

GW Physics and Astronomy Status and prospects

Yousuke Itoh (伊藤 洋介) Research Center for the Early Universe, the University of Tokyo KAGRA Collaboration LIGO Scientific Collaboration

First Workshop of the Young Researcher Association for Astroparticles, Chiba, 2016/10/29-31

Yousuke Itoh

- post-Newtonian approximation (Tohoku Univ.) with T. Futamase, 2002
- GEO600 & LIGO data anlaysis at AEI, Potsdam & UWM (2002-2006)
- iKAGRA data analysis
 - Search for Continuous gravitational waves (pulsars)
 - Data analysis subsystem





GW150914 DISCOVERY BY LIGO

Expected NS/NS event rate for aLIGO 0.1 events in O1









LIGO Laboratory Executive Director, Caltech

XAPP

TITLE: GCN CIRCULAR NUMBER: 18330 SUBJECT: LIGO/Virgo G184098: Burst candidate in LIGO engineering run data

Dear colleagues,

We would like to bring to your attention a trigger identified by the online Burst analysis during the ongoing Engineering Run 8 (ER8).







Observed GW



LSC 2016/2 PRL

Move from NS/NS to BBH



WHAT'S GW?

What emits gravitational wave (GW)?

$$L_{GW} = \frac{G}{c^5} \left\langle \hat{I}_{ij} \hat{I}_{ij} \right\rangle \sim 4 \times 10^{50} \frac{\text{erg}}{\text{s}} \left(\frac{\varepsilon}{0.1} \right) \left(\frac{GM/c^2 R}{0.1} \right)^2 \left(\frac{v/c}{0.1} \right)^6$$

Non-Spherically symmetric (large ε), Massive (large M), Fast (large v) systems will emit GWs most efficiently. Astronomy!

Indirect evidence of GW existence: Double neutron star binary

- Known to exist: PSR B1913+16
- 1993 Nobel prize



J0737-3039 (2003) NEW: PSR J1913+1102 by E@H

Gravitational Wave sources for KAGRA/LIGO/VIRGO

SNe, GRBs in local universe.

Nearby pulsars in Milkyway





NS/NS binaries coalescence within 200 Mpc. Steller mass BH/BH within z~ 0.1.

Stochastic background Probably not primordial one.

Images are taken from NASA and hubblesite.

All sky maps:

Neutrino (Icecube) Gamma-Ray >100MeV (CGRO, NASA) Gamma-Ray (N. Gehrels et.al. GSFC, EGRET, NASA) X-Ray 2-10keV (HEAO-1, NASA) X-Ray 0.25, 0.75, 1.5 keV (S. Digel et. al. GSFC, ROSAT, NASA) Ultraviolet (J. Bonnell et.al.(GSFC), NASA) Visible (Axel Mellinger) Infrared (DIRBE Team, COBE, NASA) Radio 1420MHz (J. Dickey et.al. UMn. NRAO SkyView) Radio 408MHz (C. Haslam et al., MPIfR, SkyView)



GW????



http://www.ipac.caltech.edu/outreach/Multiwave/gallery3.html

All sky maps:

Neutrino (Icecube) Gamma-Ray >100MeV (CGRO, NASA) X-Ray 0.25, 0.75, 1.5 keV (S. Digel et. al. GSFC, ROSAT, NASA) Infrared (DIRBE Team, COBE, NASA) Gamma-Ray (N. Gehrels et.al. GSFC, EGRET, NASA) Ultraviolet (J. Bonnell et.al.(GSFC), NASA) Radio 1420MHz (J. Dickey et.al. UMn. NRAO SkyView) X-Ray 2-10keV (HEAO-1, NASA) Visible (Axel Mellinger) Radio 408MHz (C. Haslam et al., MPIfR, SkyView)





http://www.ipac.caltech.edu/outreach/Multiwave/gallery3.html

It's a tiny effect at the Earth.

In essence, we just measure change of distance between two freely falling objects....



