Three generation seesaw model with a massless neutrino

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Contents

Introduction to the seesaw model

Seesaw model is a very attractive model which explains tiny neutrino mass

We suppose that Majorana masses are much heavier than Dirac masses.

a general mass term of the seesaw model

$$
\mathcal{L}_m = -\frac{1}{2}\begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} & \overline{(N_1^c)_L (N_2^c)_L} & \overline{(N_3^c)_L} \end{pmatrix} \begin{pmatrix} m_L & m_D \\ m_D^T & m_R \end{pmatrix} \begin{pmatrix} (\nu_e^c)_R \\ (\nu_\mu^c)_R \\ (\nu_\tau^c)_R \\ N_{1R} \\ N_{2R} \\ N_{3R} \end{pmatrix}
$$

$$
= -\frac{1}{2}\begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} m_L \begin{pmatrix} (\nu_e^c)_R \\ (\nu_\mu^c)_R \\ (\nu_\tau^c)_R \end{pmatrix} - \frac{1}{2}\begin{pmatrix} \overline{(N_1^c)_L (N_2^c)_L} & \overline{(N_3^c)_L} \end{pmatrix} m_R \begin{pmatrix} N_{1R} \\ N_{2R} \\ N_{3R} \end{pmatrix}
$$

$$
-\frac{1}{2}\begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} m_D \begin{pmatrix} N_{1R} \\ N_{2R} \\ N_{3R} \end{pmatrix} - \frac{1}{2}\begin{pmatrix} \overline{(N_1^c)_L (N_2^c)_L} & \overline{(N_3^c)_L} \end{pmatrix} m_D^T \begin{pmatrix} (\nu_e^c)_R \\ (\nu_\mu^c)_R \\ (\nu_\tau^c)_R \end{pmatrix}
$$

$$
= -\frac{1}{2}\begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} m_L \begin{pmatrix} (\nu_e^c)_R \\ (\nu_\mu^c)_R \\ (\nu_\tau^c)_R \end{pmatrix} - \frac{1}{2}\begin{pmatrix} \overline{(N_1^c)_L (N_2^c)_L} & \overline{(N_3^c)_L} \end{pmatrix} m_R \begin{pmatrix} N_{1R} \\ N_{2R} \\ N_{3R} \end{pmatrix}
$$

$$
-\begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} m_D \begin{pmatrix} N_{1R} \\ N_{2R} \\ N_{3R} \end{pmatrix}
$$

the mass matrix

$$M_\nu = \begin{pmatrix} m_L & m_D \\ m_D^T & m_R \end{pmatrix}$$

For the simplicity, we set m_L=0 and m_R is already diagonalized.

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 & m_{\mathrm{De}1} & m_{\mathrm{De}2} & m_{\mathrm{De}3} \\ 0 & 0 & 0 & m_{D\mu 1} & m_{D\mu 2} & m_{D\mu 3} \\ 0 & 0 & 0 & m_{D\tau 1} & m_{D\tau 2} & m_{D\tau 3} \\ m_{\mathrm{De}1} & m_{D\mu 1} & m_{D\tau 1} & M_1 & 0 & 0 \\ m_{\mathrm{De}2} & m_{D\mu 2} & m_{D\tau 2} & 0 & M_2 & 0 \\ m_{\mathrm{De}3} & m_{D\mu 3} & m_{D\tau 3} & 0 & 0 & M_3 \end{pmatrix}$$

- Motivation to the model with a massless neutrino

$$M_\nu = \begin{pmatrix} 0 & 0 & 0 & m_{\mathrm{De}1} & m_{\mathrm{De}2} & m_{\mathrm{De}3} \\ 0 & 0 & 0 & m_{D\mu 1} & m_{D\mu 2} & m_{D\mu 3} \\ 0 & 0 & 0 & m_{D\tau 1} & m_{D\tau 2} & m_{D\tau 3} \\ m_{\mathrm{De}1} & m_{D\mu 1} & m_{D\tau 1} & M_1 & 0 & 0 \\ m_{\mathrm{De}2} & m_{D\mu 2} & m_{D\tau 2} & 0 & M_2 & 0 \\ m_{\mathrm{De}3} & m_{D\mu 3} & m_{D\tau 3} & 0 & 0 & M_3 \end{pmatrix}$$

