

Neutrino deep inelastic scattering

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30th Neutrino workshop “Neutrino Interaction Physics”

IPMU, Tokyo University, Kashiwa, Japan

<https://www.icrr.u-tokyo.ac.jp/indico/event/91/>

(with the project of Unification and Development of the Neutrino Science Frontier

<http://www-he.scphys.kyoto-u.ac.jp/nufrontier/en/index.html>

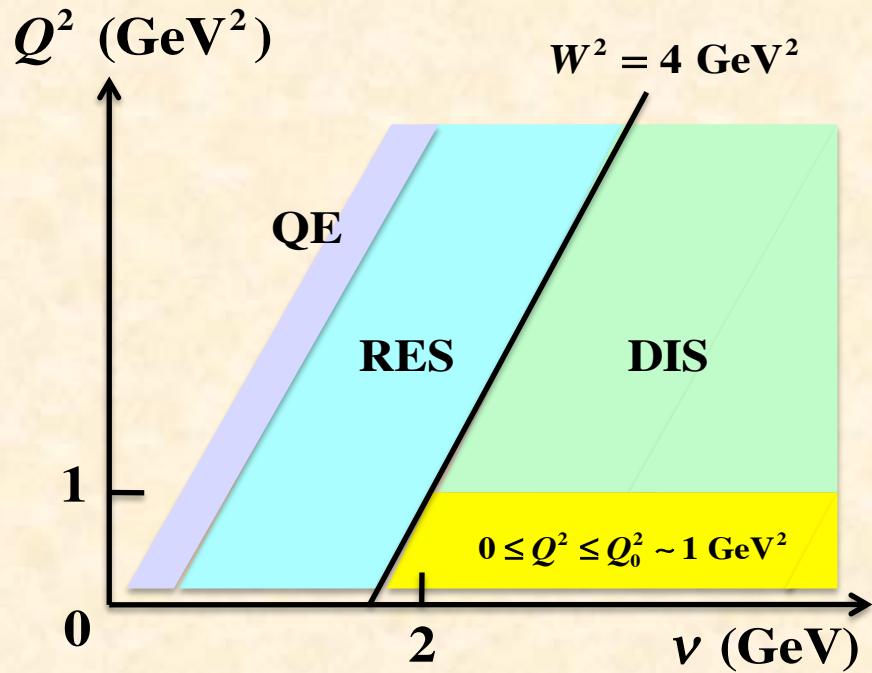
February 4, 2017

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- 2. Deep inelastic scattering (DIS)**
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Motivation

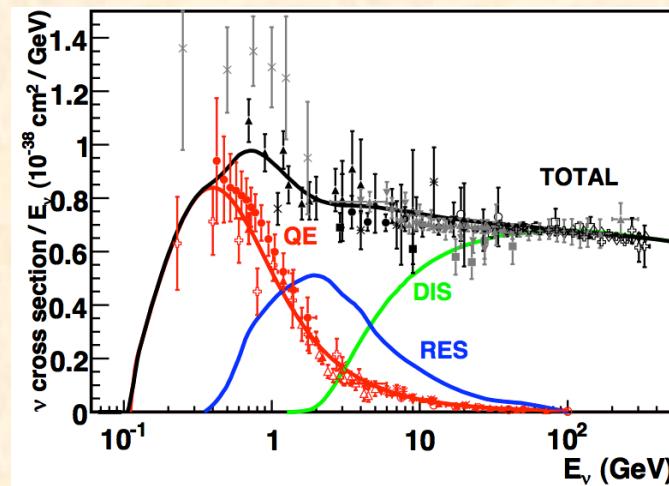
Kinematical regions of neutrino-nucleus scattering



Depending on the neutrino beam energy, different physics mechanisms contribute to the cross section.

- QE (Quasi elastic)
- RES (Resonance)
- DIS (Deep inelastic)

Activities at the J-PARC branch, KEK theory center
<http://j-parc-th.kek.jp/html/English/e-index.html>



J.L. Hewett *et al.*, arXiv:1205.2671,
 Proceedings of the 2011 workshop
 on Fundamental Physics at the Intensity Frontier

ν flux		16%
ν flux and cross section	w/o ND measurement	21.8%
	w/ ND measurement	2.7%
ν cross section due to difference of nuclear target btw. near and far		5.0%
Final or Secondary Hadronic Interaction		3.0%
Super-K detector		4.0%
total	w/o ND measurement	23.5%
	w/ ND measurement	7.7%

