Pseudo-Nambu-Goldstone Dark Matter Model Inspired by Grand Unification

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This talk is mainly based on [1, PRD**104**,035011(2021)]
collaborated with Y. Abe (Kyoto U.), T. Toma (Kanazawa U.),
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Purpose of this talk

We will discuss an $SO(10)$ GUT model which contains a dark matter (DM) candidate and satisfies gauge coupling unification and constraint from proton decay and dark matter, and reproduce neutrino masses.

Key words:

Grand unification, gauge coupling unification, proton decay, $SO(10)$, $G_{\rm PS}(= SU(4)_C \times SU(2)_L \times SU(2)_R)$, $U(1)_{B-L}$, neutrino mass, (pseudo-Nambu-Goldstone boson) dark matter.

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Motivation for Grand Unification [2, 3, R.Slansky'81;...]

The idea of grand unification has attractive features; e.g.,

- Unification of the SM gauge bosons
- Unification of the SM Weyl fermions
- 4D SM gauge anomaly cancellation
- Charge quantization for quarks and leptons

...

The SM matter content: The Standard Model

Field	Symbol	$(SU(3), SU(2), U(1))$	$SL(2,\mathbb{C})$
Quark doublet	q_j	$(\mathbf{3}, \mathbf{2}, +1/6)$	$(1/2, 0)$
Up-type quark	u_j^c	$(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$	$(1/2, 0)$
Down-type quark	d_j^c	$(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$	$(1/2, 0)$
Lepton doublet	ℓ_j	$(\mathbf{1}, \mathbf{2}, -1/2)$	$(1/2, 0)$
Charged lepton	e_j^c	$(\mathbf{1}, \mathbf{1}, +1)$	$(1/2, 0)$
Higgs	ϕ	$(\mathbf{1}, \mathbf{2}, +1/2)$	$(0, 0)$
Gluon	G_A	$(\mathbf{8}, \mathbf{1}, \pm 0)$	(1/2,1/2)
Weak	W_I	$(\mathbf{1}, \mathbf{3}, \pm 0)$	(1/2,1/2)
Hyper	B	$(\mathbf{1}, \mathbf{1}, \pm 0)$	(1/2,1/2)

The SM matter content: Grand Unification [4, Georgi,Glashow'74]

Field	Symbol	$(SU(3), SU(2), U(1))$	$SL(2,\mathbb{C})$
Quark doublet	q_j	$(\mathbf{3}, \mathbf{2}, +1/6)$	$(1/2, 0)$
Up-type quark	u_j^c	$(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$	$(1/2, 0)$
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Hyper	B	$(\mathbf{1}, \mathbf{1}, \pm 0)$	(1/2,1/2)

Unification of SM gauge bosons and symmetry

Candidate for GUT gauge group [4–7, H.Georgi,S.L.Glashow'74;...]

Type	Rank 4	Rank 5	Rank 6	Rank 7	$\cdots$
A_n	$SU(5)$	$SU(6)$	$SU(7)$	$SU(8)$	$\cdots$
B_n	$SO(9)$	$SO(11)$	$SO(13)$	$SO(15)$	$\cdots$
C_n	$USp(8)$	$USp(10)$	$USp(12)$	$USp(14)$	$\cdots$
D_n		$SO(10)$	$SO(12)$	$SO(14)$	$\cdots$
Ex.	F_4		E_6	E_7	$\cdots$

$G_{\rm SM}$ can be embedded into the above groups, but many groups are excluded by exsistence of chiral fermions [2, 8, M.Gell-Mann,P.Ramond,R.Slansky'78;...].

Unification of SM gauge bosons and symmetry

Candidate for GUT gauge group (Gray cell: no chiral fermion)

Type	Rank 4	Rank 5	Rank 6	Rank 7	$\cdots$
A_n	$SU(5)$	$SU(6)$	$SU(7)$	$SU(8)$	$\cdots$
B_n	$SO(9)$	$SO(11)$	$SO(13)$	$SO(15)$	$\cdots$
C_n	$USp(8)$	$USp(10)$	$USp(12)$	$USp(14)$	$\cdots$
D_n	▨	$SO(10)$	$SO(12)$	$SO(14)$	$\cdots$
Ex.	F_4	▨	E_6	E_7	$\cdots$

The existence of chiral fermions drastically reduces candidates for 4D GUT gauge groups.

(cf. in higher dimensional framework, see e.g. Ref. [3, N.Y.'15].)

Unification of SM fermions

Fermion	G_{SM}	$SU(5)$	$SO(10)$	$SU(16)$
q_j	$(\mathbf{3}, \mathbf{2}, +1/6)$	$\mathbf{10}$	$\mathbf{16}$	$\mathbf{16}$
u_j^c	$(\overline{\mathbf{3}}, \mathbf{1}, -2/3)$			
e_j^c	$(\mathbf{1}, \mathbf{1}, +1)$			
d_j^c	$(\overline{\mathbf{3}}, \mathbf{1}, +1/3)$	$\overline{\mathbf{5}}$		
ℓ_j	$(\mathbf{1}, \mathbf{2}, -1/2)$			
ν_j^c	$(\mathbf{1}, \mathbf{1}, 0)$	$\mathbf{1}$		

$SU(16)$ is "maximal gauged symmetry" [9, J.C.Pati,A.Salam,J.A.Strathdee'81].
Unfortunately, due to gauge anomaly, not all part of $SU(16)$ can be gauged in 4D effective theories with three chiral generations.
$SO(10)$ seems to be the best candidate from unification of fermions.