In order that all of the eigenvalues have non-zero values, in other words, all neutrinos are massive, it is required that $\det m_D \neq 0$.

$$\mathrm{m_D} = \begin{pmatrix} \mathrm{m_{De1}} & \mathrm{m_{De2}} & \mathrm{m_{De3}} \\ \mathrm{m_{D\mu 1}} & \mathrm{m_{D\mu 2}} & \mathrm{m_{D\mu 3}} \\ \mathrm{m_{D\tau 1}} & \mathrm{m_{D\tau 2}} & \mathrm{m_{D\tau 3}} \end{pmatrix}$$

If $\det m_D = 0$

we have at least one massless neutrino

proof

$$\psi_0 = \begin{pmatrix} u \\ v \end{pmatrix} \neq 0_6$$

Assuming a non-zero eigenvector that corresponds to the zero eigenvalue.

$$\begin{aligned} \boldsymbol{M}_\nu \boldsymbol{M}_\nu^\dagger \psi_0 &= 0_6 \\ h_D\, u + m_D M v &= 0_3 \\ \mathrm{Mm}_D^\dagger u + (H_D^* + M^2) v &= 0_3 \\ m_D m_D^\dagger\, u + m_D M v &= 0_3 \\ \mathrm{Mm}_D^\dagger u + (m_D^T m_D^* + M^2) v &= 0_3 \end{aligned}$$

$$\left\{ \begin{aligned} h_D &= m_D m_D^\dagger \\ H_D &= m_D^\dagger m_D \end{aligned} \right.$$

$$\begin{aligned} m_D^\dagger\, u + M v &= 0 \\ m_D^\dagger\, u &= -M v \\ \mathrm{Mm}_D^\dagger u + (m_D^T m_D^* + M^2) v &= -M^2 v + (m_D^T m_D^* + M^2) v \\ &= m_D^T m_D^*\, v \\ m_D^T m_D^*\, v &= 0 \\ v &= 0 \\ u &= 0 \end{aligned}$$

a hypothesis with self-inconsistency

- If the rank of the matrix m_D is 2,
we have just one massless neutrino.

- If the rank of the matrix m_D is 1,
we have two massless neutrinos.

- If the rank of the matrix m_D is 0, in effect, m_D is a zero matrix, we have three massless neutrinos.

Using the following unitary matrix, we can make M_ν approximately a diagonal matrix.

$$U = \begin{pmatrix} K & m_D M^{-1} \\ -M^{-1} m_D^\dagger K & 1 \end{pmatrix}$$

$$U^\dagger \boldsymbol{M}_\nu U^* = \boldsymbol{M}_{\rm diag} = \begin{pmatrix} -K^\dagger m_D \frac{1}{M} m_D^T K & 0 \\ 0 & M \end{pmatrix}$$

We define the effective mass

$$m_{\rm eff} = -m_D \frac{1}{M} m_D^T$$

One can describe 3×3 matrix whose rank is 2 by two independent column vectors and their linear combination.

