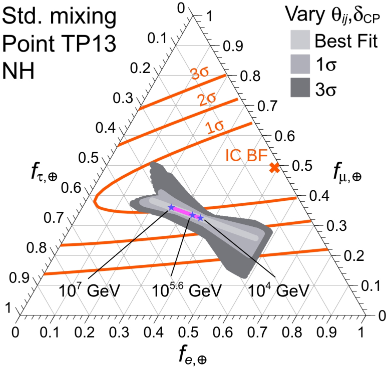


[MB, Beacom, Winter, *PRL* **115**, 161302 (2015)]

Energy dependence of the composition at the source

Different ν production channels are accessible at different energies





[MB, BEACOM, WINTER, *PRL* **115**, 161302 (2015)]

- TP13: $p\gamma$ model, target photons from co-accelerated electrons
  [HÜMMER *et al.*, *Astropart. Phys.* **34**, 205 (2010)]
- Equivalent to different sources types contributing to the diffuse flux
- Will be difficult to resolve
  [KASHTI, WAXMAN, *PRL* **95**, 181101 (2005)] [LIPARI, LUSIGNOLI, MELONI, *PRD* **75**, 123005 (2007)]

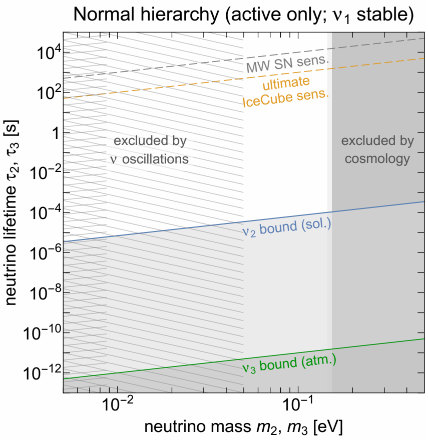
New physics: effect on the flavor composition

- New physics in the neutrino sector could affect the
  - production; and/or
  - propagation; and/or
  - detection
- Detection: probe NP in the ν interaction length via the angular dependence of the flux [Marfatia, McKay, Weiler, 1502.06337]
- NP at production and propagation could modify the incoherent mixture of ν_1, ν_2, ν_3
- Example: neutrino decay ►

[Barenboim, Quigg, *PRD* **67**, 073024 (2003)]
[Beacom, Bell, Hooper, Pakvasa, Weiler, *PRL* **90**, 181301 (2003)]
[Maltoni, Winter, *JHEP* **07**, 064 (2008)]
[Baerwald, MB, Winter, *JCAP* **1210**, 020 (2012)]
[Pagliaroli, Palladino, Vissani, Villante 1506.02624]

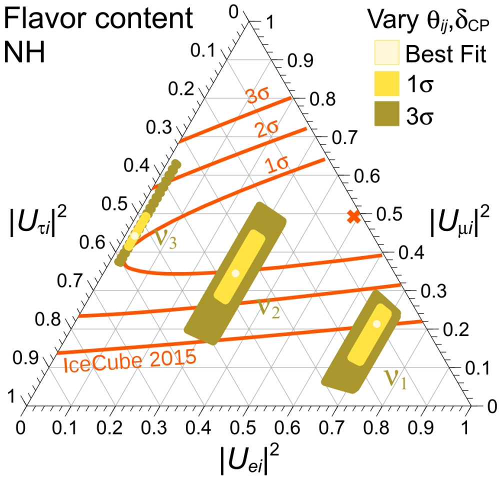
Neutrino decay

- SM: ν lifetimes are $> 10^{36}$ yr
- Via new-physics decay modes, they could be shorter
- Consider two possibilities:
  - NH: $\nu_2, \nu_3 \to \nu_1$
  - IH: $\nu_1, \nu_2 \to \nu_3$
- There are experimental bounds on the lifetime τ_i / m_i



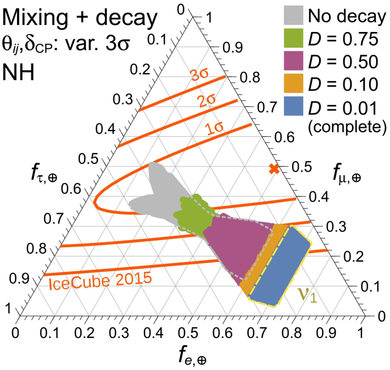
[MB, Beacom, Murase, in prep.]