$SO(10)$ GUT gauge group [6, H.Fritzsch,P.Minkowski'75]

$$SO(10)$$

$$\downarrow$$

$$G_{\rm PS} = SU(2)_L \times SU(2)_R \times SU(4)_C$$

$$\downarrow$$

$$G_{2231} = SU(2)_L \times SU(2)_R \times SU(3)_C \times \ U(1)_{B-L}$$

$$\downarrow$$

$$G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

Note: $U(1)_Y$ is a linear combination of $U(1)_R(\subset SU(2)_R)$ and $U(1)_{B-L}$.

$SO(10)$ gauge boson

Gauge	G_{SM}	G_{2231}	G_{PS}	$SO(10)$
G_A	$(\mathbf{8},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},\mathbf{8},0)$	$(\mathbf{1},\mathbf{1},\mathbf{15})$	$\mathbf{45}$
U_1	$(\mathbf{3},\mathbf{1},+2/3)$	$(\mathbf{1},\mathbf{1},\mathbf{3},+4/3)$		
U_1^c	$(\overline{\mathbf{3}},\mathbf{1},-2/3)$	$(\mathbf{1},\mathbf{1},\overline{\mathbf{3}},-4/3)$		
B_{B-L}	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},\mathbf{1},0)$		
W_{La}	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{3},\mathbf{1},\mathbf{1},0)$	$(\mathbf{3},\mathbf{1},\mathbf{1})$	
W_R^+	$(\mathbf{1},\mathbf{1},+1)$	$(\mathbf{1},\mathbf{3},\mathbf{1},0)$	$(\mathbf{1},\mathbf{3},\mathbf{1})$	
W_R^0	$(\mathbf{1},\mathbf{1},0)$			
W_R^-	$(\mathbf{1},\mathbf{1},-1)$			
V_2^c	$(\mathbf{3},\mathbf{2},-5/6)$	$(\mathbf{2},\mathbf{2},\mathbf{3},-2/3)$	$(\mathbf{2},\mathbf{2},\mathbf{6})$	
$\tilde{V}_2^c$	$(\mathbf{3},\mathbf{2},+1/6)$			
V_2	$(\overline{\mathbf{3}},\mathbf{2},+5/6)$	$(\mathbf{2},\mathbf{2},\overline{\mathbf{3}},+2/3)$		
$\tilde{V}_2$	$(\overline{\mathbf{3}},\mathbf{2},-1/6)$			

(Note)U_1 is the G_{PS} leptoquark gauge boson; V_2 is the $SU(5)$ leptoquark gauge boson X, Y; $\tilde{V}_2$ is the flipped $SU(5)$ leptoquark gauge boson X', Y'. (See, e.g., [10, I.Dorsner et al.'16].)

Proton decay via gauge bosons

The leptoquark gauge boson $V_2^{(\prime)}$ leads to proton decay such as $p \to e^+\pi^0$. The proton lifetime is roughly estimated as

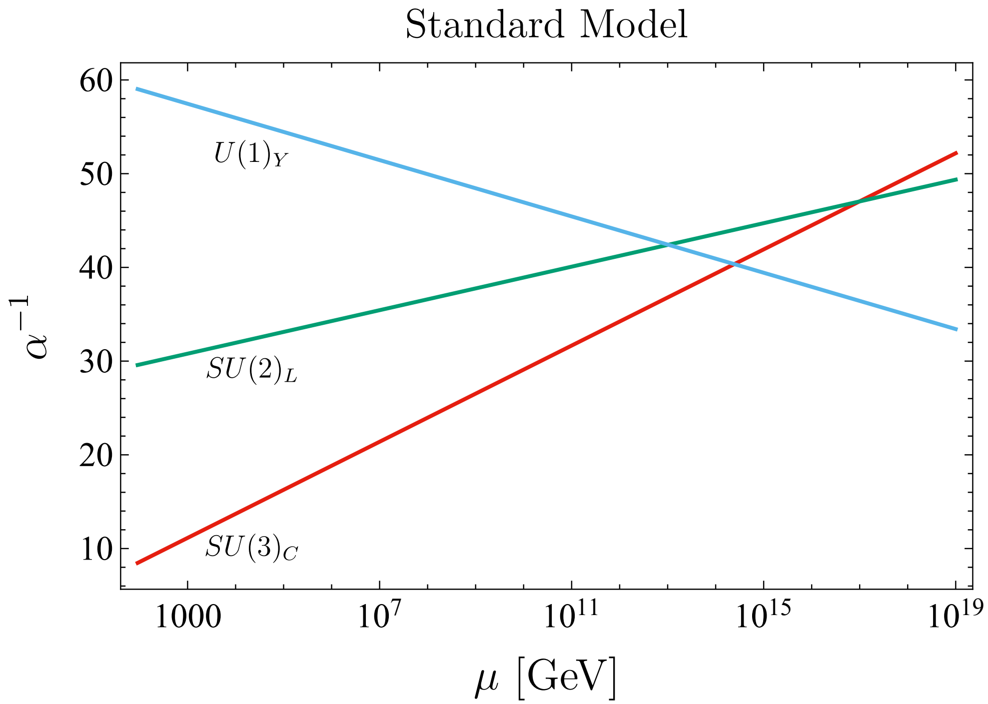
$$\tau \simeq \frac{M_U^4}{\alpha_U^2 m_p^5}.$$

Current constraint from super-Kamiokande $\tau(p \to e^+\pi^0) > 2.4 \times 10^{34}$ years at 90% CL [11, Super-Kamiokande Collaboration'20].