$$
\begin{aligned}
m_D &= \left(\boldsymbol{m}_{D1} \ \ \boldsymbol{m}_{D2} \ \ \boldsymbol{m}_{D3} \right) \\
&= \left(\boldsymbol{m}_{D1} \ \ \boldsymbol{m}_{D2} \ \ a\boldsymbol{m}_{D1} + b\boldsymbol{m}_{D2} \right) \\
&= \left(m_{D1} \ \ m_{D2} \right) \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \end{pmatrix} \\
&= \left(\boldsymbol{u}_1 \ \ \boldsymbol{u}_2 \ \ \boldsymbol{u}_3 \right) \begin{pmatrix} m_{D1} & 0 & 0 \\ 0 & m_{D2} & 0 \\ 0 & 0 & m_{D3} \end{pmatrix} \\
&= \left(\boldsymbol{u}_1 \ \ \boldsymbol{u}_2 \right) \begin{pmatrix} 1 & 0 & a_{13} \\ 0 & 1 & a_{23} \end{pmatrix} \begin{pmatrix} m_{D1} & 0 & 0 \\ 0 & m_{D2} & 0 \\ 0 & 0 & m_{D3} \end{pmatrix}
\end{aligned}
$$

$$
\left\{
\begin{aligned}
|\boldsymbol{m}_{D1}| &= m_{D1} \\
|\boldsymbol{m}_{D2}| &= m_{D2} \\
|a\boldsymbol{m}_{D1} + b\boldsymbol{m}_{D2}| &= m_{D3} \\
|\boldsymbol{u}_1| = |\boldsymbol{u}_2| = |a_{13}\boldsymbol{u}_1 &+ a_{23}\boldsymbol{u}_2| = 1
\end{aligned}
\right.
$$

In general, we can triangulate any regular matrix with the suitable unitary matrix.

$$\Delta = V_3^{\dagger} m_D = \begin{pmatrix} \Delta_{D11} & 0 & 0 \\ \Delta_{D21} & \Delta_{D22} & 0 \\ \Delta_{D31} & \Delta_{D32} & \Delta_{D33} \end{pmatrix}$$

Applying this method to m_D

$$V^{\dagger}(\boldsymbol{u}_1 \;\; \boldsymbol{u}_2) = \begin{pmatrix} \mathbf{e}_2 \times \mathbf{e}_1 \\ \mathbf{e}_2{}^{\dagger} \\ \mathbf{e}_1{}^{\dagger} \end{pmatrix} (\boldsymbol{u}_1 \;\; \boldsymbol{u}_2)$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{1 - |(\boldsymbol{u}_2{}^{\dagger}\boldsymbol{u}_1)|^2} \\ 1 & \boldsymbol{u}_1^{\dagger}\boldsymbol{u}_2 \end{pmatrix}$$

$$\left\{ \begin{aligned} \mathbf{e}_2 &= \boldsymbol{u}_2 \\ \mathbf{e}_1 &= \frac{\boldsymbol{u}_1 - (\boldsymbol{u}_2{}^{\dagger}\boldsymbol{u}_1)\,\boldsymbol{u}_2}{\sqrt{1 - |(\boldsymbol{u}_2{}^{\dagger}\boldsymbol{u}_1)|^2}} \end{aligned} \right.$$

It produces the simpler form.

by V to 2 × 2 components transformed matrix

$$V^{\dagger} m_{\mathrm{eff}} V^{*} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & Z_{22} & Z_{23} \\ 0 & Z_{23} & Z_{33} \end{pmatrix}$$

$$\begin{cases} -Z_{22} = X_1(1+R_{13}\, a_{13}^2)\cos^2\beta \\ -Z_{23} = X_1(1+R_{13}\, a_{13}^2)\sin\beta\cos\beta e^{i\gamma} + a_{13}a_{23}\sqrt{R_{13}\, R_{23}\, X_1\, X_2}\cos\beta \\ -Z_{33} = X_1(1+R_{13}\, a_{13}^2)\sin^2\beta e^{2i\gamma} + 2a_{13}a_{23}\sqrt{R_{13}\, R_{23}\, X_1\, X_2}\sin\beta e^{i\gamma} + X_2(1+R_{23}a_{23}^2) \end{cases}$$

$$\begin{cases} \sin\beta = \sqrt{1-|(\boldsymbol{u}_1{}^{\dagger}\boldsymbol{u}_2)|^2} \\ \cos\beta e^{i\gamma} = (\boldsymbol{u}_1{}^{\dagger}\boldsymbol{u}_2) \end{cases}$$

$$\begin{cases} X_1 = \dfrac{m_{D1}^2}{M_1} \qquad R_{13} = \dfrac{M_1}{M_3} \\ X_2 = \dfrac{m_{D2}^2}{M_2} \qquad R_{23} = \dfrac{M_2}{M_3} \end{cases}$$