But it is extraordinarily tiny tiny change in distance

 $\delta l = h(t)$



DETECTOR BASICS

km-class GW detectors in the world: 2015

Rein States

LIGO Hanford 4 km, desert

GEO 600 600 m, university farm





Virgo Cascina 3 km

LIGO Livingston 4 km, forest

km-class GW detectors in the world: FY2015

LIGO Hanford 4 km, desert

GEO 600 600 m, university farm





LIGO Livingston 4 km, forest

Virgo Cascina 3 km

iKAGRA Kamioka, 3 km, Michelson underground

km-class GW detectors in the world: 2023 -

LIGO Hanford 4 km, desert

GEO 600 600 m, farm







LIGO Livingston 4 km, forest

Virgo Cascina 3 km

bKAGRA Kamioka, 3 km, DRFPM cryogenic, underground

Gravitational Wave Detectors

measure time variation of spatial distance









LIGO Hanford 2Km & 4Km LIGO Livingston 4 Km GEO Hannover 600m

http://www.ligo-wa.caltech.edu/

aLIGO (Hanford) Control room



Gravitational Wave Detectors

measure time variation of spatial distance



GEO 600 Control room around 2002





Toilets available (outside of the control room) \rightarrow

20

KAGRA site



KAGRA site (Kamioka mine)



Use a Laser interferometer to detect GWs



アニメーション(LIGO)

A Schematic figure of an interferometric GW detector: taken from Einstein@Home web page.



Sensitivity curves of ground based detectors and expected GW sources



Sathyaprakash & Schutz Liv. Rev. Rel. (2009)

GW experimentalists had made it!!!



Sensitivity curve of the Ground based detectors

Lase interferometer \rightarrow measure distance

 $\delta l = h(t)L$



Joshua Smith Slide @ GWPAW2013



$$h\simeq \frac{\Delta l}{l}\sim \frac{\lambda_{laser}}{l}\sim \frac{10^{-6}m}{10^3m}\sim 10^{-9}$$



$$h\simeq \frac{\Delta l}{l_{eff}}\sim \frac{\lambda_{laser}}{\mathcal{F}l}\sim \frac{10^{-6}m}{10^{6}m}\sim 10^{-12}$$




LSC: Class.Quant.Grav. 26 (2009



2015年9月から2016年1月までおこなわれた advanced LIGOの最初の観測時の光学的 構成の簡略図。

Sensitivity curve of the Ground based detectors

Lase interferometer \rightarrow measure distance

 $\delta l = h(t)L$



Joshua Smith Slide @ GWPAW2013

KAGRA: World's first 2.5 generation GW detector



Image & seismic Data from http://gwcenter.icrr.u-tokyo.ac.jp/en/

- Japanese gravitational wave experiment.
- Fabry-Perot Michelson type Laser interferometer.
- Two 3km arms.
- Under Kamioka mine (same site as Super-Kamiokande), Gif pref. Japan to reduce seismic noise.

• 20 Kelvin mirror to reduce thermal noise.



KAGRA underground



Plot by A. Shoda et al. (JGW-G1605219-v3)

KAGRA

Aomori

Najin

KAGRA site location



KAGRA Collaboration

- PI: Takaaki Kajita
- Host: ICRR, Univ. of Tokyo, Co-Hosts: KEK and NAOJ
- 248 collaborators from more than 80 institutes
 75 researchers from ~ 38 oversea univ./inst.)



- Japan Gravitational Wave Consortium (JGWC)
- ELITES collaboration for R&D of ET-KAGRA
- Strong supports from LIGO-VIRGO collaboration
- Kipp Cannon, a new associate prof. @ RESCEU (2016/02)

MEXT New Innovative Area: "New development in astrophysics through multimessenger observations of gravitational wave sources"

http://www.gw.hep.osaka-cu.ac.jp/gwastro/

- PI: Takashi Nakamura (2012-2016)
- X-ray (N. Kawai)
- Optical Infrared (M. Yoshida)
- Neutrino (M. Vagins)
- GW data analysis (N. Kanda)
- Theory (T. Tanaka)



Photos of the KAGRA site



KAGRA status

- iKAGRA finishes successfully
 - simple Michelson
 - stable data transfer, basic calibration
 - data analysis now undertaken
- bKAGRA: Cryogenic-DRFPM (2018-).
- Some issues: (other than money & manpower)



bear threats avoidance by

man power for snow shoveling (Winter)

1000 t/h water flow by snow melting (April)





iKAGRA run

- March 26 31, April 11 25: 2 periods
- Simple Michelson configuration
- Observation shifts
- Stable data transfer, off-line calibration
- CBC Hardware injections
- Get experiences for bKAGRA 2018/03 & 2018/09