The "GUT scale" $M_U > (4.3 - 4.8) \times 10^{15}$ GeV for $40 \lesssim \alpha_U^{-1} \lesssim 50$.

(GUT scale $\simeq$ gauge coupling unification scale)

RG flow of gauge coupling constants [12, H.Georgi et al'74]



Gauge coupling unification seems to require additional particles.

Neutrino mass

"Weinberg operator" (related with Neutrino mass) [13, S.Weinberg'79]:

$$\mathcal{O}_5 = \frac{Z^{\nu}_{ij}}{\Lambda_{\rm NP}} \left(\overline{L}_{Li}\tilde{\phi}\right)\left(\tilde{\phi}^T L^C_{Lj}\right) + \text{h.c.}$$

Neutrino masses and new physics (NP) scale (rough estimation):

$$(M_\nu)_{ij} = Z^{\nu}_{ij}\frac{v^2}{\Lambda_{\rm NP}} \quad \Rightarrow \quad \Lambda_{\rm NP} \simeq Z^{\nu}\frac{v^2}{M_\nu} \lesssim O(10^{14})\,\text{GeV}$$

for $Z^\nu \lesssim O(1)$, $v \simeq O(10^2)$ GeV, and $(M_\nu)^{\rm largest} \simeq O(0.1)$ eV. [14, 15, Super-Kamiokande'98;PDG'20;...]

Observed neutrino masses seem to suggest that one of NP scales is located below "the GUT scale".

Summary for implication from GUT and neutrino

- Candidate for 4D GUT gauge group:

- Proton decay via leptoquark gauge bosons:

- Neutrino masses

- Gauge coupling unification

Summary for implication from GUT and neutrino

- Candidate for 4D GUT gauge group:
  $SU(n)(n \geq 5)$, $SO(4k+2)(k \geq 2)$, and E_6.
- Proton decay via leptoquark gauge bosons:
  $\Rightarrow$ GUT scale $M_U \gtrsim O(10^{15})$ GeV.
- Neutrino masses
  $\Rightarrow$ a NP scale $\Lambda_{\rm NP} \lesssim O(10^{14})$ GeV.
- Gauge coupling unification
  $\Rightarrow$ New particles below M_U

Dark matter

The existence of dark matter (DM) is supported from various cosmological observations such as spiral galaxies, gravitational lensing, cosmic microwave background, and collision of bullet cluster [16–20, Corbelli,Salucci'00;Sofue,Rubin'00;Massery et al'10;Planck 2018;Randall et al'07;...].

There are a lot of DM candidates such as WIMP(Weakly Interacting Massive Particle), FIMP(Feebly ...), SIMP(Strongly ...), axion, etc.

Usually, DM direct detection experiments [21, 22, e.g.,XENON1T'18;PandaX-4T'21] lead to strong constrains for DM mass and cross section.

In the talk, we consider a pseudo-Nambu-Goldstone boson (pNGB) as a WIMP-type DM candidate [23, C.Gross,O.Lebedev,T.Toma'17].

Dark matter: pseudo-Nambu-Goldstone boson

Ref. [23, C.Gross,O.Lebedev,T.Toma'17] proposed a simple pNGB DM model that contains an additional complex scalar field $S \sim s + i\chi$ with softly broken global $U(1)_S$ symmetry.

The annihilation cross sections of the pNGB DMs to SM fermions are not so weak, while the scattering cross sections of the pNGB DMs and SM fermions are strongly suppressed. So, WIMP direct detection experiments do not lead to severe constraints.

Refs. [24, 25, Y.Abe,T.Toma,K.Tsumura'20;N.Okada,D.Raut,Q.Shafi'20] proposed a pNGB DM model based on a gauged $U(1)_{B-L}$ symmetry to avoid introducing the global symmetry $U(1)_S$.

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$SO(10)$ pNGB DM model [1, Y.Abe,T.Toma,K.Tsumura,N.Y.'21]

- SM quarks and leptons are unified into $SO(10)$ **16** fermion.
- SM Higgs H is mainly contained in $SO(10)$ **10** scalar.
- pNGB scalar DM $\chi(\in S)$ is mainly contained in $SO(10)$ **16** scalar.
- Two additional scalars in $\mathbf{210}$ and $\overline{\mathbf{126}}$ of $SO(10)$ are responsible for breaking $SO(10) \to G_{\rm PS}$ and further to $G_{\rm SM}$.
- There are three scales: unification, intermediate, and electroweak scales, denoted as M_U, M_I, and $M_{\rm EW}$.

(The gauged $U(1)_{B-L}$ pNGB DM model is realized as a low-energy model, whose parameter space is limited by GUT constraints.)