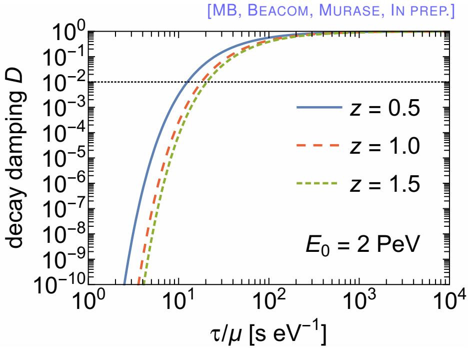
Flavor ratios are currently more sensitive to complete decay in the NH (only ν_1 survive) than in the IH (only ν_3 survive):



Flavor ratios with decay in the NH ($\nu_2, \nu_3 \to \nu_1$):

$$f_{\alpha,\oplus}\left(E_0, z, \tau_j/m_j\right) = |U_{\alpha 1}|^2 + \sum_{j=2,3} \left(|U_{\alpha j}|^2 - |U_{\alpha 1}|^2\right) f_{j,\mathrm{S}}\, D\left(E_0, z, \tau_j/m_j\right)$$





$D \lesssim 0.01$ implies a bound of $\tau_2/m_2, \tau_3/m_3 \gtrsim 10$ s eV^{-1} at $\gtrsim 2\sigma$

Decay: lifetime bounds with current IceCube data

Flavor ratios with decay in the NH ($\nu_2, \nu_3 \to \nu_1$):

$D \in [0, 1]$: exponential decay damping
[MB+ *JCAP* **1210**, 020 (2012)]

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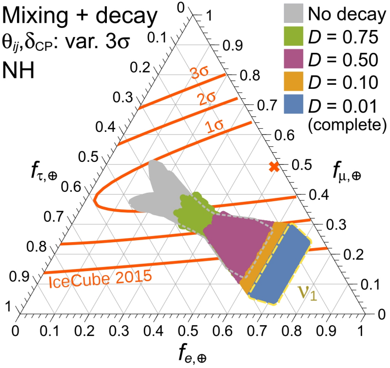
$D \to 0$: complete decay

$D \to 1$: no decay



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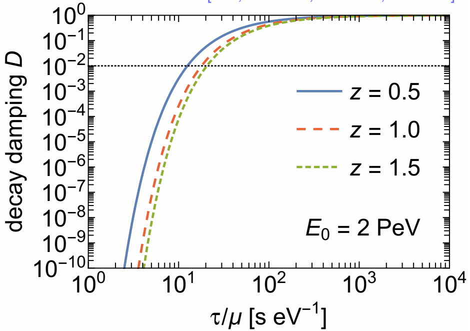

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[... 1210, 020 (2012)]

$f_{\alpha,\oplus}(E_0, z, \tau_j/ \ldots D(E_0, z, \tau_j/m_j)$

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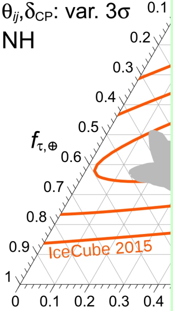
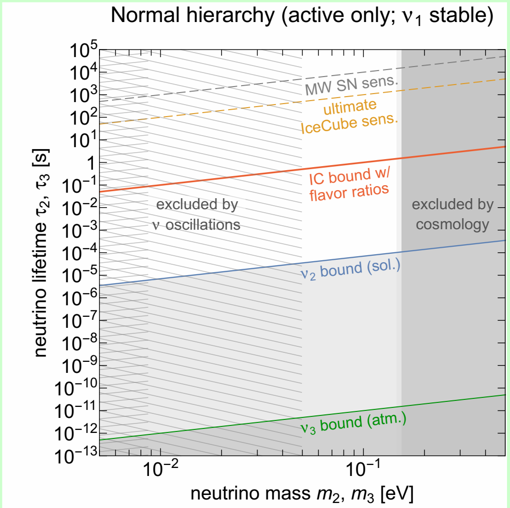
[... ACOM, MURASE, IN PREP.]

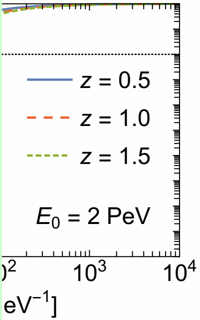
Mixing + decay

θ_{ij}, δ_{CP}: var. 3σ

NH







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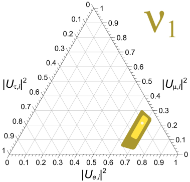
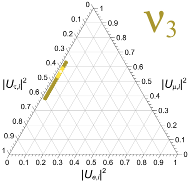
Decay: complete vs. incomplete

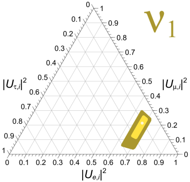
- Complete decay: only ν_1 (ν_3) reach Earth assuming NH (IH)

