

Rapid bound-state formation of Dark Matter in the Early Universe

Tobias Binder

based on arxiv:**1910.11288**, arxiv:**1911**.[in prep.],

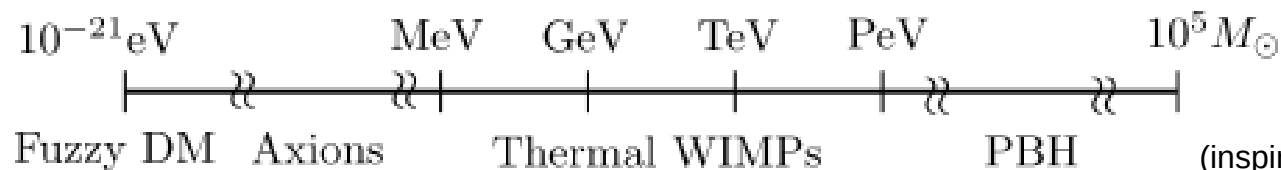
in collaboration with

Kyohei Mukaida and **Kalliopi Petraki**

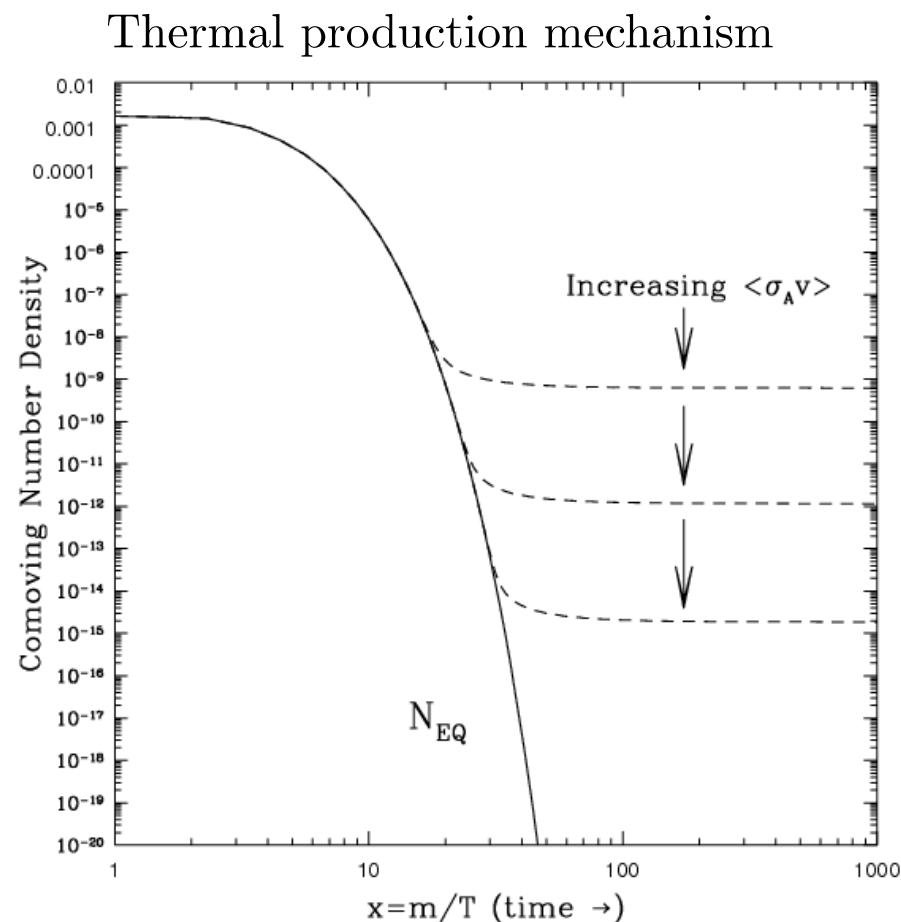
[**Burkhard Blobel**, and **Julia Harz**].



Thermally produced dark matter

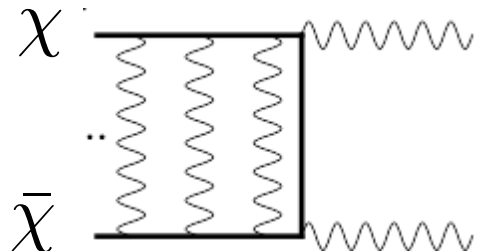


- One of the **leading hypothesis** for DM: Thermal WIMPs.
- **Testable** and final relic abundance **independent** of initial conditions.
- Strong constraints on coupling strength rule out many MeV-TeV mass realizations in thermal scenarios.
(However: "WIMPs are not dead", G. Gelmini)
- TeV-scale and above still remains attractive and much less constrained.
- **How heavy WIMPs can be?**
QM effects introduce theoretical uncertainties.



Quantum mechanical effects

QM effects induced by attractive long-range interactions:



- Sommerfeld-enhanced annihilation**

[J. Hisano *et al.* '03, '05, '06]

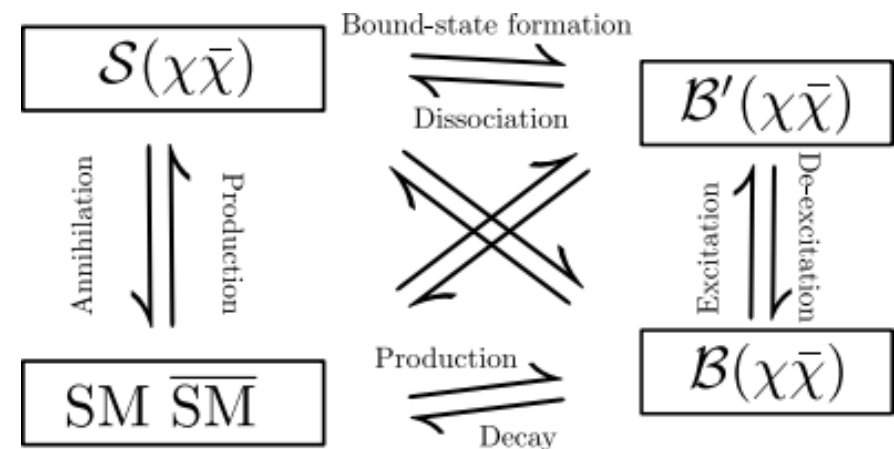
$$(\sigma v) = (\sigma v)_0 \times |\psi(r=0)|^2$$