ν interactions

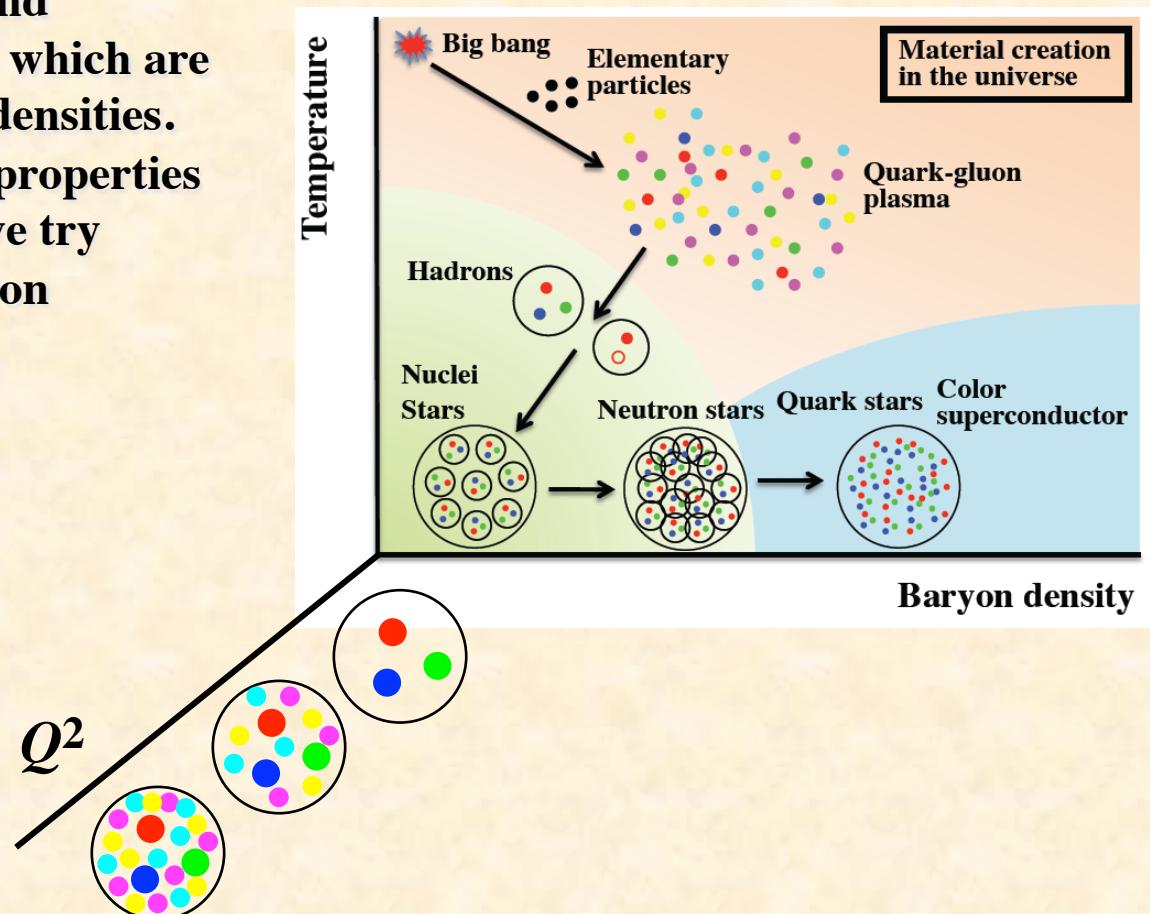
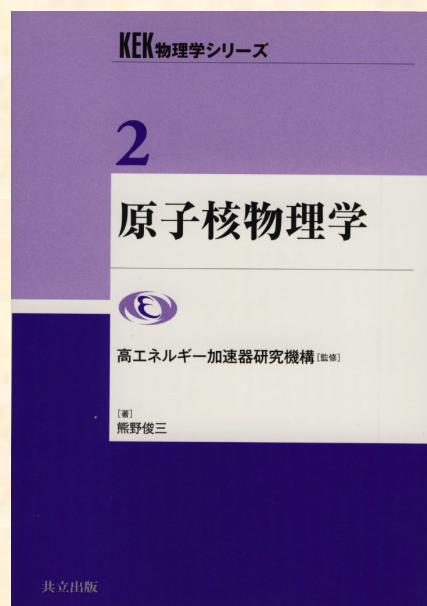
A.K.Ichikawa@KEK workshop 2015

Nuclear physics: Hadron degrees of freedom (d.o.f.) → Quark-gluon d.o.f. (QCD)