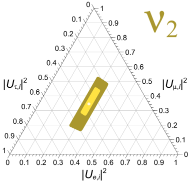


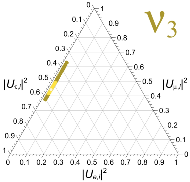
or



- Incomplete decay: incoherent mixture of ν_1, ν_2, ν_3 reaches Earth

$\alpha\Big($ 

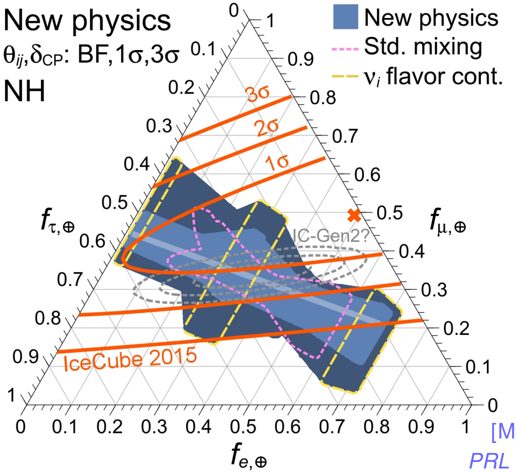
$\Big) + \beta\Big($ 

$\Big) + \gamma\Big($ 

$\Big)$

New physics that changes the ν_i mixture

Region of all linear combinations of ν_1, ν_2, ν_3:



[MB, Beacom, Winter, *PRL* **115**, 161302 (2015)]

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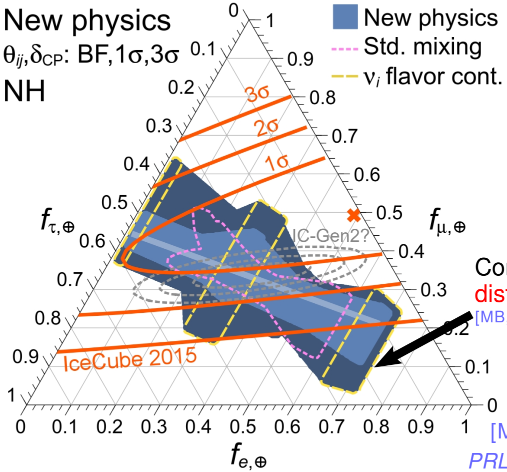

Complete decay in NH
disfavored at $> 2\sigma$
[MB, Beacom, Murase, In prep.]

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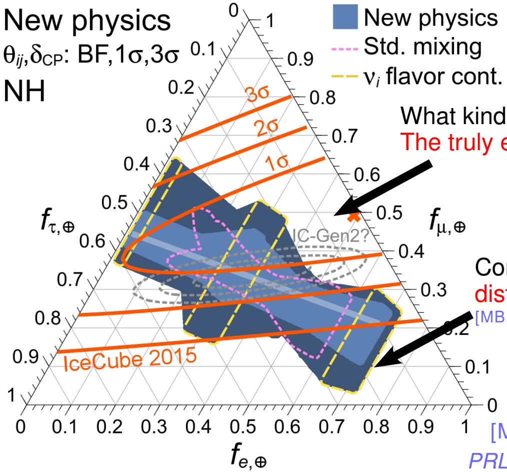

What kind of NP lives outside?
The truly exotic kind!

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[MB, BEACOM, MURASE, IN PREP.]

[MB, BEACOM, WINTER, *PRL* **115**, 161302 (2015)]

This class of NP can access *only* $\sim$ 25% of the possible combinations

New physics — of the *truly exotic* kind

What kind of NP lives outside the blue region?

- NP that changes the values of the mixing parameters, *e.g.*,
  - violation of Lorentz and CPT invariance

    [Barenboim, Quigg, *PRD* **67**, 073024 (2003)] [MB, Gago, Peña-Garay, *JHEP* **1004**, 005 (2010)]
  - violation of equivalence principle

    [Gasperini, *PRD* **39**, 3606 (1989)] [Glashow *et al.*, *PRD* **56**, 2433 (1997)]
  - coupling to a torsion field

    [De Sabbata, Gasperini, *Nuovo. Cim.* **A65**, 479 (1981)]
  - renormalization-group running of mixing parameters

    [MB, Gago, Jones, *JHEP* **1105**, 133 (2011)]
- active-sterile mixing [Aeikens *et al.*, 1410.0408]
- flavor-violating physics
- ν–$\bar{\nu}$ mixing (if ν, $\bar{\nu}$ flavor ratios are considered separately)

New physics — high-energy effects (I)

Add a new-physics term to the standard oscillation Hamiltonian:

$$H_{\text{tot}} = H_{\text{std}} + H_{\text{NP}}$$

$$H_{\text{std}} = \frac{1}{2E} U_{\text{PMNS}}^\dagger \, \text{diag}\left(0, \Delta m_{21}^2, \Delta m_{31}^2\right) U_{\text{PMNS}}$$

$$H_{\text{NP}} = \sum_n \left(\frac{E}{\Lambda_n}\right)^n U_n^\dagger \, \text{diag}\left(O_{n,1}, O_{n,2}, O_{n,3}\right) U_n$$

$n = 0$

- coupling to a torsion field
- CPT-odd Lorentz violation

$n = 1$

- equivalence principle violation
- CPT-even Lorentz violation

Experimental upper bounds from atmospheric ν's:

$O_0 \lesssim 10^{-23}$ GeV $\qquad$ $O_1/\Lambda_1 \lesssim 10^{-27}$ GeV

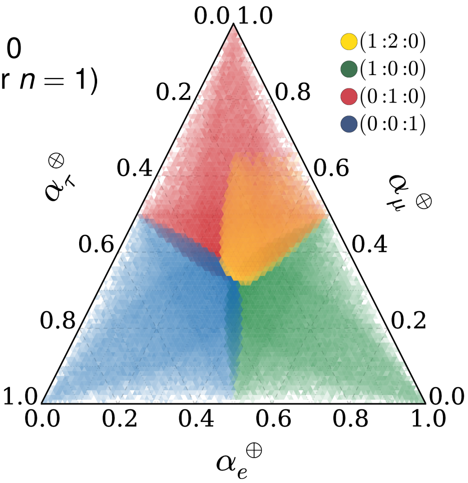
[Argüelles, Katori, Salvadó, *PRL* **115**, 161303 (2015)]
[Super-K Coll., *PRD* **91**, 052003 (2015)]
[MB, Gago, Peña-Garay, *JHEP* **1004**, 005 (2010)]
[IceCube Coll., *PRD* **82**, 112003 (2010)]

Truly exotic new physics is indeed able to populate the white region:

- use current bounds on $O_{n,i}$
- sample the unknown NP mixing angles

[Argüelles, Katori, Salvadó, *PRL* **115**, 161303 (2015)]

$n = 0$
(similar for $n = 1$)

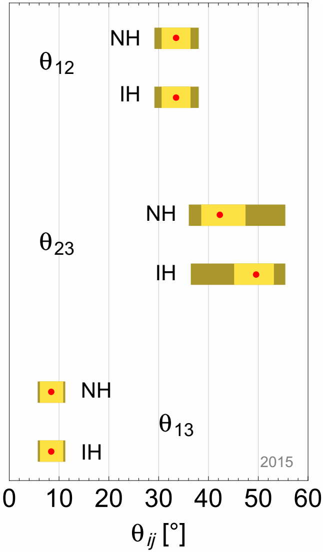


Conclusions . . . and the future

- The space of allowed flavor compositions is surprisingly small:
  - Standard mixing: $\sim$ 10% of all possibilities
  - ν_i-mixing new physics: $\sim$ 25% (*e.g.*, decay)
- Only a broader class of new physics (*e.g.*, CPT violation) can access all compositions
- IceCube can improve the lifetime bounds in the NH (now!) and IH (soon!) by several orders of magnitude
- Upcoming improvements in the determination of mixing parameters and in flavor-tagging will help identify the sources

Backup slides

Normal vs. inverted mass hierarchy



PMNS matrix U depends on θ_{12}, θ_{23}, θ_{13}, δ_{CP}.

The neutrino mass hierarchy is unknown:

- Normal hierarchy (NH): ν_1 is lightest
- Inverted hierarchy (IH): ν_3 is lightest

Using the latest fits from GONZÁLEZ-GARCÍA *et al.*, *JHEP* **1411**, 052 (2014):

- θ_{12} and θ_{13} are well-determined
- Little NH/IH difference for θ_{12} and θ_{13}
- Large error and NH/IH difference for θ_{23}
- At 3σ, NH and IH regions are equal

Why do we expect HE neutrinos?

Joint production of UHECRs, ν's, and γ's:

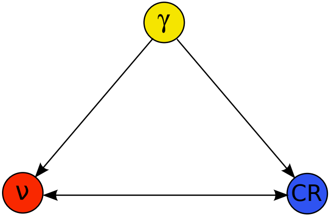
power law $\sim E^{-\alpha_p}$ broken power law

$$p \; \gamma \to \Delta^+(1232) \to \begin{cases} n\pi^+ \,, & \text{BR} = 1/3 \\ p\pi^0 \,, & \text{BR} = 2/3 \end{cases}$$

$$\pi^+ \to \mu^+ \nu_\mu \to \bar{\nu}_\mu e^+ \nu_e \nu_\mu$$

$$\pi^0 \to \gamma\gamma$$

$$n \text{ (escapes)} \to p e^- \bar{\nu}_e$$



After propagation, with flavor mixing:

$$\nu_e : \nu_\mu : \nu_\tau : p = 1 : 1 : 1 : 1$$

("one ν_μ per cosmic ray")

This neutron model of CR emission is now strongly disfavored

[AHLERS *et al.*, *Astropart. Phys.* **35**, 87 (2011)] [ICECUBE COLL., *Nature* **484**, 351 (2012)]