$$\propto (\sigma v)_0 (\alpha/v), \text{ for } v \lesssim \alpha.$$

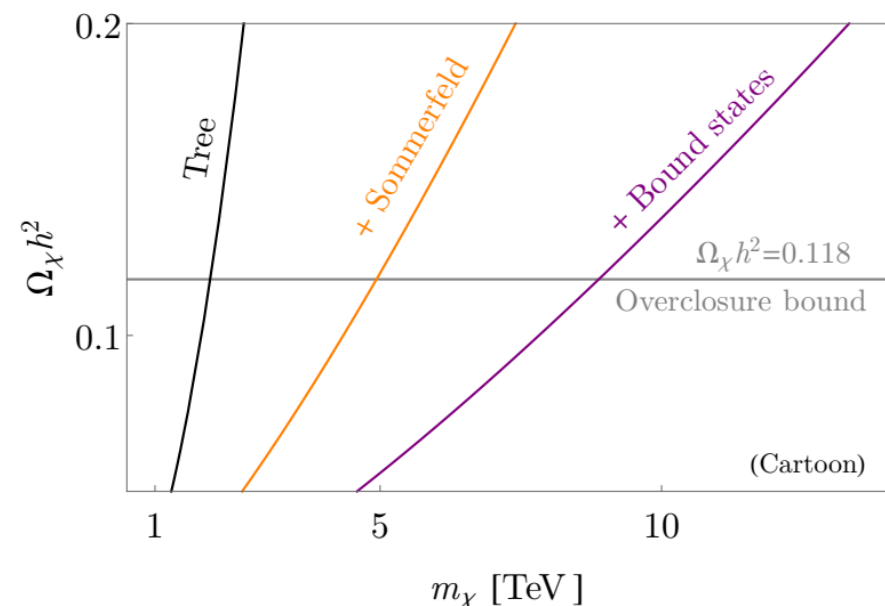
- Formation and decay of bound states**

$$\Gamma_n = (\sigma v)_0 \times |\psi_n(r=0)|^2$$

Complex chemical network:



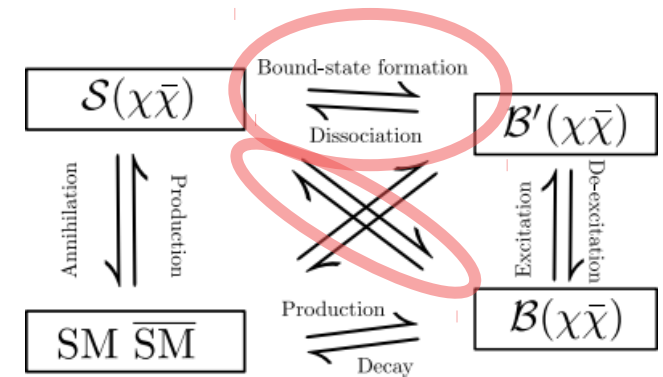
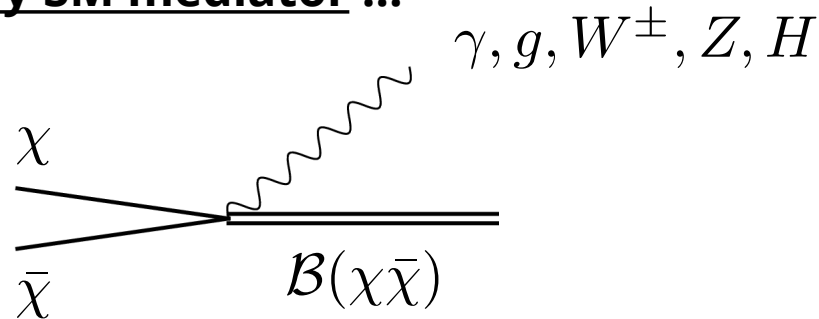
QM effects allow for **larger DM masses**:



QM effects not included in any public relic density solver. (In progress...)

Bound-state formation: model examples

For every SM mediator ...

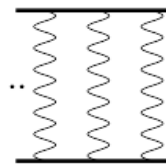


..., possible DM models have been found where QM effects are relevant to include:

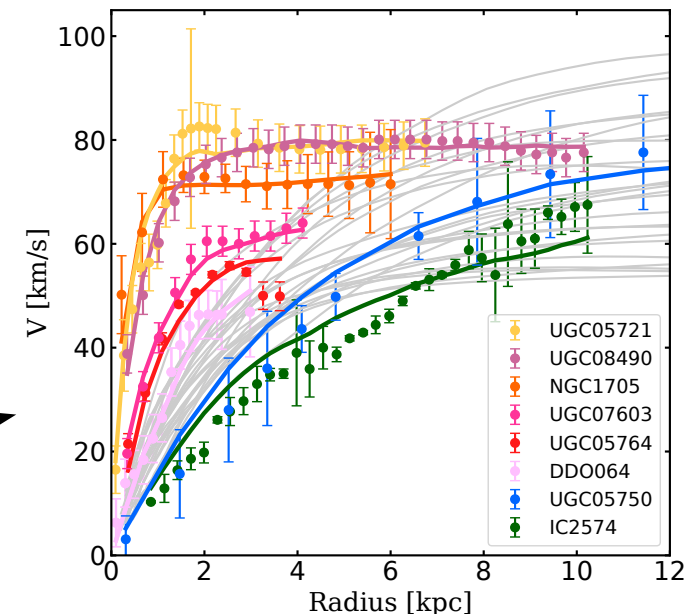
- Minimal DM (includes Wino)
..., [Cirelli *et al.* '07], [Mitridate *et al.* '17]
- Co-annihilation with color-charged particles
[J. Ellis *et al.* '16], [Kim&Laine '17], ...
[Harz&Petraki '18], [S. Biodini *et al.* '19,'19,'19], ...
- Higgs mediated bound states
[Harz&Petraki '18], [S. Biodini '18], ...