MULTIBAND GWASTRONOMY

 $\frac{3Gm}{R^3}$ Characteristic GW freq. $f_{gw} \sim \frac{1}{2\pi} \sqrt{\pi G \rho} \sim \frac{1}{4\pi} \sqrt{\pi}$



Multi-frequency GW Science



http://www.ast.cam.ac.uk/~rhc26/sources/



http://www.ast.cam.ac.uk/~rhc26/sources/

PTA: Background from SMBH Binaries

Sesana 2013 arxiv 1211.5375

Binary Stochastic BGGW

$$h_c(f) \simeq \left(\frac{1}{f^2} \int \frac{dz dM_{\bullet,1} dq_{\bullet}}{1+z} \frac{d^3 n}{dz dM_{\bullet,1} dq_{\bullet}} \frac{d^3 n}{d \ln f_r} \right)^{1/2} \equiv A \left(\frac{f}{1 \mathrm{yr}^{-1}}\right)^{-2/3}$$

• Assuming SMBHB coalescence just after host galaxies merges. Find galaxy merger rate

$$\frac{d^3 n_G}{dz dM dq} = \frac{\phi(M,z)}{M \ln 10} \frac{\mathcal{F}(z,M,q)}{\tau(z,M,q)} \frac{dt_r}{dz}$$

- Galactic mass function number of galaxy pairs/volume from Observation
- Merger time scale from Millenium simulation (Kitzbichler & White (2008))
- SMBH mass from the Buldge-mass relation



PTA: Background from SMBH Binaries



Shannon et al (2015) PPTA



http://relativity.livingreviews.org/Articles/Irr-2014-3/articlese10.html

eLISA (2034?) verification binary



GW ASTRONOMY

Data analysis



Detector output time series



Detector power spectrum:
Huge power in low frequencies
✓ High-pass filter (typ. Hann)
✓ Whitening

Line noise notch filter

Data analysis



Want to extract physical information Compute correlation with theoretical expectation Find the model that maximizes the correlation



Data analysis



Want to extract physical information Compute correlation with theoretical expectation Find the model that maximizes the correlation



Luminosity distance, sky location \vec{n} , masses, spin angular momentums of the black holes of the binary



Luminosity distance, sky location \vec{n} , masses, spin angular momentums of the black holes of the binary



Luminosity distance, sky location \vec{n} , masses, spin angular momentums of the black holes of the binary



Numerical Relativity: Merger of two black holes What happened to event horizons? How much energy was radiated?



GW150914:FACTSHEET

BACKGROUND IMAGES: TIME-FREQUENCY TRACE (TOP) AND TIME-SERIES (BOTTOM) IN THE TWO LIGO DETECTORS; SIMULATION OF BLACK HOLE HORIZONS (MIDDLE-TOP), BEST FIT WAVEFORM (MIDDLE-BOTTOM)

first direct detection of gravitational waves (GW) and first direct observation of a black hole binary

observed by	LIGO L1, H1	duration from 30 Hz	~ 200 ms
source type	black hole (BH) binary	# cycles from 30 Hz	~10
date	14 Sept 2015	peak GW strain	1 x 10 ⁻²¹
time	09:50:45 UTC	peak displacement of	+0.002 fm
likely distance	0.75 to 1.9 Gly 230 to 570 Mpc	interferometers arms frequency/wavelength	150 Hz, 2000 km
redshift	0.054 to 0.136	neak speed of BHs	~ 0.6 c
signal-to-noise ratio	24	peak GW luminosity	3.6 x 10 ⁵⁶ erg s ⁻¹
false alarm prob.	< 1 in 5 million	radiated GW energy	2.5-3.5 M⊙
false alarm rate	< 1 in 200,000 yr	remnant ringdown free	g. ~ 250 Hz
Source Masses Mo		remnant damping time ~ 4 ms	
total mass primary BH	60 to 70 32 to 41	remnant size, area 180 km, 3.5 x 10 ⁵ km ²	
secondary BH	25 to 33	consistent with passes all tests general relativity? performed	
remnant BH	58 to 67	graviton mass bound	< 1.2 x 10 ⁻²² eV
mass ratio primary BH spin	0.6 to 1 < 0.7	coalescence rate of binary black holes	2 to 400 Gpc ⁻³ yr ⁻¹
secondary BH spin	< 0.9	online trigger latency	~ 3 min
remnant BH spin	0.57 to 0.72	# offline analysis pipeli	nes 5
signal arrival time delay	arrived in L1 7 ms before H1	CPU hours consumed PCs run for 100 days)	
likely sky position likely orientation resolved to	face-on/off ~600 sq. deg.	papers on Feb 11, 2016 # researchers	13 ~1000, 80 institutions in 15 countries