But we can do better by letting the p's escape without interacting

[BAERWALD, MB, WINTER, *ApJ* **768**, 186 (2013)] [BAERWALD, MB, WINTER, *Astropart. Phys.* **62**, 66 (2015)]
[MB, BAERWALD, MURASE, WINTER, *Nat. Commun.* **6**, 6783 (2015)]

Flavor mixing in high-energy astrophysical neutrinos

Probability of $\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta$ transition:

$$P_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} = \delta_{\alpha\beta} - 4 \sum_{k>j} \text{Re}\left(J_{\alpha\beta jk}\right) \sin^2\left(\frac{\Delta m_{kj}^2 L}{4E}\right) \pm 2 \sum_{k>j} \text{Im}\left(J_{\alpha\beta jk}\right) \sin\left(\frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$\text{For } \begin{cases} E \sim 1 \text{ PeV} \\ \Delta m_{kj}^2 \sim 10^{-4} \text{ eV}^2 \end{cases} \Rightarrow L_{\text{osc}} \sim 10^{-10} \text{ Mpc} \ll L = 10 \text{ Mpc} - \text{few Gpc}$$

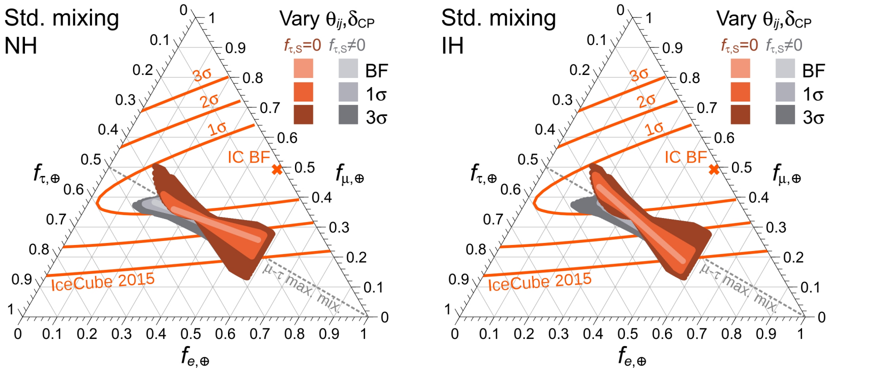
- Therefore, oscillations are very rapid
- They average out after only a few oscillations lengths:

$$\sin^2(\ldots) \to 1/2 \,, \quad \sin(\ldots) \to 0$$

Hence, for high-energy astrophysical neutrinos:

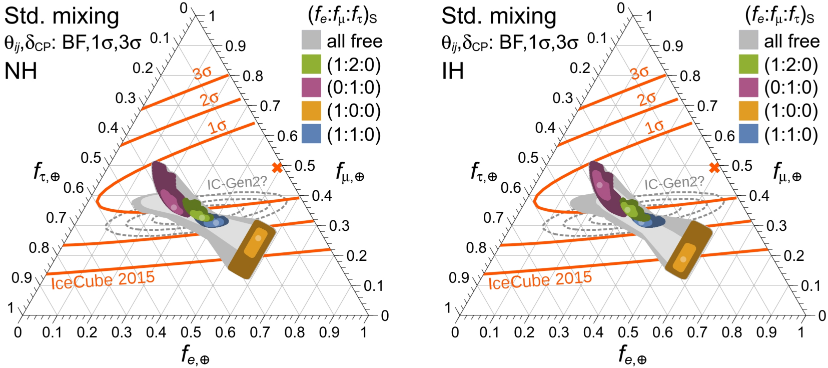
$$P_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} = \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2$$ ◀ incoherent mixture of mass eigenstates

Flavor combinations from std. flavor mixing: NH vs. IH



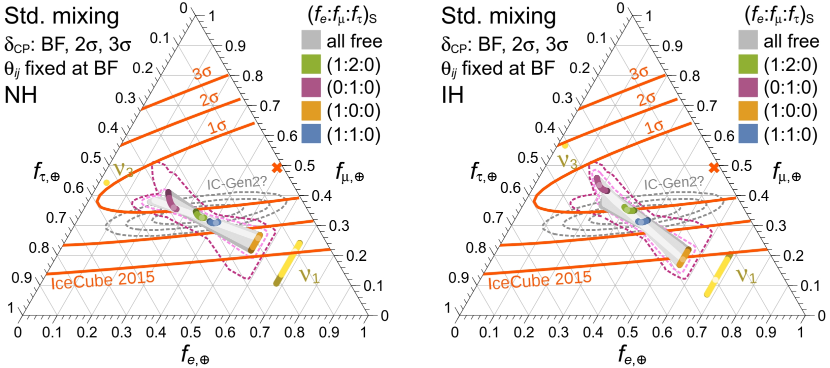
[MB, Beacom, Winter, *PRL* **115**, 161302 (2015)]

Selected source compositions: NH vs. IH



[MB, Beacom, Winter, *PRL* **115**, 161302 (2015)]