Or bottom-up motivated scenarios
with exotic mediators:

- Self-Interacting DM** with light mediators
[J. L. Feng *et al.* '10], [von Harling&Petraki '14], ... [many]



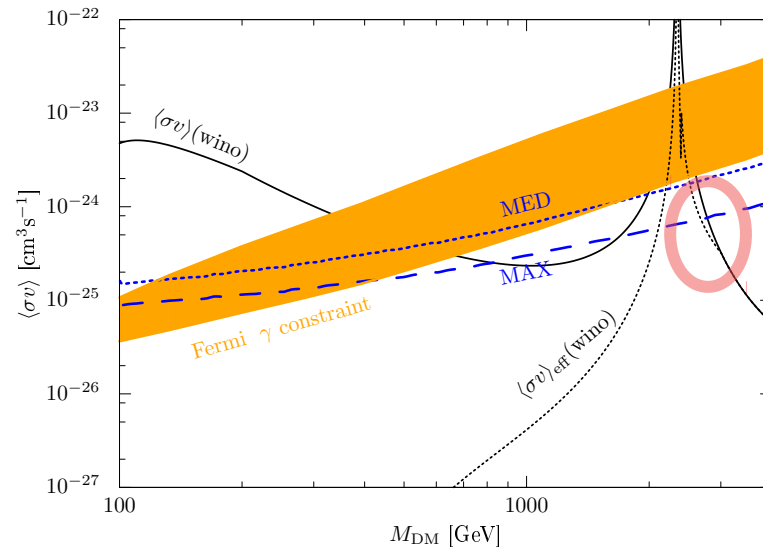
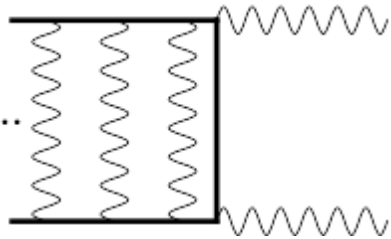
SIDM solves Diversity problem:
[Kamada *et al.* '16], ..., [Kaplinghat *et al.* '19]



One particular motivation: Wino

For traditional Wino neutralino, bound states exist!

[J. Hisano *et al.* '03, '05, '06]



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- Indirect detection **sensitive to DM mass** due to Sommerfeld resonances. For constraining WIMPs reliably, we need to theoretical predict the relic abundance precisely!

(10% change in the mass would result in 100 % change in the flux! see Satoshi's talk)

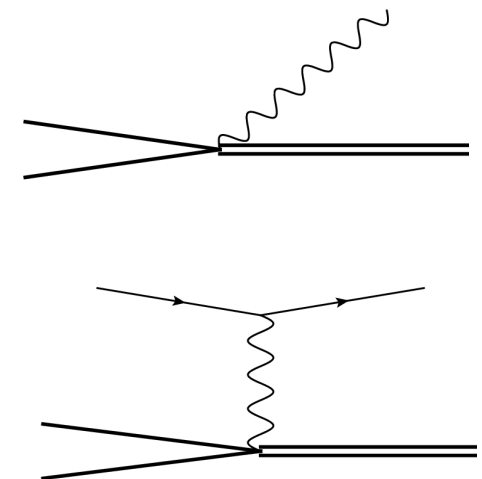
- BSF via on-shell mediator emission** has essentially no impact on the traditional Wino relic abundance.

(see "Cosmological implications of dark matter bound states" [Mitridate *et al.* '17])

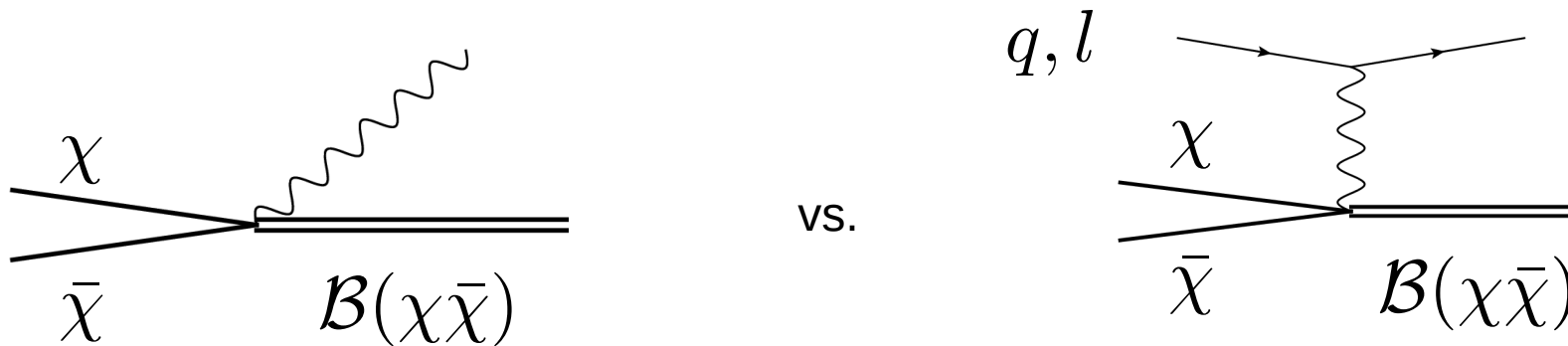
- Virtual mediator** contributions?

E.g.: **BSF via bath particle scattering.**

(no kinematic block, no phase-space suppression, number-density dependent)



BSF: On-shell or off-shell mediator?

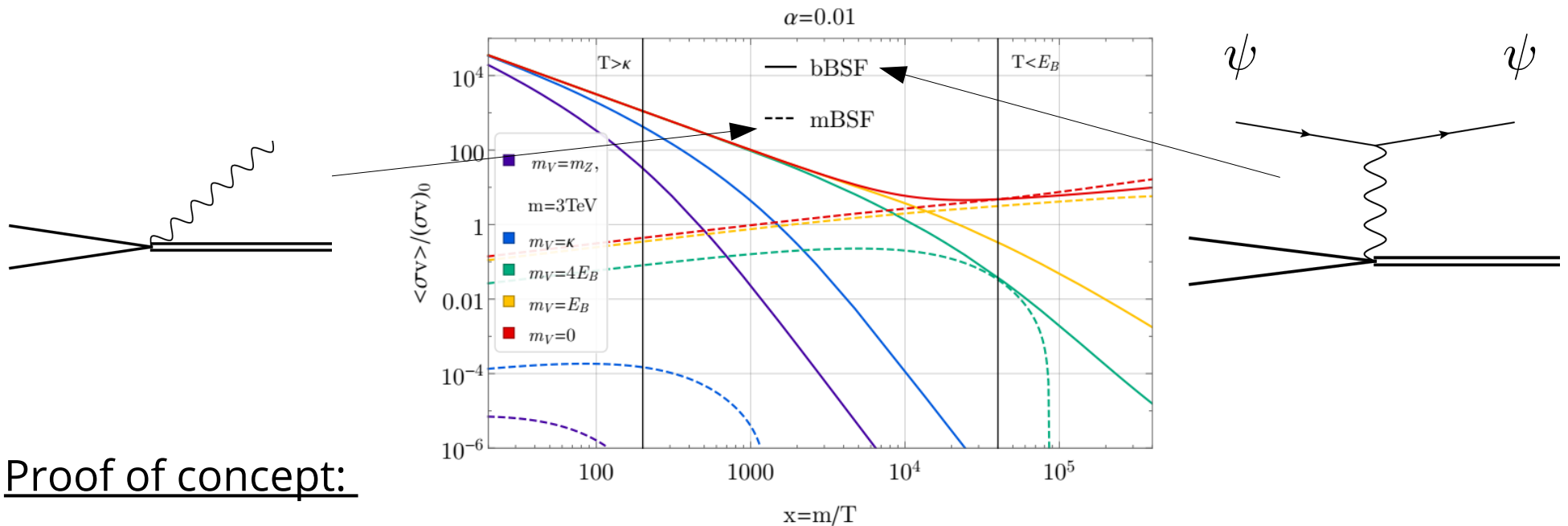


Which one dominates in the Early Universe?