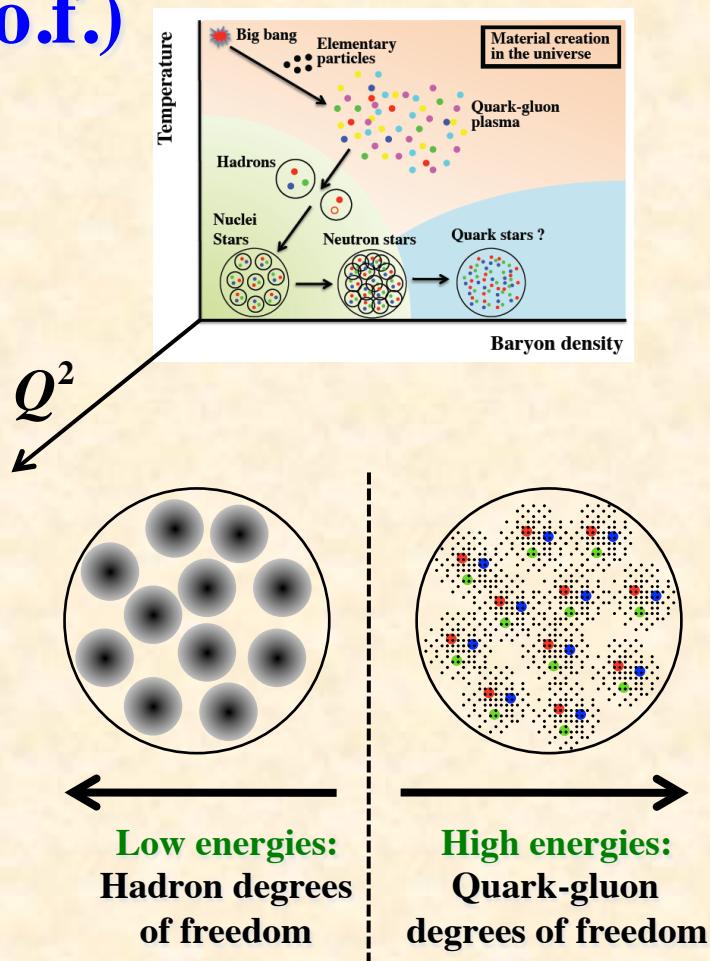
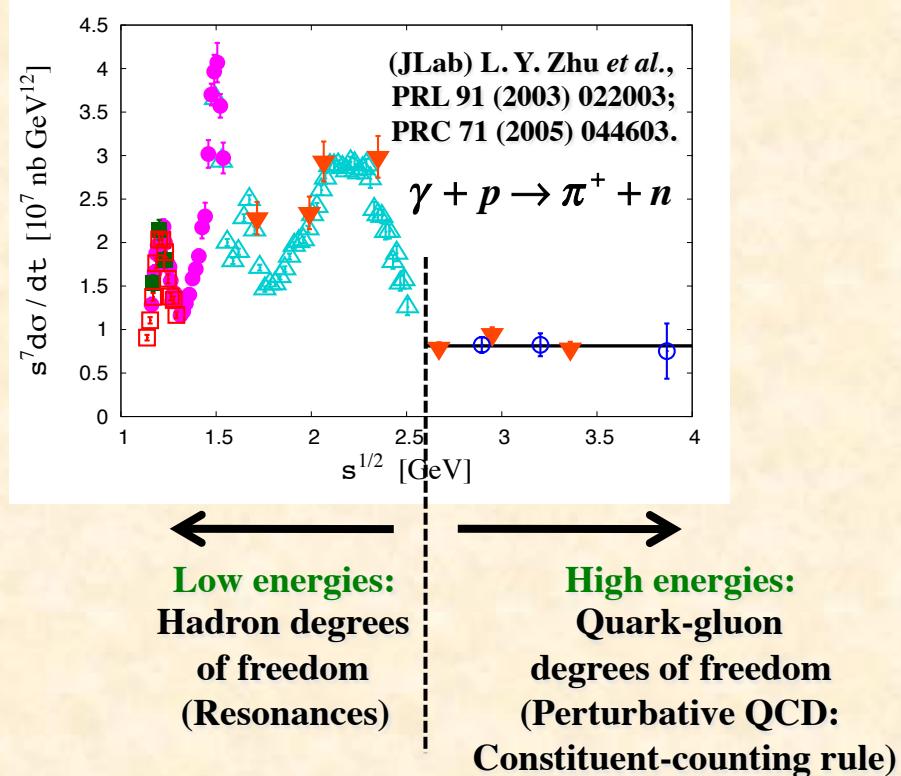
Nuclear physics is the field of science to study

- (1) material creation in the universe and
- (2) quark-hadron many-body systems which are quantum materials with ultimate densities.

Clarifying the origin of elements and properties of quark-gluon many-body systems, we try to understand our universe and creation of materials.



Hadron degrees of freedom (d.o.f.) \Leftrightarrow Quark d.o.f.