- The structure of MNSP-matrix

- $V_{MNSP} =$

$$V^{\dagger} \times \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta e^{(-i\phi)} \\ 0 & -\sin\theta e^{i\phi} & \cos\theta \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{(-i\alpha)} \end{pmatrix}}$$

the matrix which diagonalize the 2 × 2 component.

Comparison between the seesaw model with two right-handed neutrinos and the model with three right-handed neutrinos

- (3,2) model

three left-handed neutrinos with two right-handed neutrinos

- (3,3)model + rank of m_D =2

A Common feature of two models A massless neutrino

- What about the other features ?

CP violation and neutrino-less double beta decay, etc.

Dirac mass structure

$$(\boldsymbol{u_1} \quad \boldsymbol{u_2} \quad \boldsymbol{u_3})=(\boldsymbol{u_1} \quad \boldsymbol{u_2})\begin{pmatrix} \mathbf{1} & \mathbf{0} & a_{13} \\ \mathbf{0} & \mathbf{1} & a_{23} \end{pmatrix}$$

- **Normalization;** $\overrightarrow{\boldsymbol{m_{Di}}} = \overrightarrow{\boldsymbol{u_i}}\boldsymbol{m_{Di}}$

$$|\boldsymbol{u_i}| = 1(i = 1,2,3)$$

- **Rank**$(\boldsymbol{m_D}) = \boldsymbol{2}$ **; Linearly dependent**

$$\boldsymbol{u_3} = a_{13}\boldsymbol{u_1} + a_{23}\boldsymbol{u_2}$$

- **Convention; Im(**u_1**)=0**

Low energy effective mass for three neutrinos

- $-m_{eff}=$

$$(u_1 \quad u_2)\begin{pmatrix}1 & 0 & a_{13}\\ 0 & 1 & a_{23}\end{pmatrix}\begin{pmatrix}\frac{{m_{D1}}^2}{M_1} & 0 & 0\\ 0 & \frac{{m_{D2}}^2}{M_2} & 0\\ 0 & 0 & \frac{{m_{D3}}^2}{M_3}\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 1\\ a_{13} & a_{23}\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix}$$

- $= (u_1 \quad u_2)\begin{pmatrix}X_1 + {a_{13}}^2 X_3 & a_{13}a_{23}X_3\\ a_{13}a_{23}X_3 & X_2 + {a_{23}}^2 X_3\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix},$

  $X_i = \frac{{m_{Di}}^2}{M_i}$

- **Two limits $M_3 \to \infty$, or $a_{13}, a_{23} \to 0$ lead to the same effective mass as (3,2) Model.**

Two cases which lead to m_{eff} similar to (3,2) model.

1. $(a_{13}, a_{23}) = (e^{i\frac{\varphi_{13}}{2}}, 0)$; $(u_1 \quad u_2 \quad u_3) = (u_1 \quad u_2)\begin{pmatrix} 1 & 0 & e^{i\frac{\varphi_{13}}{2}} \\ 0 & 1 & 0 \end{pmatrix}$

- $m_{eff} = -(u_1 \quad u_2)\begin{pmatrix} X_1 + X_3 e^{i\varphi_{13}} & 0 \\ 0 & X_2 \end{pmatrix}\begin{pmatrix} {u_1}^T \\ {u_2}^T \end{pmatrix}$