- On-shell scenario resembles situation of SM neutral hydrogen recombination in matt. dom. era.
- Heavy Quarkonia in quark-gluon plasma:
Parton dissociation dominates on-shell gluon absorption for temperature larger than binding energy.
- By argument of detailed balance: **BSF via parton scattering must dominate** as well!
- Heavy Quarkonia system similar to DM in the Early Universe (>100 rel. d.o.f).
- This insight might already have profound implications for models where light or massless mediators in co-annihilation scenarios are involved.

First estimate: Capture into the ground state

Toy model: $\mathcal{L} \supset g\bar{\chi}\gamma^\mu\chi V_\mu + g\bar{\psi}\gamma^\mu\psi V_\mu$



- Proof of concept:

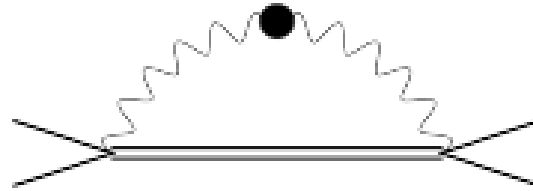
BSF via bath particle scattering can dominate over on-shell mediator emission.

- Forward scattering divergence regularized by **Debye screening mass**.
- HTL resummation **not applicable** for temperature smaller than binding energy!



Requires **thermal field theory analysis**.

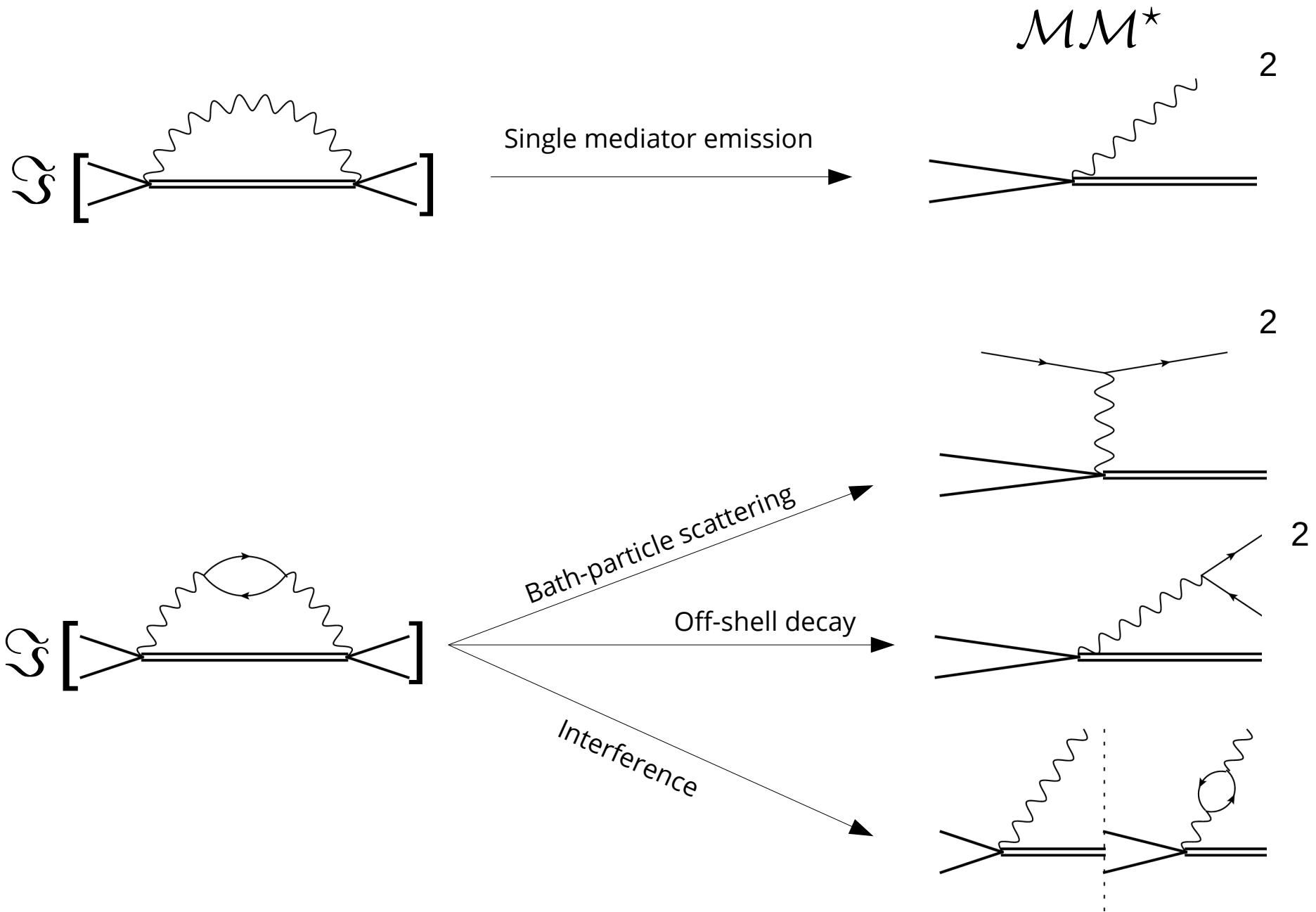
Generalized bound-state formation cross section



$$\sigma_{nlm}^{\text{BSF}} v_{\text{rel}} = \int \frac{d^3 p}{(2\pi)^3} D_{\mu\nu}^{-+}(\Delta E, \mathbf{p}) \sum_{\text{spins}} \mathcal{T}_{\mathbf{k},nlm}^{\mu}(\Delta E, \mathbf{p}) \mathcal{T}_{\mathbf{k},nlm}^{\nu*}(\Delta E, \mathbf{p}).$$

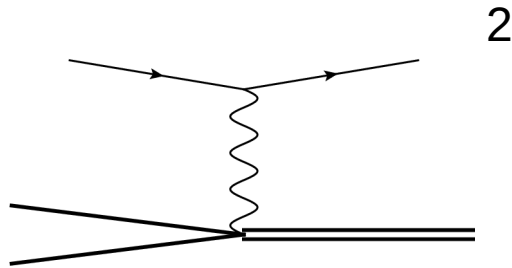
arxiv:**1911**.(in prep.)