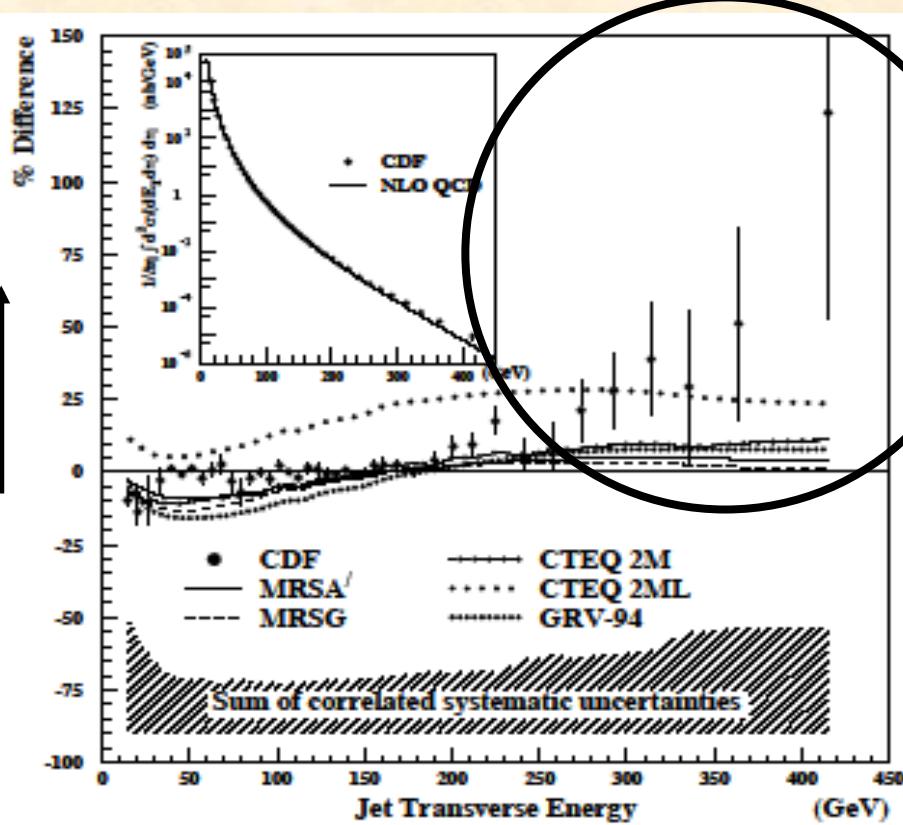


Nuclei should be described by quark and gluon degrees of freedom at high energies.

New discovery or not?

Difference between theory and experiment

Comparison of theoretical calculations with CDF experimental data.



Jet transverse energy

CDF experiment: PRL, 77 (1996) 438.

$$p + \bar{p} \rightarrow \text{jet} + X$$

$$\sqrt{s} = 1.8 \text{ TeV}, E_T^{\text{jet}} = 15 - 400 \text{ GeV}$$

New physics ???

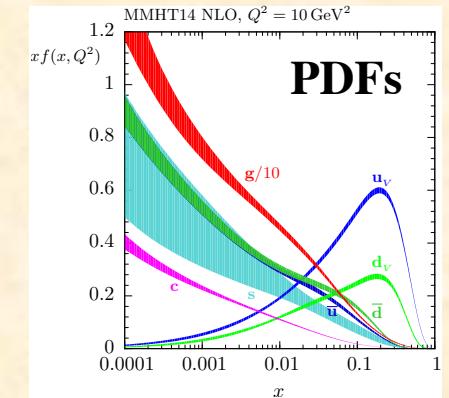
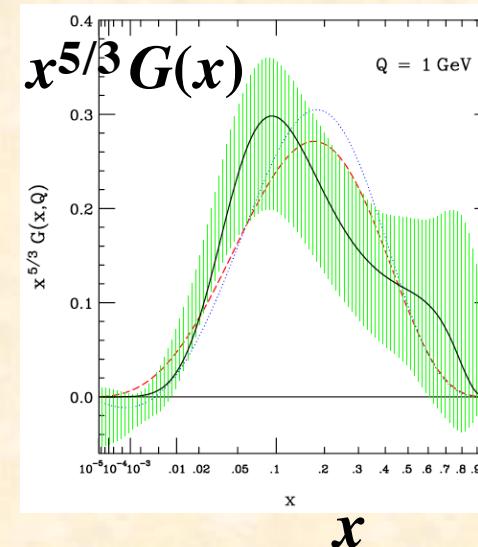
Could be explained
without new physics

(importance of accurate PDFs)
JET cross sections are “explained”
by the CTEQ6 PDFs.