2. $(a_{13}, a_{23}) = (0, e^{i\frac{\varphi_{23}}{2}})$; $(u_1 \quad u_2 \quad u_3) = (u_1 \quad u_2)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & e^{i\frac{\varphi_{23}}{2}} \end{pmatrix}$

- $m_{eff} = -(u_1 \quad u_2)\begin{pmatrix} X_1 & 0 \\ 0 & X_2 + X_3 e^{i\varphi_{23}} \end{pmatrix}\begin{pmatrix} {u_1}^T \\ {u_2}^T \end{pmatrix}$

Mixing matrix and mass with orthogonality assumption $(\boldsymbol{u_2}^{+}\boldsymbol{u_1}=0)$

- MNSP matrix $;V_{MNSP} = (\boldsymbol{u_2}^{*} \times \boldsymbol{u_1}^{*} \quad , \boldsymbol{u_2}, \quad \boldsymbol{u_1})P$

- $$\boldsymbol{V_{MNS}}^{+}\,\boldsymbol{m_{eff}}\boldsymbol{V_{MNSP}}^{*} = \boldsymbol{P}^{+}\begin{pmatrix} 0 & 0 & 0 \\ 0 & -X_2 & 0 \\ 0 & 0 & -(X_1 + X_3 e^{i\varphi_{13}}) \end{pmatrix}\boldsymbol{P}^{*} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \mathbf{P} = \mathbf{diag.}(1, \mathrm{e}^{\mathrm{i}\frac{\pi}{2}}, \mathrm{e}^{\mathrm{i}\frac{\alpha+\pi}{2}}) \text{ ,tan } \alpha = \frac{X_3 \sin\varphi_{13}}{X_1 + X_3 \cos\varphi_{13}}$$

$$m_3 = \sqrt{X_1{}^2 + 2X_1\, X_3 \cos\varphi_{13} + X_3{}^2}\,,\; m_2 = X_2$$

Mixing Matrix

- $V_{MNSP} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix}$

- $(u_2{}^* \times u_1{}^* \quad u_2 \quad u_1) = \begin{pmatrix} u_{\mu 2}{}^* u_{\tau 1}{}^* - u_{\tau 2}{}^* u_{\mu 1}{}^* & u_{e2} & u_{e1} \\ u_{\tau 2}{}^* u_{e1}{}^* - u_{e2}{}^* u_{\tau 1}{}^* & u_{\mu 2} & u_{\mu 1} \\ u_{e2}{}^* u_{\mu 1}{}^* - u_{\mu 2}{}^* u_{e1}{}^* & u_{\tau 2} & u_{\tau 1} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\pi}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha+\pi}{2}} \end{pmatrix}$

(Our convention; $\mathbf{Im.\,u_1}$**=0)**

Tuning the Standard notation form of MNSP to our convention

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \times \mathrm{diag}(1,\ e^{i\frac{\alpha_{21}}{2}},\ e^{i\frac{\alpha_{31}}{2}})\ .$$

$$V_{MNSP} = \begin{pmatrix} C_{12}C_{13}e^{i\delta} & S_{12}C_{13}e^{i\delta} & S_{13} \\ -S_{12}C_{23} - C_{12}S_{23}S_{13}\,e^{i\delta} & C_{12}C_{23} - S_{12}S_{23}S_{13}e^{i\delta} & S_{23}C_{13} \\ S_{12}S_{23} - C_{12}C_{23}S_{13}e^{i\delta} & -C_{12}S_{23} - S_{12}C_{23}S_{13}e^{i\delta} & C_{23}C_{13} \end{pmatrix} \times diag.(1,\quad e^{i\alpha_{21}/2},\quad e^{i\alpha_{31}/2})$$