https://losc.ligo.org/events/GW150914/

Localization



~ 600 square degrees

 $\delta\theta \sim \frac{0.3 \text{ rad}}{\text{SNR}} \left(\frac{f}{100 \text{Hz}}\right)^{-1} \left(\frac{D}{10,000 \text{km}}\right)^{-1}$

Gamma-ray detection by Fermi GBM



- ✓ 0.4sec after GW150914
- ✓ False alarm probability 0.002
- ✓ 1sec flash: SGRB?
- ✓ 2 x 10⁴⁹ erg s⁻¹ @410Mpc
 - one order dimmer?
- ✓ If SGRB we should find 10 events akin to GW150914 .
- ✓ NO detection by INTEGRAL/SWIFT
- Collaboration issues Arxiv: 1606.00314

Can find (1+z)M. No redshift info.



degeneracy between distance & inclination



$$h_{+}(t) = \frac{1 + \cos^{2} \iota}{2} \frac{GM}{c^{2}r} \left(\frac{t_{c} - t}{5GM/c^{3}} \right)^{-1/4} \cos \left[\varphi_{c} - \left(\frac{t_{c} - t}{5GM/c^{3}} \right)^{3/6} \right]$$
$$h_{\times}(t) = -\cos \iota \frac{GM}{c^{2}r} \left(\frac{t_{c} - t}{5GM/c^{3}} \right)^{-1/4} \sin \left[\varphi_{c} - \left(\frac{t_{c} - t}{5GM/c^{3}} \right)^{5/8} \right]$$

Spins



質量比
$$q = m_2/m_1 < 1$$

有効スピン $\chi_{eff} = \frac{1}{m_1 + m_2} \hat{L} \cdot \left(\frac{\vec{S}_1}{m_1} + \frac{\vec{S}_2}{m_2}\right)$

Aligned or intrinsically small.

Test of GR



- Mass/spin of final BH estimated from pre-merger and during/after merger are consistent.
- Not contradictory to GR waveform

$$\begin{split} \tilde{h}_{k}^{\mathrm{M}}(f;\vec{\vartheta}) &= \tilde{h}_{k}(f;\vec{\vartheta}) \left[1 + \delta A_{k}(f;\vec{\vartheta}) \right] \\ &\times \exp \left[i \delta \phi_{k}(f;\vec{\vartheta}) \right], \end{split}$$



Final BH Mass and spin



Relatively well-determined Determination from X-ray obs may be controversial.

Test of GR



- Parameters from Inspiral and post-inspiral are consistent.
- No deviation from GR found


Progenitor



Star's Metallicity (as a fraction of the solar metallicity)

✓ Unknown

 may have much smaller metalicity

too much metals:

- (a) interstellar gas cools
 - cloud splits into small pieces
 - small stars form
- (b) Large opacity
 - Large stellar wind
 - Large mass loss
- ✓ May be Pop III ??
 (Kinugawa et al. 2014)
 ✓ B-DECIGO science target

http://ligo.org/science/Publication-GW150914Astro/index.php

ASTROPHYSICAL IMPLICATIONS

ASTROPHYSICAL IMPLICATIONS OF THE BINARY BLACK-HOLE MERGER GW150914

- Facts:
 - Existence of BH with mass larger than 25 M_{sun}
 - Nearly equal mass
 - Small spin or aligned spin
 - Event rate: 2-53 (400) Gpc⁻³ yr⁻¹
- Two scenario
 - Isolated BBH in galactic fields: isolated scenario
 - Young and/or old dense stellar environments: dynamical scenario
- Suggestions
 - Mass indicates low metalicity: Z < 0.5 Z_{sun}
 - From old universe or low mass (low metal) galaxy
 - Cannot distinguish two scenarios