$SO(10)$ pNGB DM model [1, Y.Abe,T.Toma,K.Tsumura,N.Y.'21]

Matter content in the $SO(10)$ pNGB DM model

	A_μ	$\Psi_{\mathbf{16}}^{(a=1,2,3)}$	$\Phi_{\mathbf{10}}$	$\Phi_{\mathbf{16}}$	$\Phi_{\overline{\mathbf{126}}}$	$\Phi_{\mathbf{210}}$
$SO(10)$	$\mathbf{45}$	$\mathbf{16}$	$\mathbf{10}$	$\mathbf{16}$	$\overline{\mathbf{126}}$	$\mathbf{210}$
$SL(2,\mathbb{C})$	$(1/2,1/2)$	$(1/2,0)$	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
	Gauge bosons	SM fermions	SM Higgs H	pNGB DM S	$\cancel{U(1)_{B-L}}$ Φ	$\cancel{SO(10)}$

Symmetry breaking pattern

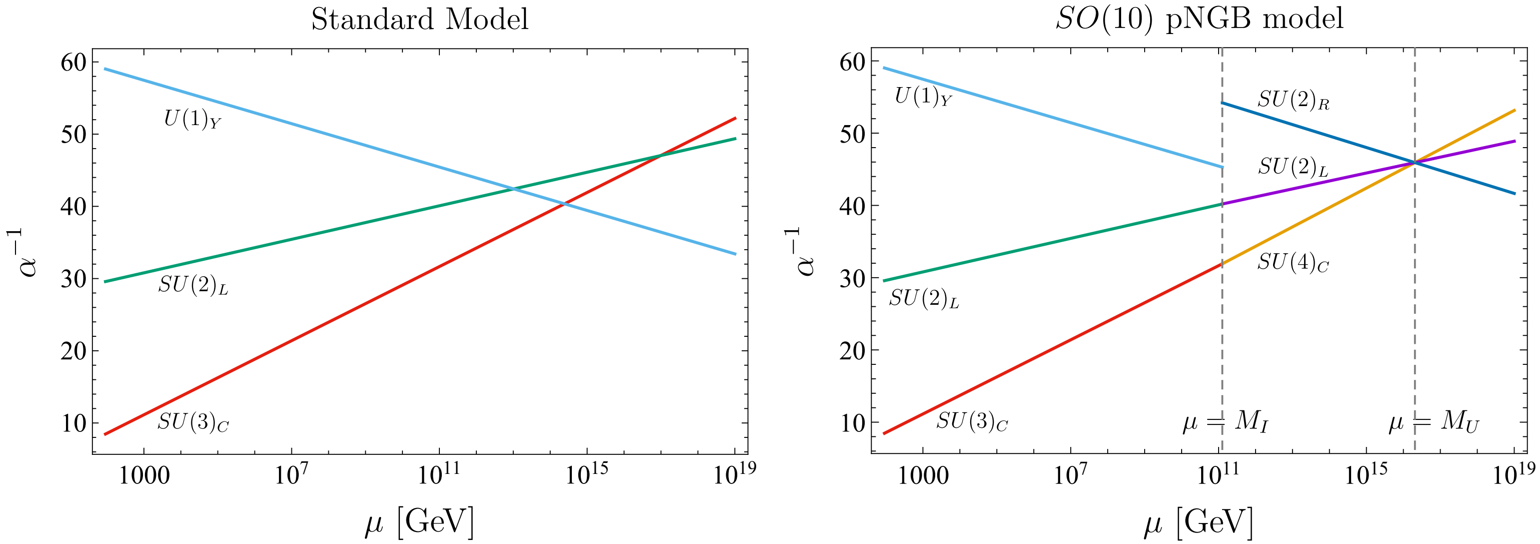
$$SO(10) \xrightarrow{\langle\Phi_{\mathbf{210}}\rangle = v_{\mathbf{210}} \simeq M_U \neq 0} G_{\mathrm{PS}}\ (\supset G_{\mathrm{SM}} \times U(1)_{B-L})$$

$$\xrightarrow{\langle\Phi_{\overline{\mathbf{126}}}\rangle = v_\Phi \simeq M_I \neq 0} G_{\mathrm{SM}}$$

$$\xrightarrow{\langle\Phi_{\mathbf{10}}\rangle = v \simeq M_{\mathrm{EW}} \neq 0} SU(3)_C \times U(1)_{\mathrm{EM}}.$$

(Note: To change breaking pattern, we need to introduce a different GUT breaking scalar field.)