Perfect knowledge of mixing angles: NH vs. IH



[MB, Beacom, Winter, *PRL* **115**, 161302 (2015)]

Decay: effect on flavor ratios

$$f_{\alpha,\oplus}\left(E_0, z, \kappa_j^{-1}\right) = |U_{\alpha l}|^2 + \sum_{j \neq l} \left(|U_{\alpha j}|^2 - |U_{\alpha l}|^2\right) f_{j,S}\, D\left(E_0, z, \kappa_j^{-1}\right)$$

$l = 1$ (NH), 3 (IH)

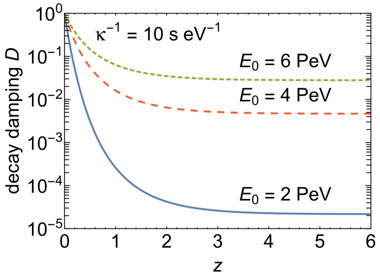
$\nu_{1,2,3}$ ratios at source

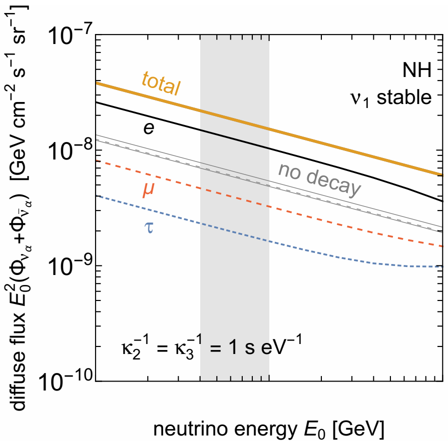
- Damping due to decay:

$$0 < D < 1$$

- Complete decay:

$$D \to 0 \Rightarrow f_{\alpha,\oplus} = |U_{\alpha l}|^2$$

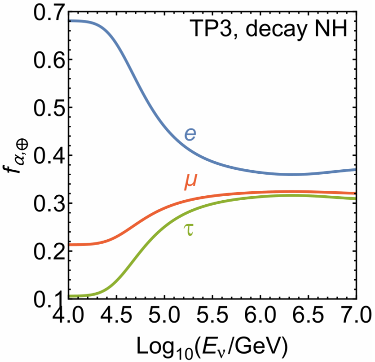


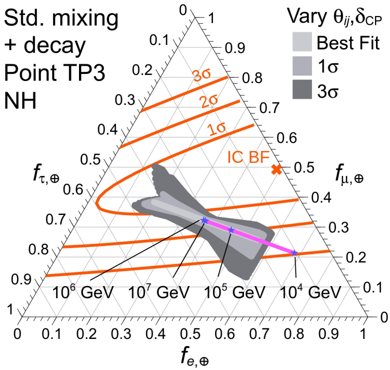


[MB, Beacom, Murase, in prep.]

Decay: seeing the energy dependence?

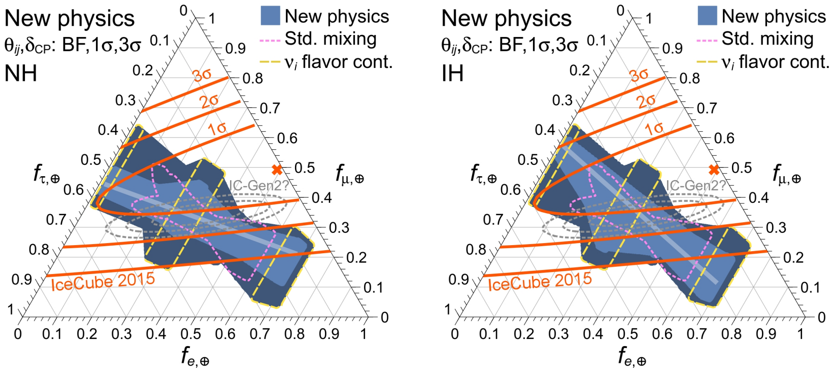
- The effect of decay shows up at low energies
- *e.g.*, for a model of AGN cores [HÜMMER *et al.*, *Astropart. Phys.* **34**, 205 (2010)],





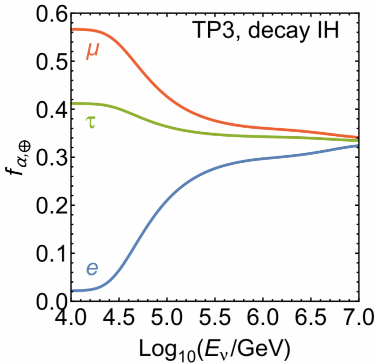
[MB, BEACOM, WINTER, *PRL* **115**, 161302 (2015)]