CTEQ6

CTEQ5

MRST2001



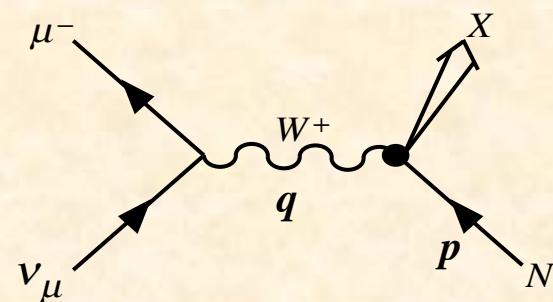
Structure functions of nucleon

Introduction to neutrino deep inelastic scattering (DIS)

Deep inelastic scattering (DIS)

A nucleon is broken up by a high-energy neutrino.

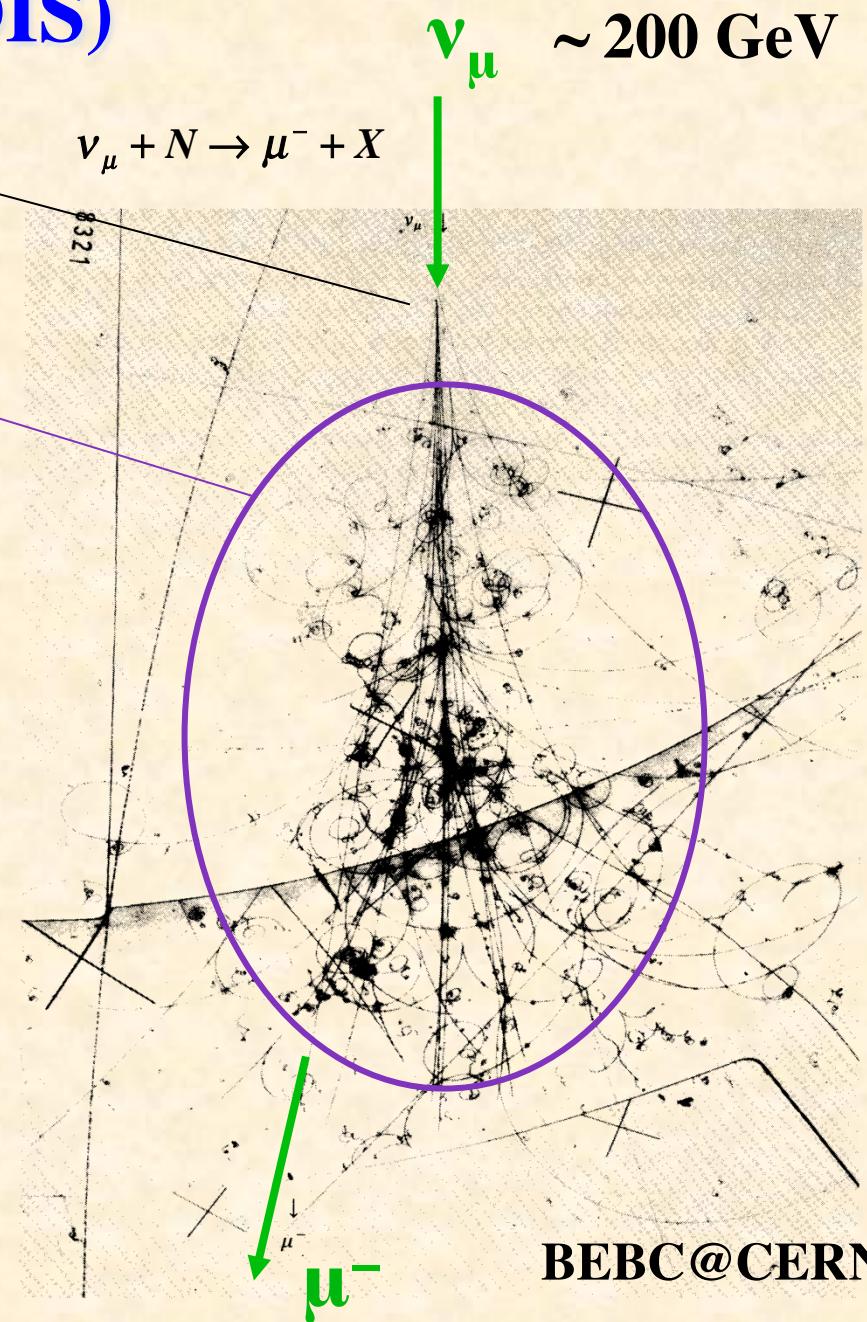
Hadrons are produced; however, these are not usually measured.
(inclusive reaction)