Gauge coupling unification: $SO(10) \to G_{\mathrm{PS}} \to G_{\mathrm{SM}}$

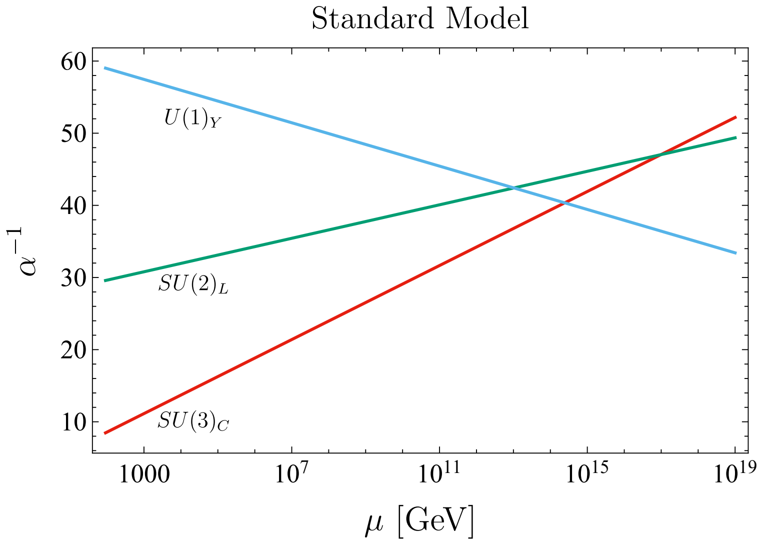


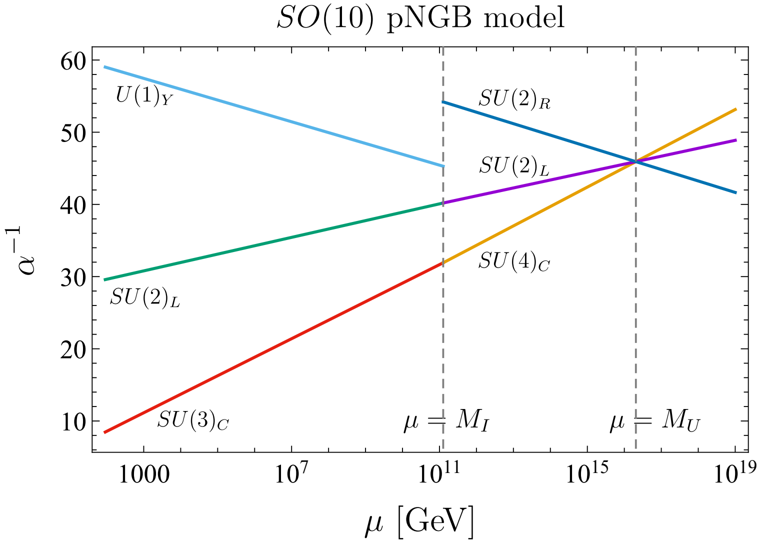
The RGE for $\alpha_i(\mu) := g_i^2(\mu)/4\pi$ at 1-loop level [2, 3, e.g.,R.Slansky'81]

$$\frac{d}{d\log(\mu)}\alpha_i^{-1}(\mu) = -\frac{b_i}{2\pi}, \quad b_i = -\frac{11}{3}\sum_{\text{Vector}} T(R_V) + \frac{2}{3}\sum_{\text{Weyl}} T(R_F) + \frac{1}{6}\sum_{\text{Real}} T(R_S).$$

Matching conditions at $\mu = M_I, M_U$ [26, e.g.,R.N.Mohapatra'02]

Gauge coupling unification: $SO(10) \to G_{\mathrm{PS}} \to G_{\mathrm{SM}}$





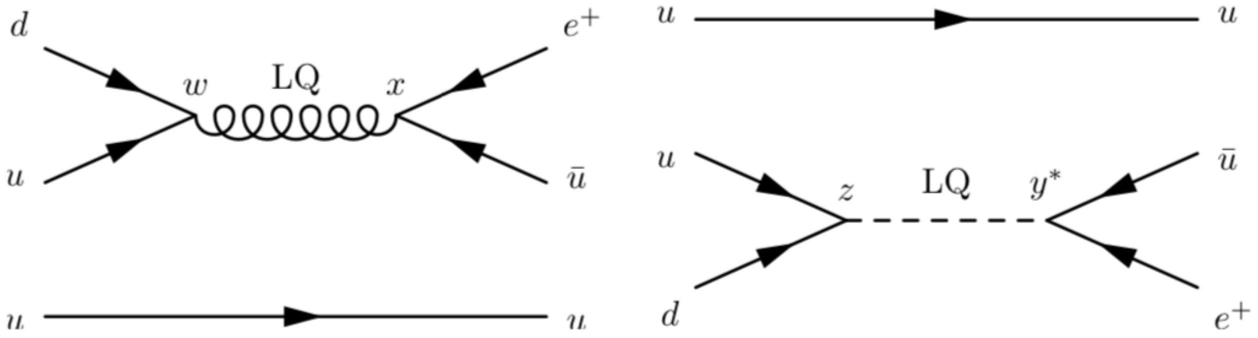
SM gauge coupling constants at $\mu = M_Z$ and matching conditions at $\mu = M_I, M_U$

$$\Rightarrow \begin{cases} M_I = (1.261 \pm 0.242) \times 10^{11}\,\text{GeV}, \quad M_U = (2.057 \pm 0.688) \times 10^{16}\,\text{GeV}, \\ g_{B-L}(M_I) = 0.3843 \pm 0.0009, \quad \alpha_U^{-1} = 45.92 \pm 0.50, \quad \ldots \end{cases} .$$

$U(1)_{B-L}$ is a part of $SO(10)$, so g_{B-L} is fixed by matching conditions.

Proton decay

Proton decay $p \to e^+\pi^0$ via leptoquark vector and scalar mediation:



From Figure 1 in Ref. [10, I.Dorsner et al.'16]

Proton lifetime $\tau(p \to \{V_{LQ}, S_{LQ}\} \to e^+\pi^0)$ is roughly

$$\tau \simeq \frac{m_{\rm LQ}^4}{(xw)^2 m_p^5} \quad \text{or} \quad \frac{m_{\rm LQ}^4}{|y|^2|z|^2 m_p^5}.$$

Proton decay via leptoquark vector & scalar bosons

Proton lifetime $\tau(p \to \{V_{LQ}\} \to e^{+}\pi^{0})$ in the model is roughly

$$\tau \simeq \frac{m_{\mathrm{LQ}}^{4}}{(xw)^{2}m_{p}^{5}} \simeq \frac{M_{U}^{4}}{\alpha_{U}^{2}m_{p}^{5}} \simeq 1.1\times 10^{37}\,\text{years.}$$

Current constraint from super-Kamiokande $\tau(p \to e^{+}\pi^{0}) > 2.4\times 10^{34}$ years at 90% CL [11, Super-Kamiokande Collaboration'20].