High mass indicates low metalicity

- Mass:
 - X-Ray Binary (XRB) 22 systems : $M_{BH} \sim 5-20 M_{sun}$
 - Stellar wind:
 - low metallicity -> weak wind -> low mass-loss -> high mass BH
 - But stellar wind from low metal star is not well-known: extraporation to $\rm Z_{sun} \, ^{\sim} \, 0.02$



distinguish two scenarios

- Spin measurement
 - orbital angular momentum does not need to align with BH spins in dynamical scenarios
 - Could not constrain from the GW observation.
- Eccentricity
 - Eccentricity would be large in dynamical scenarios
 - But we cannot determine e if e < 0.1</p>
 - Eccentricity is always small in the aLIGO band

Redshift distribution and B-DECIGO?





geocentric orbit

Maybe we will call it "B-DECIGO" instead of "Pre-DECIGO".

nakamura et al. 1607.00897

Redshift distribution and "B-DECIGO"?



With "B-DECIGO", we can distinguish three scenarios: PoP III, Pop I/II, and PBBH

nakamura et al. 1607.00897

"B-DECIGO" forecasts



DETAILS

Operation time-line and sensitivity



Kiwamu Izumi (和泉究)、天文学会誌6月号 http://www.asj.or.jp/geppou/archive_open/2016_109_06/109_381.pdf Keiko Kokeyama (苔山圭子)、高エネルギーニュース Cannon et al., 情報処理学会誌5月号

TITLE: GCN CIRCULAR NUMBER: 18330 SUBJECT: LIGO/Virgo G184098: Burst candidate in LIGO engineering run data

Dear colleagues,

We would like to bring to your attention a trigger identified by the online Burst analysis during the ongoing Engineering Run 8 (ER8).

G184098 (2015-09-16 06:39 UT) 2015-09-14 09:50:45 UTC

- FAR below ~1/month
 - < $1/(2.7yr) \rightarrow$ < $1/(100yr) \rightarrow$ < $1/(22500yr) \rightarrow$ < 1/(203000yr)
- gstlal/pycbc pipeline were turned-off.
- ER8: O1 not yet started
 - Calibration not fixed.
 - Manual alert to EM follow-up within 3 mins of discovery.
- 200 deg² (50 % CL) or 750 deg² (90 % CL)
- Distance not announced.
- Mass not announced.

direction



 98 deg^2 308 deg^2 0.55 0.55

 208 deg^2 746 deg^2

 $101 \text{ deg}^2 634 \text{ deg}^2$

 $140 \text{ deg}^2 590 \text{ deg}^2$

LALNoCE 48 deg^2 150 deg²

0.51

0.68

0.68

0.45

-

_

-

0.50

0.28

0.87

0.81

cWB

LIB

BW

LALInf

	÷.		n
c.		1	
	٩.	/	
		-	

Calibration



$$R(f) = \frac{1 + A(f) D(f) C(f)}{C(f)}$$

$$L_{\text{ext}}(f) = R(f)d_{\text{err}}(f)$$





Another event?



Another event? Not really



False alarms: Omicron



Abbott et al. P1500238

89



False alarms: Omicron

Abbott et al. P1500238





False alarm probability



- Green (Right)/Black (Left) are assuming GW150914 were a noise: essentially one detector background.
- Scaled to 16 days
- Actually hard to see significance. Perhaps, much significant than "5.1 sigma"

FUTURE

Plan and prospects



Prospects based on real events



Why KAGRA useful in 2018?

CBC/Burst Search: we do not know when it visits us.

Importance is in not only the sensitivity, but also in the duty cycle.

Duty cycle in O1 LHO (windy desert) LLO (forestry) 33 % coincidence

VIRGO (VSR2/4) 80% KAGRA (expected) 80%

We need as many detectors as possible that are actually running.
 Underground is ideal place (low seismic noise)

Stochastic foreground



O5 (2022?) science Obstacles to inflationary GWs?

BNS: Expected sensitivity curves & inspiral ranges of aLIGO/aVIRGO Arxiv: 1304.0670



	Estimated	$E_{\rm GW} =$	$10^{-2} M_{\odot} c^2$			Number	% BNS	Localized
	Run	Burst Range (Mpc)		BNS Range (Mpc)		of BNS	within	
Epoch	Duration	LIGO	Virgo	LIGO	Virgo	Detections	$5\mathrm{deg}^2$	$20\mathrm{deg}^2$
2015	3 months	40 - 60	_	40 - 80	-	0.0004 - 3	-	_
2016 - 17	6 months	60 - 75	20 - 40	80 - 120	20 - 60	0.006 - 20	2	5 - 12
2017 - 18	9 months	75 - 90	40 - 50	120 - 170	60 - 85	0.04 - 100	1 - 2	10 - 12
2019 +	(per year)	105	40 - 80	200	65 - 130	0.2 - 200	3 - 8	8 - 28
2022+ (India)	(per year)	105	80	200	130	0.4 - 400	17	48

BNS: Expected sensitivity curves & inspiral ranges of aLIGO/aVIRGO Arxiv: 1304.0670



KAGRA has similar future prospects.