Non-equilibrium QFT approach

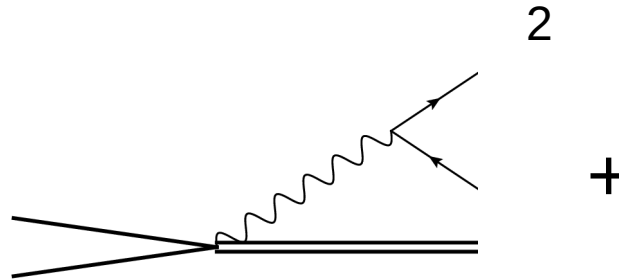


NLO contributions

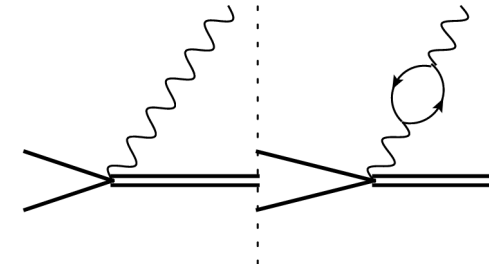
UV finite,
collinear divergent.



UV finite,
collinear divergent.



Vacuum part UV
divergent,
collinear divergent.



= Finite in collinear direction, and UV finite after vacuum renormalization.

- Provide mathematical proof for cancellation of collinear divergences.
- Holds even for arbitrary phase-space distribution of bath particles, i.e. bath particles do not have to be in thermal equilibrium in order to guarantee finiteness in the forward scattering direction.
- (Bloch-Nordsieck theorem does not help here)

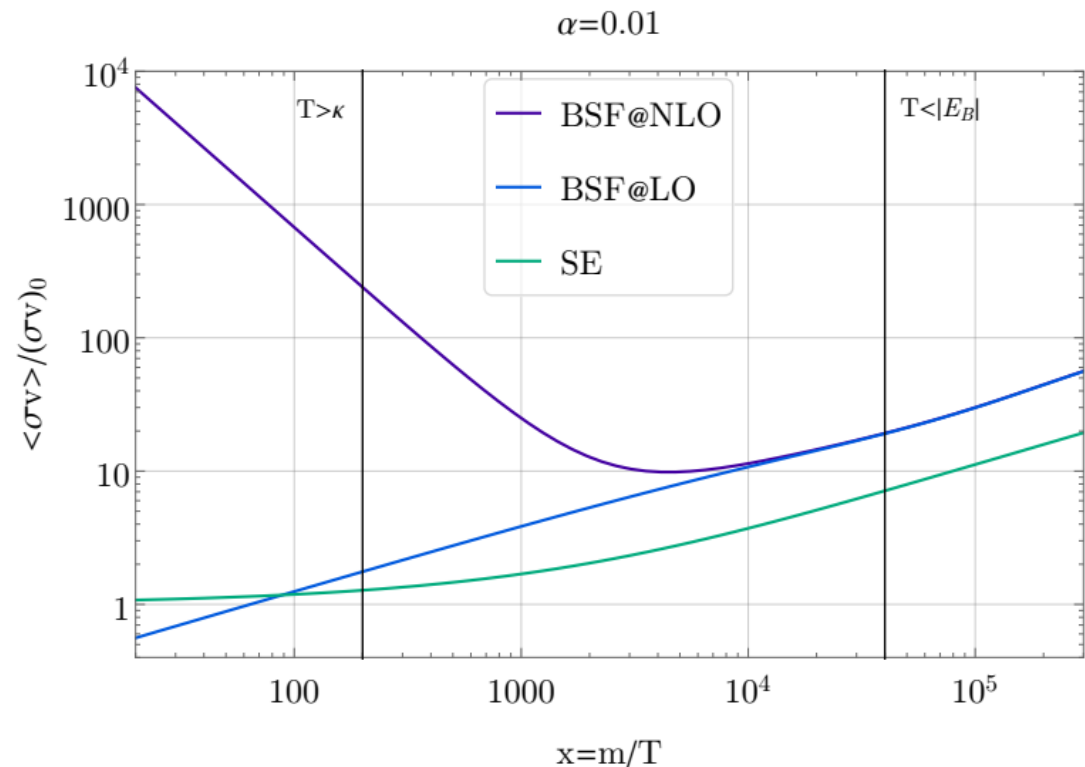
arxiv:1911.(in prep.)

Bound-state formation at NLO: massless case

- Interference terms **cancel collinear divergences**, resulting in a finite cross section.
- At high temperature ($T \gtrsim E_B$) BSF via bath particle scattering **dominates** over single mediator emission.
- Variation of renormalization scale between DM mass and binding energy doesn't affect plot visually, hence Log-contributions are under control.

$$(\sigma v)_{\text{BSF}}^{\text{NLO}} \equiv \Im \left[\text{Diagram 1} + \text{Diagram 2} \right]$$

The diagrams represent the imaginary parts of the NLO corrections to the bound-state formation cross-section. Diagram 1 shows a wavy line (mediator) connecting two fermion lines, with a self-energy loop on the wavy line. Diagram 2 shows a similar setup but with a different loop configuration.



arxiv:1911.(in prep.)