For the scalar sector, there are three lepto-quark scalars denoted as S_1 in Ref. [10, I.Dorsner et al.'16], which belong to $(\mathbf{3},\mathbf{1},1/3)$ under G_{SM}. Under our assumption, their contribution is much smaller than the current experimental bound. Constraint for Yukawa coupling constants are $|y_{\mathbf{10}}^{(11)}|, |y_{\mathbf{126}}^{(11)}| \lesssim O(1)$.

Neutrino mass

The right-handed neutrino masses comes from the Yukawa coupling terms $y_{\mathbf{126}}^{(ab)}\Psi_{\mathbf{16}}^{(a)}\Psi_{\mathbf{16}}^{(b)}\Phi_{\overline{\mathbf{126}}}$.

The right-handed neutrino masses are given by $M_N^{(ab)} = y_{\mathbf{126}}^{(ab)} v_\phi$, where $\langle\Phi\rangle = v_\phi = M_I \simeq 10^{11}$ GeV. For $10^{-5} \lesssim y_{\mathbf{126}}^{(ab)} \lesssim 1$, 10^6 GeV $\lesssim M_N^{(ab)} \lesssim 10^{11}$ GeV.

From the Type-I see-saw mechanism, a light neutrino mass is roughly $m_\nu^{(aa)} \simeq |y_{\mathbf{10}}^{(aa)} v|^2/M_N^{(aa)}$ (ignoring the off-diagonal part of $M_N^{(ab)}$).
For $|y_{\mathbf{126}}^{(11)}| \simeq |y_{\mathbf{10}}^{(11)}| \simeq 10^{-5}$ and $v \simeq 10^2$ GeV, $m_\nu^{(11)} \simeq 10^{-3}$ eV

The proton decay constraints only a part of the Yukawa coupling constants $y_{\mathbf{126}}^{(ab)}$, so it is expected that the observed neutrino masses can be reproduced.

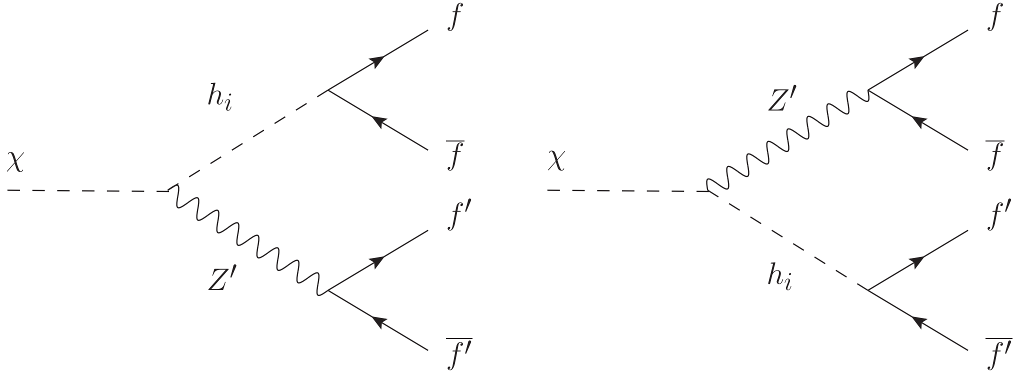
DM decay

DM lifetime: $\tau_{\rm DM} \gtrsim 10^{17}$ sec (the age of the universe)

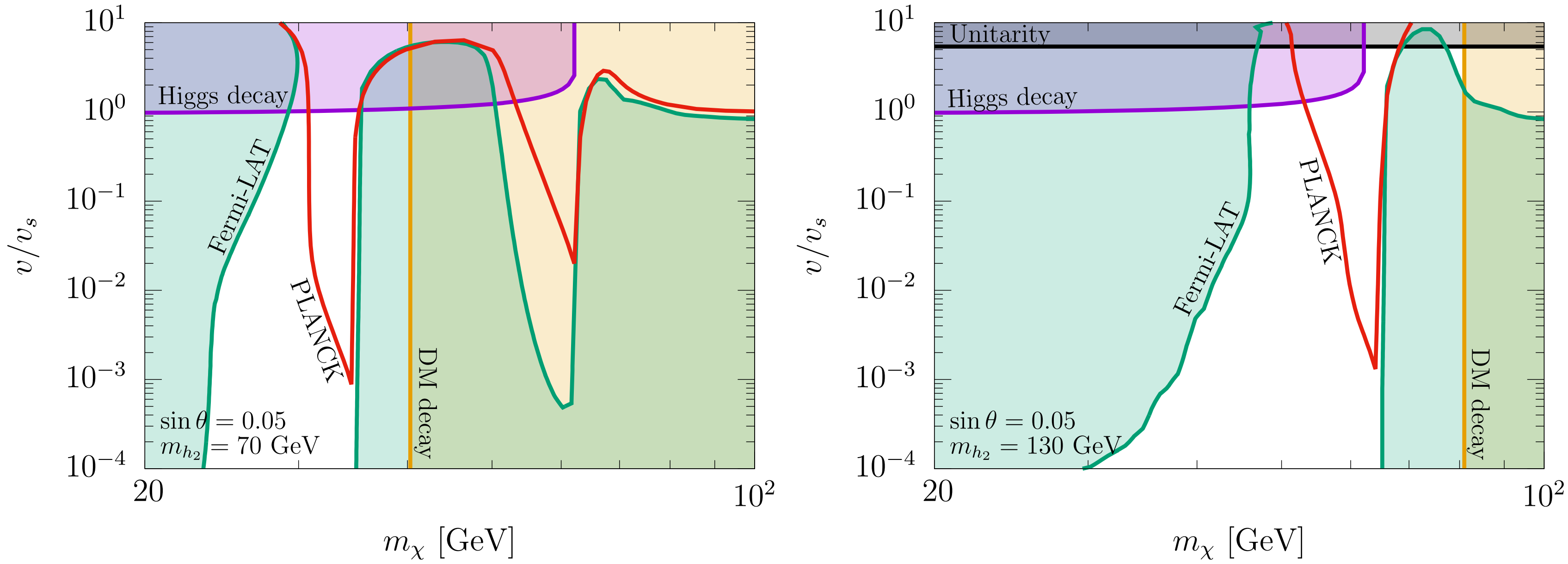
Cosmic ray observations give stronger limits: $\tau_{\rm DM} \gtrsim 10^{27}$ sec. [27, Baring et al'16].