10 ¹	10 ² frequency (H	10 ³	10 ¹	10 ² frequency (H	10 ³ Hz)	
	inequency (i	(2)		inequency (i	12)	

	Estimated	$E_{\rm GW} = 10^{-2} M_{\odot} c^2$				Number	% BNS Localized	
	Run	Burst Range (Mpc)		BNS Range (Mpc)		of BNS	within	
Epoch	Duration	LIGO	Virgo	LIGO	Virgo	Detections	$5 \mathrm{deg}^2$	$20\mathrm{deg}^2$
2015	3 months	40 - 60	_	40 - 80	_	0.0004 - 3	_	_
2016 - 17	6 months	60 - 75	20 - 40	80 - 120	20 - 60	0.006 - 20	2	5 - 12
2017 - 18	9 months	75 - 90	40 - 50	120 - 170	60 - 85	0.04 - 100	1 - 2	10 - 12
2019 +	(per year)	105	40 - 80	200	65 - 130	0.2 - 200	3 - 8	8 - 28
2022 + (India)	(per year)	105	80	200	130	0.4 - 400	17	48

Prospects



and duty cycle?

NS-BH? \bullet

Non CBC

Typical GW: Double Neutron star



Including tidal effects



2014 April KAGRA Data analysis school "Numerical Relativity" (By Prof. Masaru Shibata)



Radius (km)

• Mystery of generation sites of the r-process elements



Follow up



M. Tanaka, 1605.07235

Angular resoluton (90 % CL) Arxiv: 1304.0670



Virgo plans to join the LSC O2b from 2017 March. Need more to test polization!!!

Localization



Need multiple detectors, better to have many. Baseline D and arrival time determination are important

Diffraction limited angular localization is bad....

$$\delta\theta \sim \frac{0.3 \text{ rad}}{\text{SNR}} \left(\frac{f}{100 \text{Hz}}\right)^{-1} \left(\frac{D}{10,000 \text{km}}\right)^{-1}$$

Localization (simple time delay)


Localization (simple time delay)



Follow-up: time resolution & FOV



circle size: limiting magnitude

Follow-up: macronova



Tanaka & Hotokezaka (2013)

SUMMARY

Summary: 2016

- Existence of stellar mass BBHs
 - GW150914, LVT151012, GW151226
 - Progenitors unknown
- Localization is important
 - KAGRA

Summary: 202x

- Localization > 10 square degrees (With KAGRA,India)
- High event rate era: 1 event / day ?
 - Distributions: Mass, Spins, direction, redshift
 - Test of GR
 - Strange object?
 - RESCEU Symposium on GW science "High Event Rate Era", Dec. 5-6
- "Big(?)data ": 300k channels, 1PB/yr
- Detection of NS-BH (or NS/NS), NS EOS
- Non CBC

<u>GW2016 RESCEUで検索!</u>

http://www.resceu.s.u-tokyo.ac.jp/workshops/GW2016/index.php

Gravitational-Wave Astrophysics in the High Event Rate Regime

From foundations of gravity to astrophysics, a discussion of the tests and measurements made possible by an abundance of gravitational-wave sources.

University of Tokyo December 5 & 6, 2016 Followed by KAGRA Face-to-Face, Dec. 7 & 8

Scientific Organizing Committee Kipp Cannon Chad Hanna Yousuke Itoh Frederique Marion Masaru Shibata

Website: http://www.resceu.s.u-tokyo.ac.jp/ workshops/GW2016 Registration deadline: November 20, 2016







BREAK

Want to draw the following curve.