BSF via bath-particle scattering: massive case

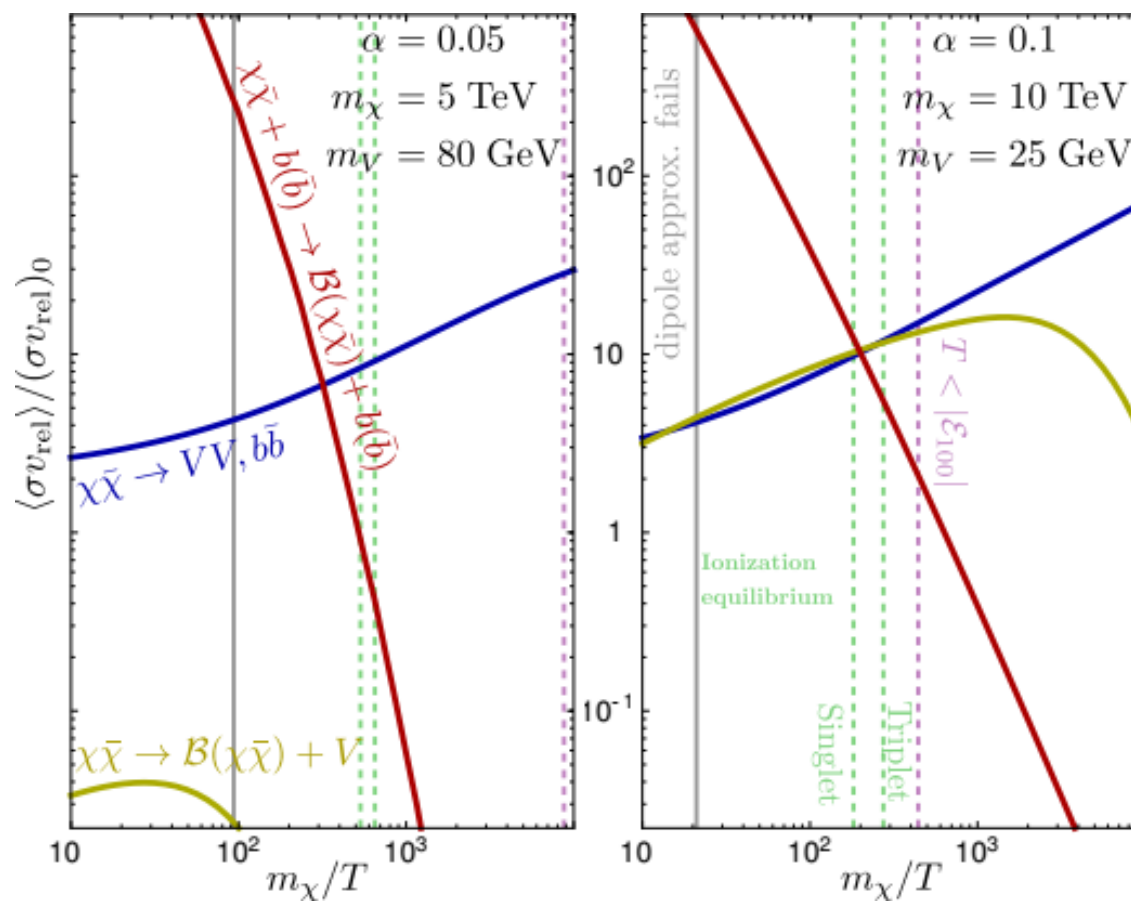
arxiv:1910.11288

Parametrically resembles:

no kinematical block

Wino

Co-annihilation with
colored charged particles



Summary and conclusion

Formal achievements:

- Non-equilibrium QFT analysis shows that **collinear divergences cancel** in the case of a massless mediator, even for arbitrary phase-space distributions of bath particles.
- Combining with previous lit., we achieved more **complete description** of the DM freeze-out, ranging from melting effects of bound states at high T down to far below the decoupling from ionization equilibrium.

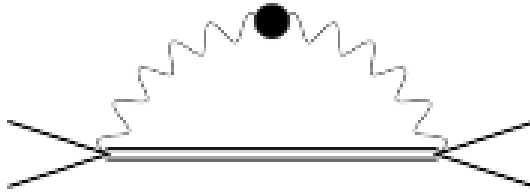
Phenomenological results and their implications:

- **Previous literature** considered BSF via **on-shell mediator emission** only.
- For temperature larger binding energy, we find in the massless case that the **dominant BSF channel** is **via bath-particle scattering** (in agreement with lit. about heavy quarkonia in QGP).
- BSF cross section via bath-particle scattering can exceed the single mediator emission by many orders of magnitude.
- Statement expected to be true also for non-abelian gauge or Yukawa theories.
- **Consequently, DM mass could be heavier than previously expected.**
(Eventually informs indirect searches and construction of future colliders)

➡ **Ready to (re-)analyse multi-TeV scale thermal relics!**

Thank you!

Generalized bound-state formation cross section



$$\sigma_{nlm}^{\text{BSF}} v_{\text{rel}} = \int \frac{d^3 p}{(2\pi)^3} D_{\mu\nu}^{-+}(\Delta E, \mathbf{p}) \sum_{\text{spins}} \mathcal{T}_{\mathbf{k},nlm}^{\mu}(\Delta E, \mathbf{p}) \mathcal{T}_{\mathbf{k},nlm}^{\nu*}(\Delta E, \mathbf{p}).$$

arxiv:1911.(in prep.)

$$\mathcal{L} \supset g \bar{\chi} \gamma^{\mu} \chi V_{\mu} + g \bar{\psi} \gamma^{\mu} \psi V_{\mu}$$

arxiv:1910.11288

Interacting two-point correlation fct.:

$$D_{\mu\nu}^{-+}(x, y) \equiv \langle V_{\mu}(x) V_{\nu}(y) \rangle$$

$$\langle \dots \rangle = \text{Tr}[e^{-H_{\text{env}}/T} \dots]$$

Kubo-Martin-Schwinger relation:

$$D_{\mu\nu}^{-+}(\Delta E, \mathbf{p}) = [1 + f_V^{\text{eq}}(\Delta E)] D_{\mu\nu}^{\rho}(\Delta E, \mathbf{p})$$

$$D_{\mu\nu}^{\rho} = 2\Im [i D_{\mu\nu}^R]$$

$$D_{\mu\nu}^R = D_{\mu\nu}^{R,0} + D_{\mu\alpha}^{R,0} \Pi_R^{\alpha\beta} D_{\beta\nu}^{R,0} + \dots$$

S-B transition matrix elements:

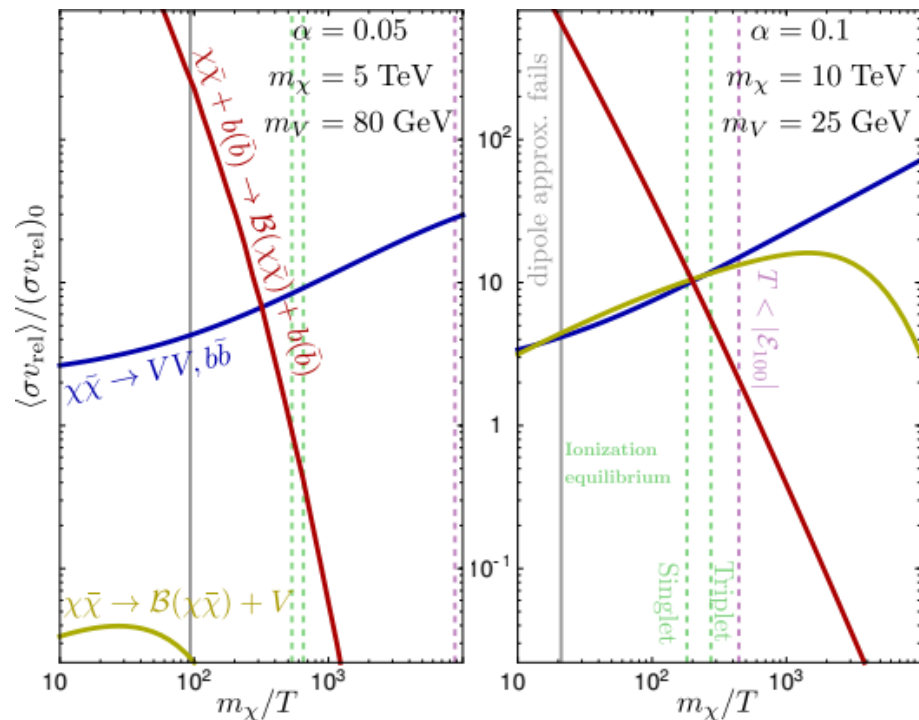
$$\mathcal{T}_{\mathbf{k},nlm}^{\mu}(P) \equiv (g_{\chi} g_{\bar{\chi}} 4m_{\chi}^2 2M)^{-1/2} \mathcal{M}_{\mathbf{k},nlm}^{\mu} \Big|_{\text{dip}}^{\text{NR}}$$

$$\delta^4 \mathcal{M}_{\mathbf{k},nlm}^{\mu} = \int d^4 x e^{iPx} \langle \mathcal{B}_{nlm} | g \bar{\chi}(x) \gamma^{\mu} \chi(x) | \mathcal{S}_{\mathbf{k}} \rangle$$

Well developed, see, e.g., Kallias works.

Implications of strongly enhanced BSF

arxiv:1910.11288



Approx. number density eq. [von Harling&Petraki '14]:

$$\dot{n}_s + 3Hn_s = - \left[\langle\sigma v\rangle_{\text{an}} + \frac{\Gamma_1 \langle\sigma v\rangle_{\text{BSF}}}{\Gamma_1 + \Gamma_{1\rightarrow s}} \right] (n_s^2 - n_s^{\text{eq}} n_s^{\text{eq}})$$

$$\Gamma_{1\rightarrow s} = \langle\sigma v\rangle_{\text{BSF}} \frac{n_s^{\text{eq}} n_s^{\text{eq}}}{n_1^{\text{eq}}}$$

$$\Gamma_1 \ll \Gamma_{1\rightarrow s}$$

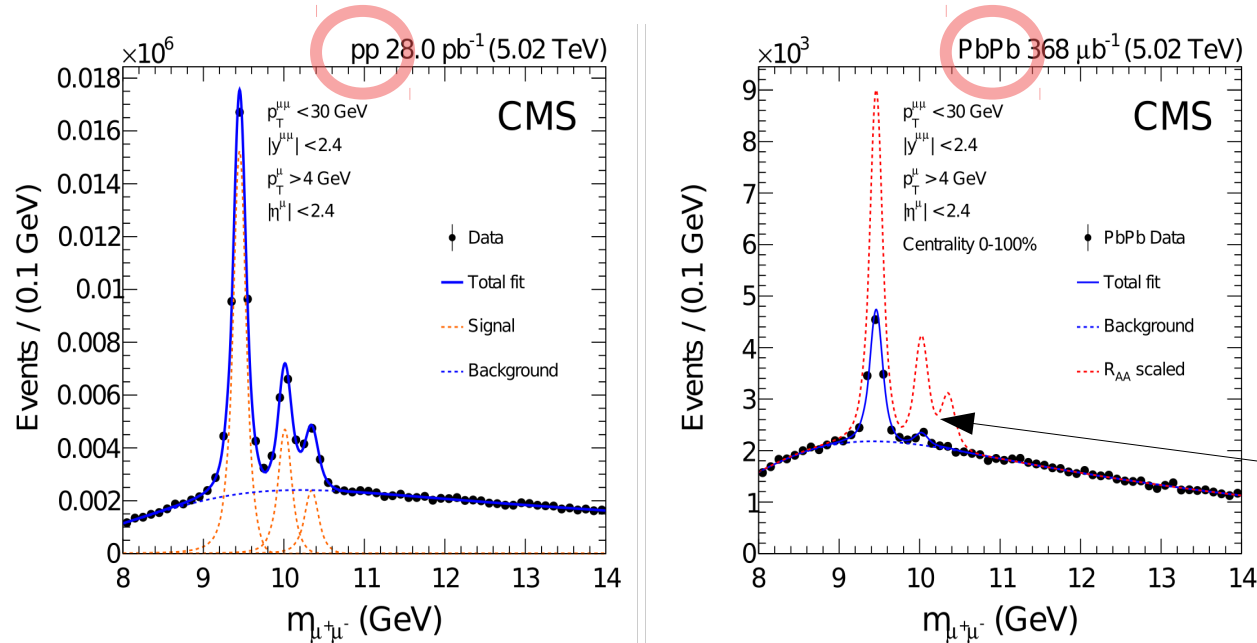
(Saha-) Ionization equilibrium [TB, Covi, Mukaida '18]:

$$\dot{n}_s + 3Hn_s = - \left[\langle\sigma v\rangle_{\text{an}} + \Gamma_1 \frac{n_1^{\text{eq}}}{n_s^{\text{eq}} n_s^{\text{eq}}} \right] (n_s^2 - n_s^{\text{eq}} n_s^{\text{eq}})$$

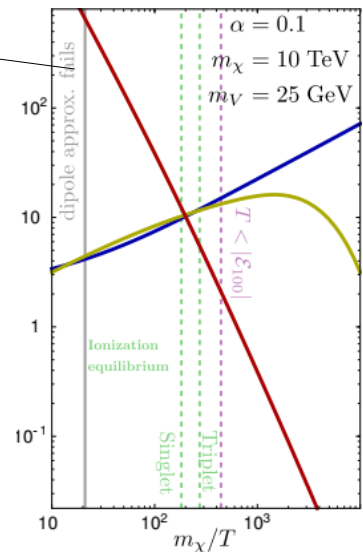
➡ Strongly enhanced BSF via bath-particle scattering leads to **ionization equilibrium**, where i) collision term is **independent of BSF cross section and DISS rate**, and ii) **effective depletion cross section takes maximum value** for fixed T.