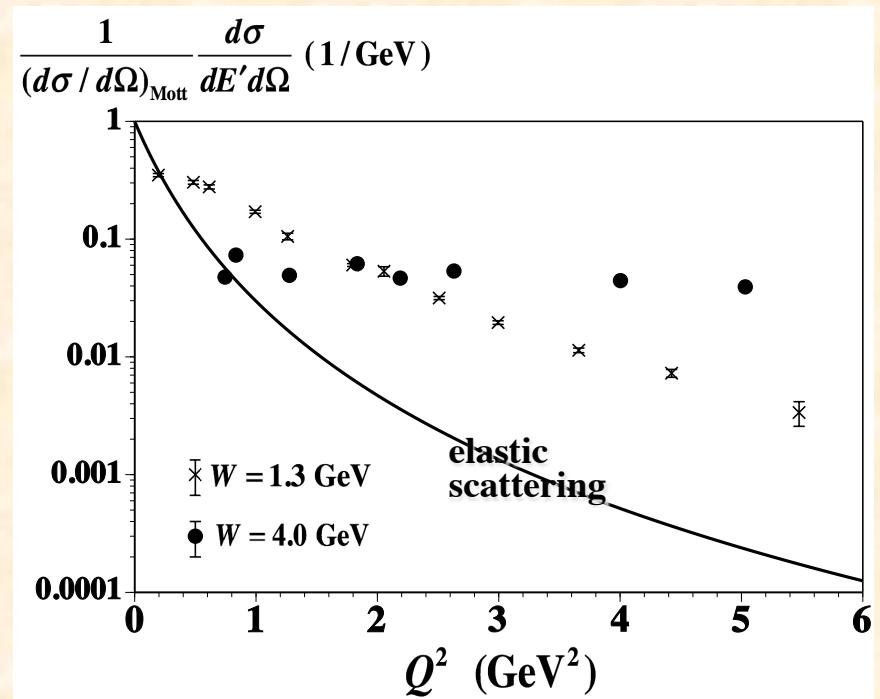
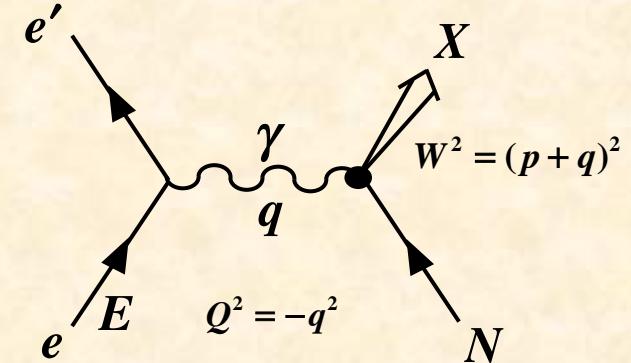
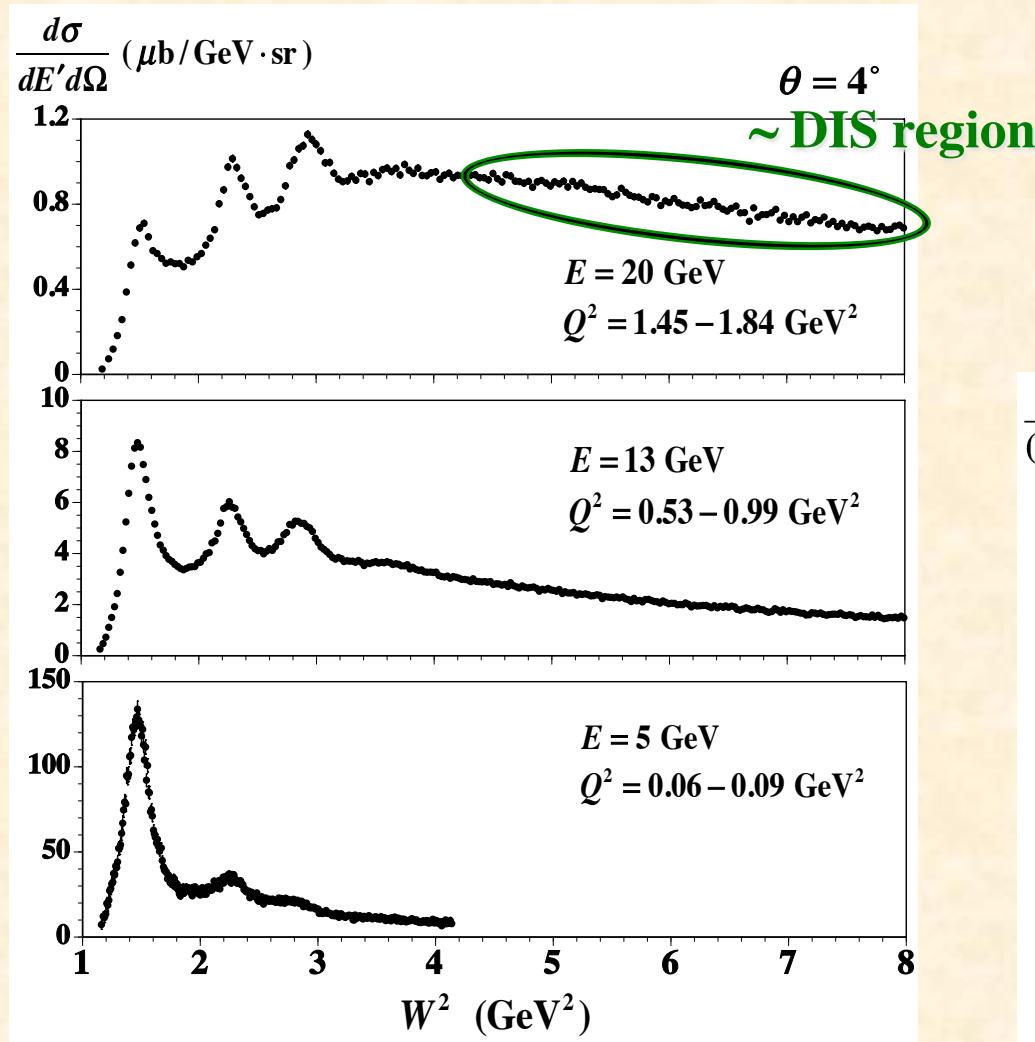
Momentum transfer: $q^2 = (k - k')^2 = -Q^2$