$$\vec{x}_{1}(t) = \frac{m_{2}}{m_{1} + m_{2}} a(\cos(\omega_{o}t), \sin(\omega_{o}t), 0),$$

$$\vec{x}_{2}(t) = -\frac{m_{1}}{m_{1} + m_{2}} a(\cos(\omega_{o}t), \sin(\omega_{o}t), 0),$$

$$I_{xx} = \frac{1}{2}\mu a (1 + \cos(2\omega_o t)),$$
$$I_{xy} = \frac{1}{2}\mu a \sin(2\omega_o t),$$
$$I_{yy} = \frac{1}{2}\mu a (1 - \cos(2\omega_o t)),$$

$$\begin{split} \ddot{I}_{xx} &= -2\omega_o^2 \mu a \cos(2\omega_o t), \\ \ddot{I}_{xy} &= -2\omega_o^2 \mu a \sin(2\omega_o t), \\ \ddot{I}_{yy} &= 2\omega_o^2 \mu a \cos(2\omega_o t)), \end{split}$$

Binary with orbital radius a , angular frequency ω_{o}

Compute quadrupole moment. Let µ denote reduced mass

$$\mu = m_1 m_2 / (m_1 + m_2)$$

Take temporal derivatives twice, compute transversetraceless part.

$$h_{ij}^{TT} = \frac{2G\ddot{I}_{ij}^{TT}}{c^4 r} = \frac{4G\mu a\omega_o^2}{c^4 r} \begin{pmatrix} -\cos(2\omega_o t) & -\sin(2\omega_o t) & 0\\ -\sin(2\omega_o t) & \sin(2\omega_o t) & 0\\ 0 & 0 & 0 \end{pmatrix}$$





How can we compute time series h(t) (scalar) from tensor h^{TT}_{ii}(t) ?



Detector output time series

Linear perturbation solution to the Einstein equations

$$h_{ij}^{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{+} & h_{\times} & 0 \\ 0 & h_{\times} & -h_{+} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}_{ij} = e_{ij}^{+}h_{+} + e_{ij}^{\times}h_{\times}$$

Polarization tensors

$$e_{ij}^{+} = \hat{x}_i \hat{x}_j - \hat{y}_i \hat{y}_j,$$

$$e_{ij}^{\times} = \hat{y}_i \hat{x}_j + \hat{x}_i \hat{y}_j$$

Detector arm vectors p & q, Antenna pattern function F_+ , F_x

$$\begin{split} h(t) &= \frac{1}{2} (\hat{p}^i \hat{p}^j - \hat{q}^i \hat{q}^j) h_{ij}^{TT}(t) \\ &= F_+(\vec{n}, \psi) h_+(t) + F_\times(\vec{n}, \psi) h_+(t) \end{split}$$

121



Detector output time series

Linear perturbation solution to the Einstein equations

$$h_{ij}^{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{+} & h_{\times} & 0 \\ 0 & h_{\times} & -h_{+} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}_{ij} = e_{ij}^{+}h_{+} + e_{ij}^{\times}h_{\times}$$

Polarization tensors

$$e_{ij}^{+} = \hat{x}_i \hat{x}_j - \hat{y}_i \hat{y}_j,$$

$$e_{ij}^{\times} = \hat{y}_i \hat{x}_j + \hat{x}_i \hat{y}_j$$

Detector arm vectors p & q, Antenna pattern function F_+ , F_x

$$\begin{split} h(t) &= \frac{1}{2} (\hat{p}^i \hat{p}^j - \hat{q}^i \hat{q}^j) h_{ij}^{TT}(t) \\ &= F_+(\vec{n}, \psi) h_+(t) + F_\times(\vec{n}, \psi) h_+(t) \end{split}$$

$$L_{\rm gw} = \frac{G}{5c^5} \langle \ddot{I}_{kl} \ddot{I}^{kl} \rangle = \frac{32c^5}{5G} \left(\frac{G\mathcal{M}_c \omega_o}{c^3} \right)^{10/3}$$
$$\mathcal{M}_c = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5}$$

 $E_{\rm orbit} = \frac{1}{2}\mu v^2 - \frac{G\mu m_t}{R} = -\frac{\mu c^2}{2} \left(\frac{Gm_t \omega_o}{c^3}\right)^{2/3}$

 $\dot{\omega}_o = \frac{96}{5} \left(\frac{G\mathcal{M}_c}{c^3}\right)^{5/3} \omega_o^{11/3}$

Compute GW energy using the quadrupole formula. Mc is a chirp mass

L_{gw} is from the orbital energy.