Limitation

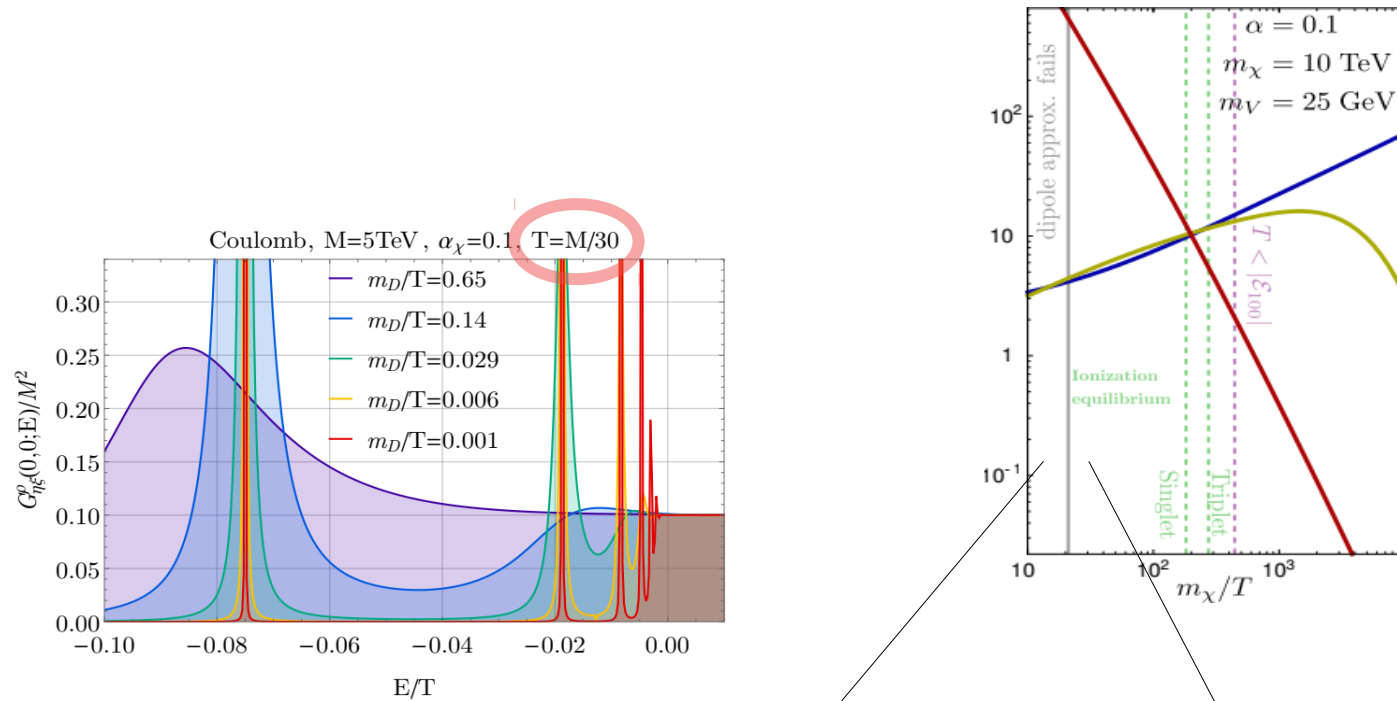
CMS collaboration, Phys.Lett. **B790** (2019) 270-293



Melting of bound states inside plasma environment observed.



Complete picture



[TB, Covi, Mukaida '18]

$$\dot{n} + 3Hn = -2(\sigma v)_0 G_{\eta\xi}^{++--}(x, x, x, x)|_{\text{eq}} [e^{\beta 2\mu} - 1],$$

$$G_{\eta\xi}^{++--}|_{\text{eq}} = e^{-2\beta M} \int \frac{d^3\mathbf{P}}{(2\pi)^3} e^{-\beta \mathbf{P}^2/(4M)} \int_{-\infty}^{\infty} \frac{dE}{(2\pi)} e^{-\beta E} G_{\eta\xi}^\rho(\mathbf{0}, \mathbf{0}; E).$$

arxiv:1911.(in prep.)

arxiv:1910.11288

$$\sigma_{nlm}^{\text{BSF}} v_{\text{rel}} = \int \frac{d^3p}{(2\pi)^3} D_{\mu\nu}^{-+}(\Delta E, \mathbf{p}) \sum_{\text{spins}} \mathcal{T}_{\mathbf{k},nlm}^\mu(\Delta E, \mathbf{p}) \mathcal{T}_{\mathbf{k},nlm}^{\nu*}(\Delta E, \mathbf{p}).$$

Backup

1 BS+R.h.s of BS equation vanishes

$$\dot{n}_s + 3Hn_s = - \left[\langle \sigma v \rangle_{\text{an}} + \frac{\Gamma_1 \langle \sigma v \rangle_{\text{BSF}}}{\Gamma_1 + \Gamma_{1 \rightarrow s}} \right] (n_s^2 - n_s^{\text{eq}} n_s^{\text{eq}})$$

Coupled BEs:

$$\begin{aligned} \dot{n}_s + 3Hn_s &= - \langle \sigma v \rangle_{\text{an}} [n_s^2 - (n_s^{\text{eq}})^2] \\ &\quad - \sum_i \langle \sigma^i v \rangle_{\text{BSF}} [n_s^2 - n_i (n_s^{\text{eq}})^2 / n_i^{\text{eq}}], \\ \dot{n}_i + 3Hn_i &= - \Gamma_i [n_i - n_i^{\text{eq}}] \\ &\quad + \langle \sigma^i v \rangle_{\text{BSF}} [n_s^2 - n_i (n_s^{\text{eq}})^2 / n_i^{\text{eq}}] \\ &\quad - \sum_j \Gamma_{i \rightarrow j} [n_i - n_j n_i^{\text{eq}} / n_j^{\text{eq}}] \end{aligned}$$

$$\Gamma_1 \ll \Gamma_{1 \rightarrow s}$$

[TB, Covi, Mukaida '18]

Radiative processes much faster
than total number violating
processes -> **ionization equilibrium**

$$\dot{n} + 3Hn = - \left[\langle \sigma v \rangle_{\text{an}} + \sum_i \Gamma_i \frac{n_i^{\text{eq}}}{n_s^{\text{eq}} n_s^{\text{eq}}} \right] (\alpha^2 n^2 - n_s^{\text{eq}} n_s^{\text{eq}})$$