Determination of seesaw parameters

$$m_3 = \sqrt{X_1^{\,2} + 2X_1\, X_3 \cos\varphi_{13} + X_3^{\,2}}\ ;\ \ m_2 = X_2$$

$$(\mathbf{u}_1 \quad \mathbf{u}_2 \quad \mathbf{u}_3) = (\mathbf{u}_1 \quad \mathbf{u}_2)\begin{pmatrix} 1 & 0 & e^{i\frac{\varphi_{13}}{2}} \\ 0 & 1 & 0 \end{pmatrix}$$

$$(\mathbf{u}_1 \quad \mathbf{u}_2) = \begin{pmatrix} S_{13} & S_{12}C_{13}e^{i\delta} \\ S_{23}C_{13} & C_{12}C_{23} - S_{12}S_{23}S_{13}e^{i\delta} \\ C_{23}C_{13} & -C_{12}S_{23} - S_{12}C_{23}S_{13}e^{i\delta} \end{pmatrix}$$

$$\tan\frac{X_3 \sin\varphi_{13}}{X_1 + X_3\cos\varphi_{13}} = \tan\alpha, \alpha_{21} = \pi, \alpha_{31} = \pi + \alpha$$

Determination of seesaw parameters from the observables

- Mixings , S_{12}=0.55, , S_{23}=0.71, , S_{13}=0.15
- C_{12}=0.83, , C_{23}=0.70, , C_{13}=0.99
- Mass,
- Normal hierarchy ;
- $m_1 = 0, m_2 = \sqrt{\Delta\ {m_{21}}^2}$= 0.00867756(eV),
- $m_3 = \sqrt{\Delta\ {m_{21}}^2 + \Delta\ {m_{32}}^2} = 0.0501528(\text{eV})$

CP violation of neutrino oscillation and neutrino-less double beta decay

- CP violation of neutrino oscillation

$$P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e) \qquad J = \mathrm{Im}(V_{e1}^{\mathrm{MNS}} V_{\mu 1}^{\mathrm{MNS}*} V_{e2}^{\mathrm{MNS}*} V_{\mu 2}^{\mathrm{MNS}})$$

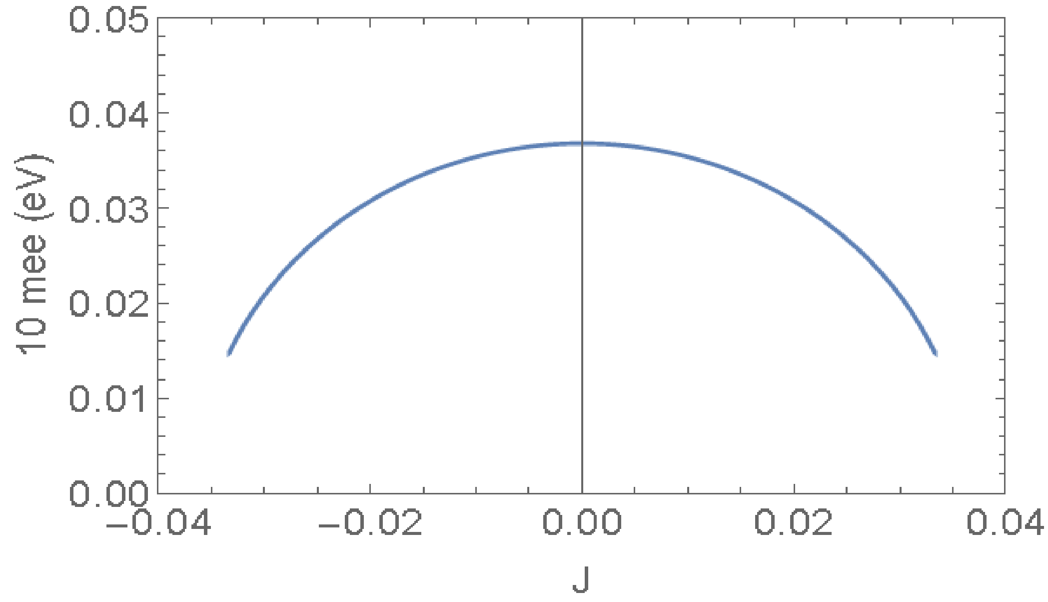
$$= 4J\left(\sin\frac{\Delta m_{12}^2 L}{2E} + \sin\frac{\Delta m_{23}^2 L}{2E} + \sin\frac{\Delta m_{31}^2 L}{2E}\right)$$