$$dE_{\rm orbit}/dt + L_{\rm gw} = 0$$

Get a differential equation for ω_o

Solve it, find the phase evolution equation.

$$\Phi_o(t) = -\int_t^{t_c} \omega_o(t)dt = -\left(\frac{5G\mathcal{M}_c}{c^3(t_c - t)}\right)^{-5/8}$$

 $\omega_o(t) = \frac{5^{3/8}}{2} \left(\frac{G\mathcal{M}_c}{2}\right)^{-5/8} (t_c - t)^{-3/8},$

$$h_{+}(t) = -A(t)\frac{1}{2}(1+\cos^{2}\iota)\cos(2\Phi_{o}(t))),$$
$$h_{\times}(t) = -A(t)\cos\iota\sin(2\Phi_{o}(t))),$$
$$A(t) = \frac{G\mathcal{M}_{c}}{c^{2}r}\left(\frac{5G\mathcal{M}_{c}}{c^{3}(t_{c}-t)}\right)^{1/4}$$



$$\tilde{h}_{+}(f_{\rm gw}) = \int_{-\infty}^{\infty} dt e^{-2\pi i f_{\rm gw} t} h_{+}(t) = \frac{1}{2} \int_{-\infty}^{t_c} dt e^{-2\pi i f_{\rm gw} t} A_{+}(t) e^{i\Phi_{\rm gw}(t)}$$

Use stationary phase approximation. Note that the integrand oscillates so rapidly that it amounts to zero for any frequencies other than

$$d\Phi_{\rm gw}(t_f)/dt = f_{\rm gw}$$

$$\tilde{h}_{+}(f_{\rm gw}) \simeq \frac{1}{2} \int_{-\eta}^{\eta} dt A_{+}(t) \exp\left[i\Phi_{\rm gw}(t_{f}) - 2\pi i f_{\rm gw}t_{f} + \frac{i}{2}\ddot{\Phi}_{\rm gw}(t_{f})(t-t_{f})^{2}\right]$$
$$\simeq \frac{1}{2} A_{+}(t_{f}) e^{i\Phi_{\rm gw}(t_{f}) - 2\pi i f_{\rm gw}t_{f}} \int_{-\eta}^{\eta} dt \exp\left[\frac{i}{2}\ddot{\Phi}_{\rm gw}(t_{f})t^{2}\right]$$
$$\simeq \frac{1}{2} \sqrt{\frac{2\pi}{|\ddot{\Phi}_{\rm gw}(t_{f})|}} A_{+}(t_{f}) e^{i\Phi_{\rm gw}(t_{f}) - 2\pi i f_{\rm gw}t_{f} + \frac{i\pi}{4}}$$
(1.3.106)



$$h_{+}(t) = -A(t)\frac{1}{2}(1+\cos^{2}\iota)\cos(2\Phi_{o}(t))),$$

$$h_{\times}(t) = -A(t)\cos\iota\sin(2\Phi_{o}(t))),$$

$$A(t) = \frac{G\mathcal{M}_{c}}{c^{2}r}\left(\frac{5G\mathcal{M}_{c}}{c^{3}(t_{c}-t)}\right)^{1/4}$$



$$\begin{split} \check{h}_{+}(f) &= -A(f)\frac{1}{2}(1+\cos^{2}\iota)\exp\left[-i\Psi(f)-2i\phi\right], \\ \check{h}_{\times}(f) &= -A(f)\cos\iota\exp\left[-i\Psi(f)-\frac{i\pi}{2}-2i\phi\right], \\ A(f) &\equiv \left(\frac{5\pi}{24}\right)^{1/2}\frac{c}{r}\left(\frac{G\mathcal{M}_{c}}{c^{3}}\right)^{2}\left(\frac{\pi G\mathcal{M}_{c}f}{c^{3}}\right)^{-7/6}, \\ \Psi(f) &\equiv 2\pi ft_{c}-\frac{\pi}{4}-\phi_{c}+\frac{3}{128}\left(\frac{\pi G\mathcal{M}_{c}f}{c^{3}}\right)^{-5/3} \end{split}$$

Power =
$$\int f S_h(f) d \ln f$$
,
 $\sqrt{f S_h(f)} = \sqrt{f |\tilde{h}(f)|^2} \propto f^{-2/3}$

Sathyaprakash & Schutz Liv. Rev. Rel. (2009)

RESUME