If 3-body decays $\chi \to f\bar{f}h_i, f\bar{f}Z$ are allowed ($m_\chi \gtrsim m_{h_i}, m_Z$), the VEV of Φ must be $v_\phi \gtrsim 10^{13}$ GeV to satisfy the gamma-ray observations [24, 25, Y.Abe,T.Toma,K.Tsumura'20;N.Okada,D.Raut,Q.Shafi'20] (In the model, $v_\phi \simeq 10^{11}$ GeV).

We consider $m_\chi \lesssim 100$ GeV to forbid the three body decays kinetically.
In the case, the following four body decay modes are dominant.

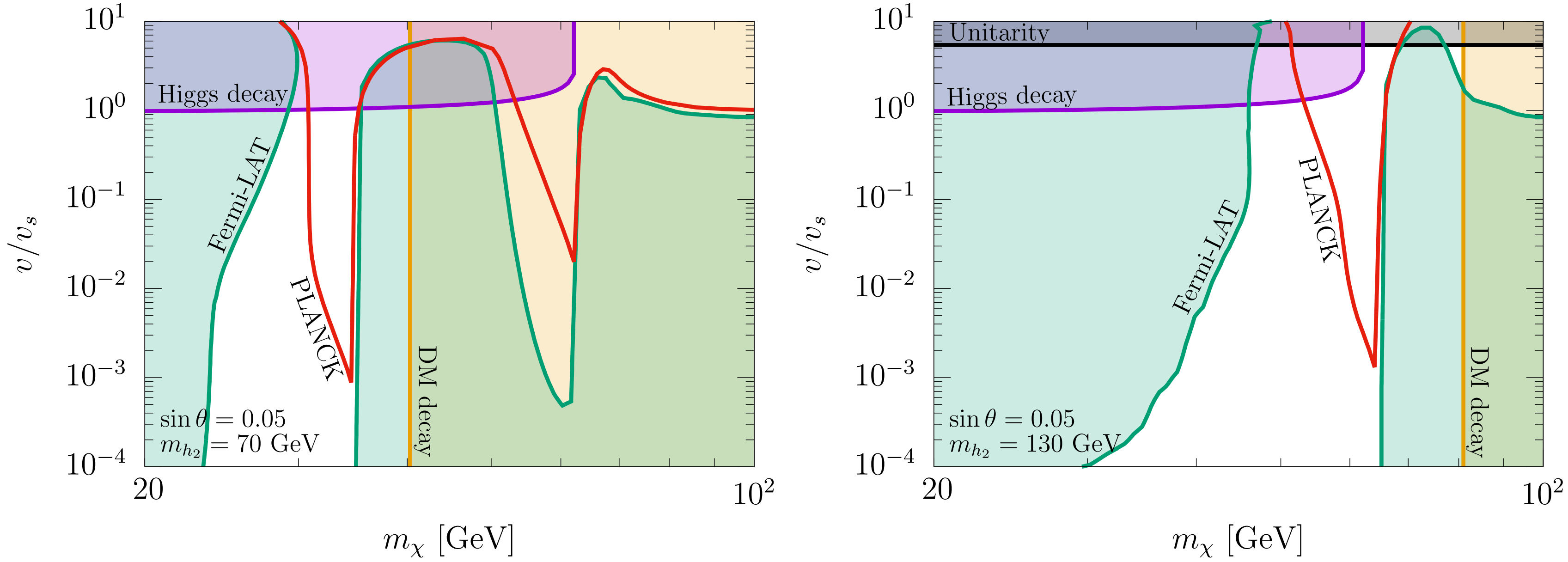


DM decay: summary plots



Red line: reproducing the observed relic abundance of DM $\Omega_\chi h^2 \simeq 0.12$ [19, Planck'20].
Purple region: excluded by the Higgs invisible decay [28, 29, CMS, ATLAS'19].
Gray region: excluded by the perturbative unitarity bound $\lambda_S < 8\pi/3$ [30, C.Y.Chen et al'15].
Green and Orange region: excluded by the gamma-ray observations for DM annihilation [31, Fermi-LAT,DES'17] and four body decays [27, M.G.Baring et al'16], respectively.

DM decay: summary plots



In the $SO(10)$ pNGB DM model, from GUT constraints and etc., we find

> the pNGB can be a DM candidate when the DM mass m_χ is slightly below half of the second Higgs mass m_{h_2}: $m_\chi \simeq m_{h_2}/2$.