Bjorken scaling variable: $x = \frac{Q^2}{2p \cdot q}$

Invariant mass: $W^2 = p_X^2 = (p + q)^2$



Lepton scattering

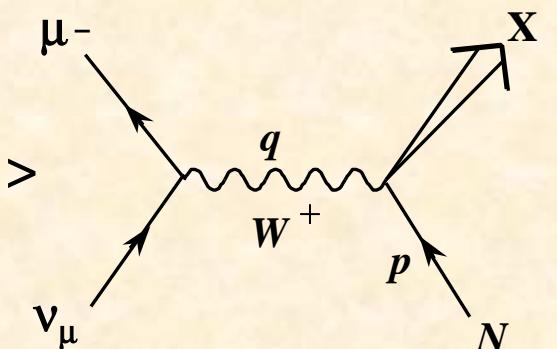


Neutrino deep inelastic scattering (CC: Charged Current)

$$d\sigma = \frac{1}{4k \cdot p} \frac{1}{2} \sum_{spins} \sum_X (2\pi)^4 \delta^4(k + p - k' - p_X) |M|^2 \frac{d^3 k'}{(2\pi)^3 2E'} \quad \mu^-$$

$$M = \frac{1}{1+Q^2/M_W^2} \frac{G_F}{\sqrt{2}} \bar{u}(k', \lambda') \gamma^\mu (1-\gamma_5) u(k, \lambda) \langle X | J_\mu^{CC} | p, \lambda_p \rangle$$

$$\frac{d\sigma}{dE' d\Omega} = \frac{G_F^2}{(1+Q^2/M_W^2)^2} \frac{k'}{32\pi^2 E} L^{\mu\nu} W_{\mu\nu}$$



$$L^{\mu\nu} = 8 \left[k^\mu k'^\nu + k'^\mu k^\nu - k \cdot k' g^{\mu\nu} + i \epsilon^{\mu\nu\rho\sigma} k_\rho k'_\sigma \right], \quad \epsilon_{0123} = +1$$

$$W_{\mu\nu} = -W_1 \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + W_2 \frac{1}{M^2} \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) + \frac{i}{2M^2} W_3 \epsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma$$

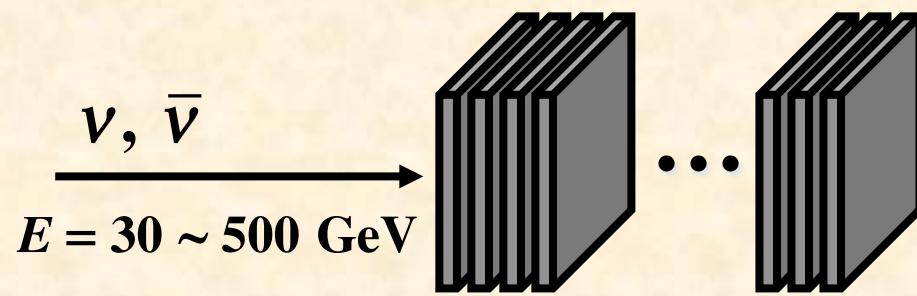
$$MW_1 = F_1, \quad \nu W_2 = F_2, \quad \nu W_3 = F_3, \quad x = \frac{Q^2}{2p \cdot q}, \quad y = \frac{p \cdot q}{p \cdot k}$$

$$\frac{d\sigma_{\nu,\bar{\nu}}^{CC}}{dx dy} = \frac{G_F^2 (s - M^2)}{2\pi (1+Q^2/M_W^2)^2} \left[x y^2 F_1^{CC} + \left(1 - y - \frac{M}{2E} \frac{x y}{x + y} \right) F_2^{CC} \pm x y \left(1 - \frac{y}{2} \right) F_3^{CC} \right]$$