$$\boldsymbol{J = C_{12} S_{12} C_{23} S_{23} {C_{13}}^2 S_{13} \sin \delta}$$

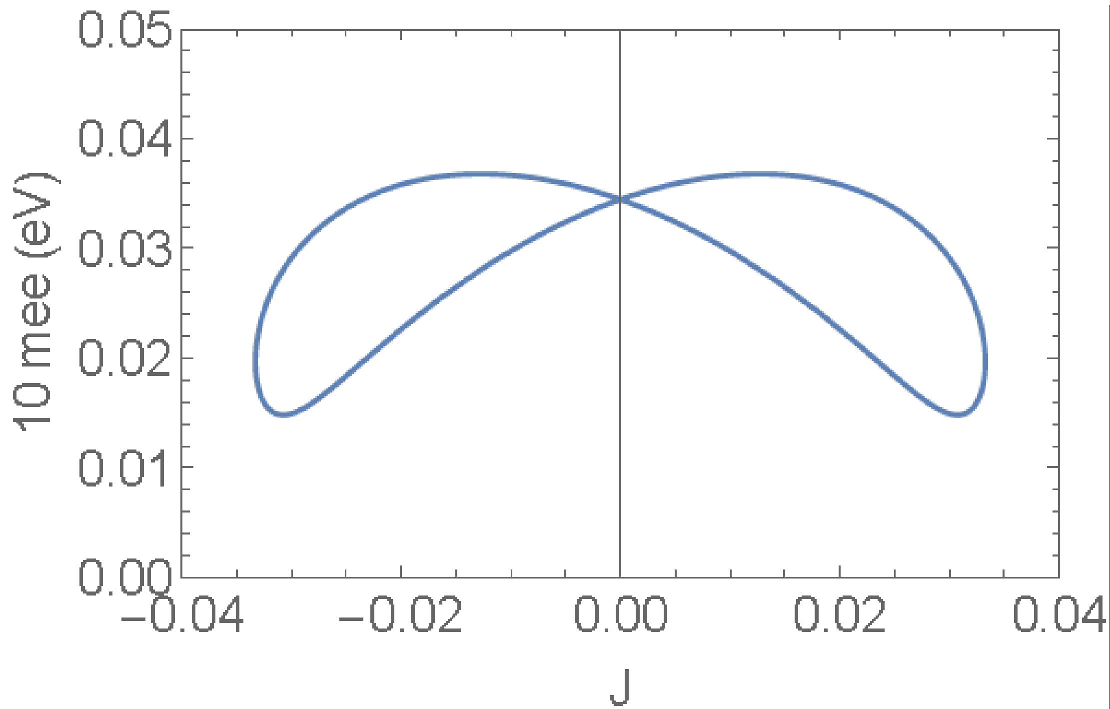
- Neutrino-less double beta decay

$$\boldsymbol{|m_{eff}| = \left|{V_{MNSe2}}^2\ m_2 + {V_{MNSe3}}^2\ m_3\right|}$$
$$\boldsymbol{= \left|{S_{12}}^2 {C_{13}}^2 m_2 + {S_{13}}^2 m_3 e^{i(\alpha - 2\delta)}\right|}$$

$\alpha = 0$



$\alpha = \frac{\pi}{4}$



Summary

- Three generation seesaw model with a massless neutrino is studied.
- Condition realizing a massless neutrino

  in the light neutrino mass spectrum

  Rank(m_D)=2
- A common structure with (3,2) model giving rise

to a massless neutrino is pointed out.
- A specific case for m_D is numerically studied.

  A correlation bet. J (CPV of oscillation) and

  mee of neutrino-less double beta decay