Summary

We proposed an $SO(10)$ pNGB DM model in the framework of an $SO(10)$ GUT. $SO(10)$ is broken to G_{PS} at $\mu = M_U$, and further to G_{SM} at $\mu = M_I$.

The $SO(10)$ pNGB DM model

- Gauge coupling unification is realized because of the existence of an intermediate scale $M_I \simeq 10^{11}$ GeV.
- Constraint from proton decay is satisfied due to $M_U \gtrsim 10^{16}$ GeV.
- pNGB can be a DM candidate when the DM mass m_χ is slightly below half of the second Higgs mass.

$SO(10)$ pNGB DM model

Matter content in the $SO(10)$ pNGB DM model

	$\Psi_{\mathbf{16}}$						$\Phi_{\mathbf{10}}$	$\Phi_{\mathbf{16}}$	$\Phi_{\overline{\mathbf{126}}}$
$SO(10)$	$\mathbf{16}$						$\mathbf{10}$	$\mathbf{16}$	$\overline{\mathbf{126}}$
$SL(2,\mathbb{C})$	$(1/2,0)$						$(0,0)$	$(0,0)$	$(0,0)$
	$\psi_{(\mathbf{2},\mathbf{1},\mathbf{4})}$		$\psi_{(\mathbf{1},\mathbf{2},\overline{\mathbf{4}})}$				$\phi_{(\mathbf{2},\mathbf{2},\mathbf{1})}$	$\phi_{(\mathbf{1},\mathbf{2},\overline{\mathbf{4}})}$	$\phi'_{(\mathbf{1},\mathbf{3},\overline{\mathbf{10}})}$
G_{PS}	$(\mathbf{2},\mathbf{1},\mathbf{4})$		$(\mathbf{1},\mathbf{2},\overline{\mathbf{4}})$				$(\mathbf{2},\mathbf{2},\mathbf{1})$	$(\mathbf{1},\mathbf{2},\overline{\mathbf{4}})$	$(\mathbf{1},\mathbf{3},\overline{\mathbf{10}})$
	Q_L	L	u_R^c	d_R^c	e_R^c	ν_R^c	H	S	Φ
$SU(3)_c$	$\mathbf{3}$	$\mathbf{1}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$+\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{2}{3}$	$+\frac{1}{3}$	$+1$	0	$+\frac{1}{2}$	0	0
$U(1)_{B-L}$	$+\frac{1}{3}$	-1	$-\frac{1}{3}$	$-\frac{1}{3}$	$+1$	$+1$	0	$+1$	$+2$

Gauge coupling unification: other cases

Group G_I	Scalars at $\mu = M_I$	b_j	$\log_{10}(M/1[\text{GeV}])$		α_U^{-1}
			M_I	M_U	
G_{PS}	$(\mathbf{1},\mathbf{2},\mathbf{2})_{\mathbf{10}}$ $(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_{\mathbf{16}}$ $(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})_{\overline{\mathbf{126}}}$	$\begin{pmatrix} b_{4C} \\ b'_{2L} \\ b_{2R} \end{pmatrix} = \begin{pmatrix} -\frac{22}{3} \\ -3 \\ +\frac{13}{3} \end{pmatrix}$	11.10 ± 0.08	16.31 ± 0.15	45.92 ± 0.50
$G_{\text{PS}} \times D$	$(\mathbf{1},\mathbf{2},\mathbf{2})_{\mathbf{10}}$ $(\mathbf{4},\mathbf{2},\mathbf{1})_{\mathbf{16}}$ $(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_{\mathbf{16}}$ $(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})_{\overline{\mathbf{126}}}$ $(\mathbf{10},\mathbf{3},\mathbf{1})_{\overline{\mathbf{126}}}$	$\begin{pmatrix} b_{4C} \\ b'_{2L} \\ b_{2R} \end{pmatrix} = \begin{pmatrix} -4 \\ +\frac{13}{3} \\ +\frac{13}{3} \end{pmatrix}$	13.71 ± 0.03	15.22 ± 0.04	40.82 ± 0.13
G_{LR}	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_{\mathbf{10}}$ $(\mathbf{1},\mathbf{1},\mathbf{2},1)_{\mathbf{16}}$ $(\mathbf{1},\mathbf{1},\mathbf{3},2)_{\overline{\mathbf{126}}}$	$\begin{pmatrix} b'_{3C} \\ b'_{2L} \\ b_{2R} \\ b_{B-L} \end{pmatrix} = \begin{pmatrix} -7 \\ -3 \\ -\frac{13}{6} \\ +\frac{23}{4} \end{pmatrix}$	8.57 ± 0.06	16.64 ± 0.13	46.13 ± 0.41
$G_{\text{LR}} \times D$	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_{\mathbf{10}}$ $(\mathbf{1},\mathbf{1},\mathbf{2},1)_{\mathbf{16}}$ $(\mathbf{1},\mathbf{2},\mathbf{1},1)_{\mathbf{16}}$ $(\mathbf{1},\mathbf{1},\mathbf{3},2)_{\overline{\mathbf{126}}}$ $(\mathbf{1},\mathbf{3},\mathbf{1},-2)_{\overline{\mathbf{126}}}$	$\begin{pmatrix} b'_{3C} \\ b'_{2L} \\ b_{2R} \\ b_{B-L} \end{pmatrix} = \begin{pmatrix} -7 \\ -\frac{13}{6} \\ -\frac{13}{6} \\ +\frac{15}{2} \end{pmatrix}$	10.11 ± 0.04	15.57 ± 0.09	43.38 ± 0.30

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