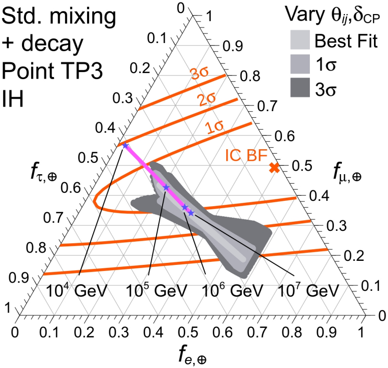
New physics: NH vs. IH



[MB, Beacom, Winter, *PRL* **115**, 161302 (2015)]

New physics: decay in the IH





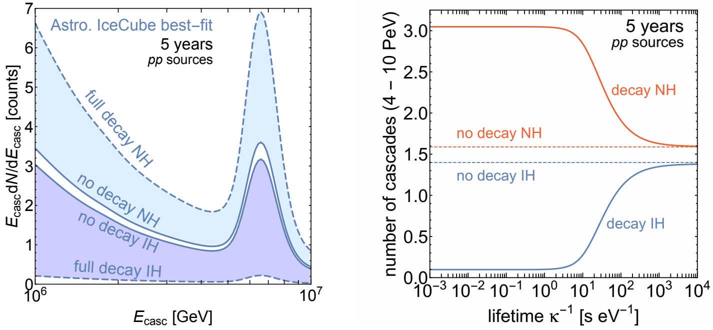
[MB, Beacom, Winter, *PRL* **115**, 161302 (2015)]

Decay: cascade rate probes the IH

- Around 6.3 PeV, the Glashow resonance is accessible:

$$\bar{\nu}_e + e \to W \to \text{ hadronic shower (BR} = 67\%)$$

- Three scenarios:
  - Neutrinos are stable: we see the GR as a bump in the cascade rate
  - Neutrinos decay in the NH: the bump is larger ($|U_{e1}|^2$ is large)
  - Neutrinos decay in the IH: no or almost no cascades ($|U_{e3}|^2$ is tiny)



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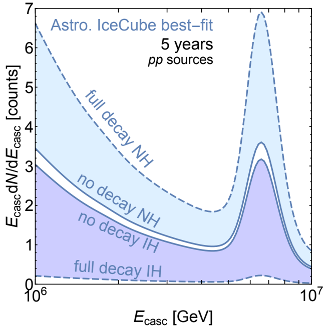

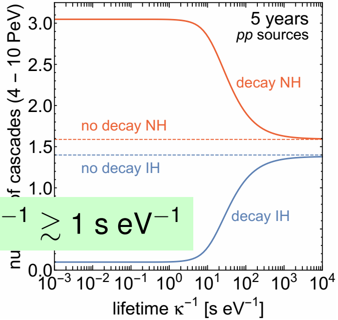

$$\kappa^{-1} \gtrsim 1 \text{ s eV}^{-1}$$

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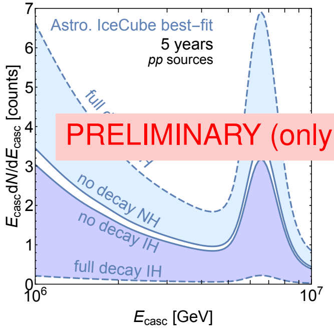

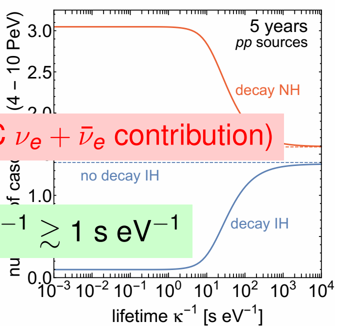

PRELIMINARY (only CC $\nu_e + \bar{\nu}_e$ contribution)

$\kappa^{-1} \gtrsim 1$ s eV^{-1}

[MB, BEACOM, MURASE, IN PREP.]

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  $\bar{\nu}$ … 67%)

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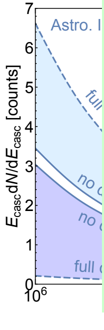
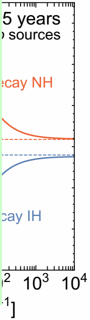
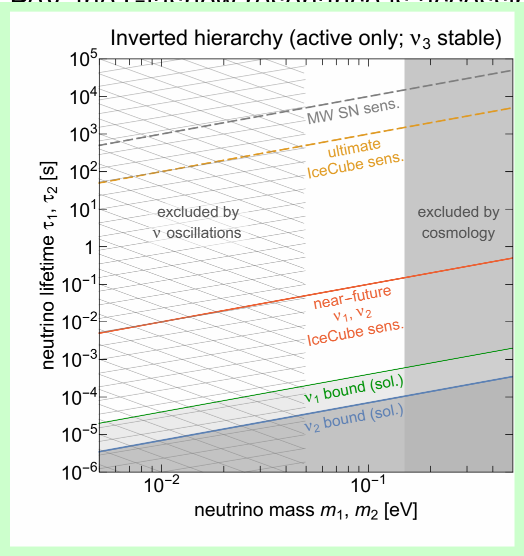




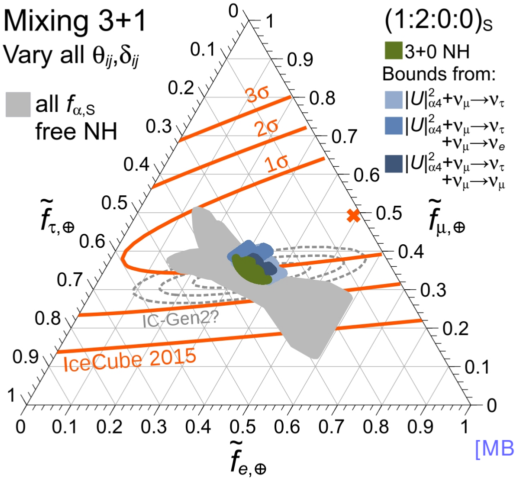
[MB, BEACOM, MURASE, IN PREP.]



New physics — active-sterile mixing

Mixing with a sterile neutrino (3+1) changes the flavor ratios:

- standard parameters: θ_{12}, θ_{23}, θ_{13}, δ_{13}
- sterile parameters: θ_{14}, θ_{24}, θ_{34}, δ_{24}, δ_{34}



Bounds from T2K, SK, Daya Bay

[MB, Coloma]

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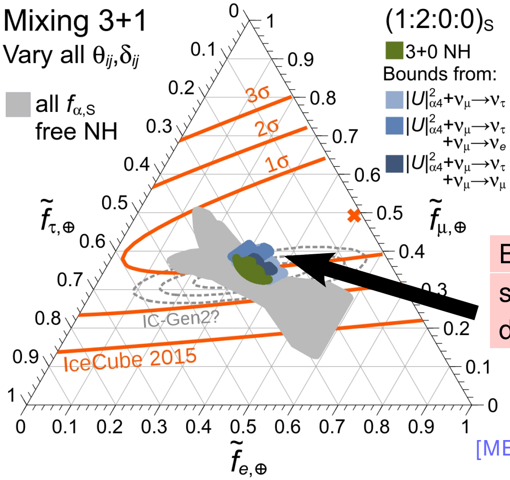

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Bounds are too strong for large deviations

[MB, Coloma]

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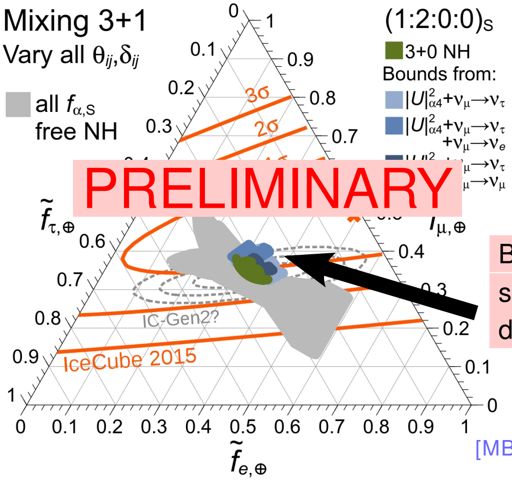

Bounds from T2K, SK, Daya Bay

Bounds are too strong for large deviations

[MB, Coloma]

The need for km-scale neutrino telescopes

Expected ν flux from cosmological accelerators (WAXMAN & BAHCALL 1997–1998):

$$E^2\Phi_\nu \sim 10^{-8}\frac{f_\pi}{0.2}\left(\frac{\dot{\varepsilon}_{\mathrm{CR}}^{[10^{10},10^{12}]}}{10^{44}~\mathrm{erg~Mpc}^{-3}~\mathrm{yr}^{-1}}\right)~\mathrm{GeV~cm}^{-2}~\mathrm{s}^{-1}~\mathrm{sr}^{-1}$$

Integrated flux above 1 PeV:

$$\Phi_\nu\,(>1~\mathrm{PeV}) \sim \int_{1~\mathrm{PeV}}^{\infty}\frac{10^{-8}}{E^2}\,dE \sim 10^{-20}~\mathrm{cm}^{-2}~\mathrm{s}^{-1}~\mathrm{sr}^{-1}$$

Number of events from half of the sky (2π):

$$N_\nu \simeq 2\pi\cdot\Phi_\nu\,(>1~\mathrm{PeV})\cdot 1~\mathrm{yr}\cdot A_{\mathrm{eff}} \approx \left(2.4\times 10^{-10}~\mathrm{cm}^{-2}\right)A_{\mathrm{eff}}~,$$

where A_{eff} is the effective area of the detector

To detect $N_\nu > 1$ events per year, we need an area of

$$A_{\mathrm{eff}} \gtrsim 0.4~\mathrm{km}^2$$

Therefore, we need km-scale detectors, like IceCube