Neutrino DIS experiments

• CDHS,	H. Abramowics <i>et al.</i> ,	Z. Phys. C 25 (1984) 29
• WA25,	D. Allasia <i>et al.</i> ,	Z. Phys. C 28 (1985) 321
• WA59,	K. Varvell <i>et al.</i> ,	Z. Phys. C 36 (1987) 1
• CDHSW,	P. Berge <i>et al.</i> ,	Z. Phys. C 49 (1991) 187
• Serpukhov,	A. V. Sidorov <i>et al.</i> ,	Eur. Phys. J. C 10 (1999) 405
• CCFR,	U.-K. Yang <i>et al.</i> ,	PRL 86 (2001) 2742
• NuTeV/CCFR $\mu^+\mu^-$,	M. Goncharov <i>et al.</i> ,	PRD 64 (2001) 112006
• CHORUS,	G. Onengut <i>et al.</i> ,	PLB 632 (2006) 65
• NuTeV,	M. Tzanov <i>et al.</i> ,	PRD 74 (2006) 012008
• Minerva,	J. Mousseau <i>et al.</i> ,	PRD 93 (2016) 071101, in progress

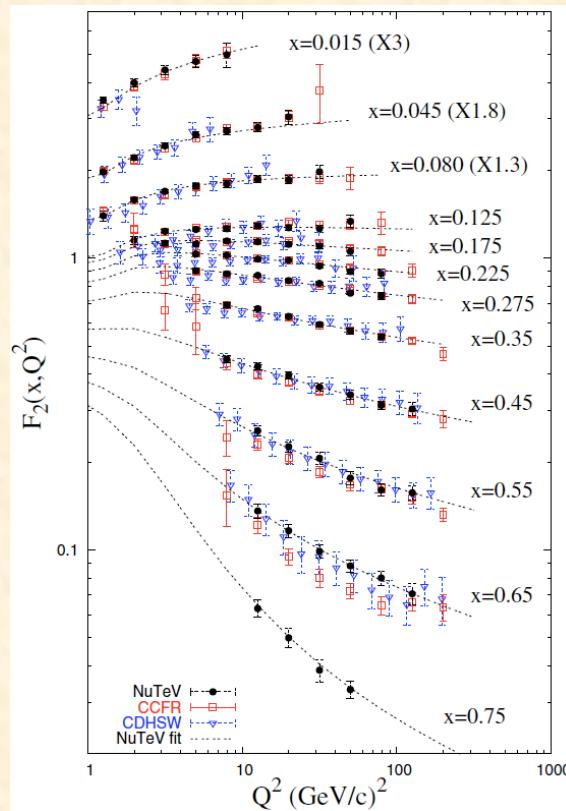
Neutrino DIS experiments



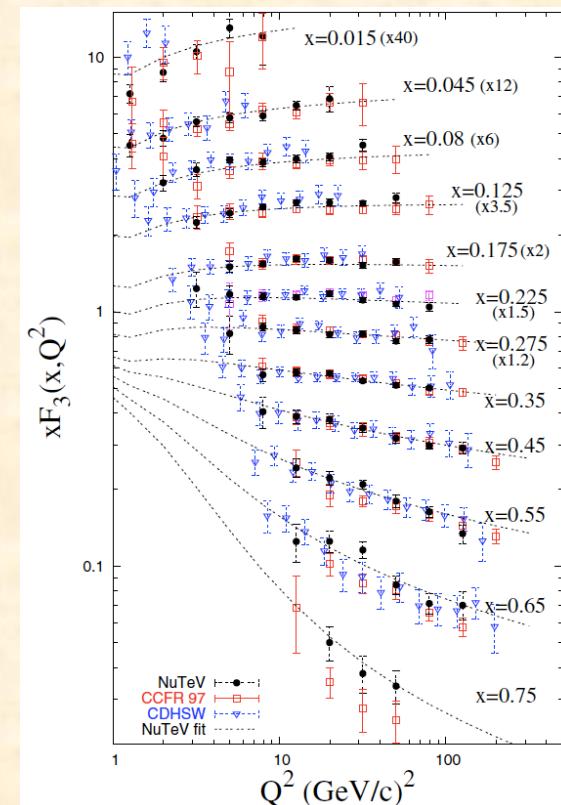
Huge Fe target (690 ton)

Experiment	Target	ν energy (GeV)
CCFR	Fe	30-360
CDHSW	Fe	20-212
CHORUS	Pb	10-200
NuTeV	Fe	30-500

M. Tzanov *et al.* (NuTeV),
PRD74 (2006) 012008.

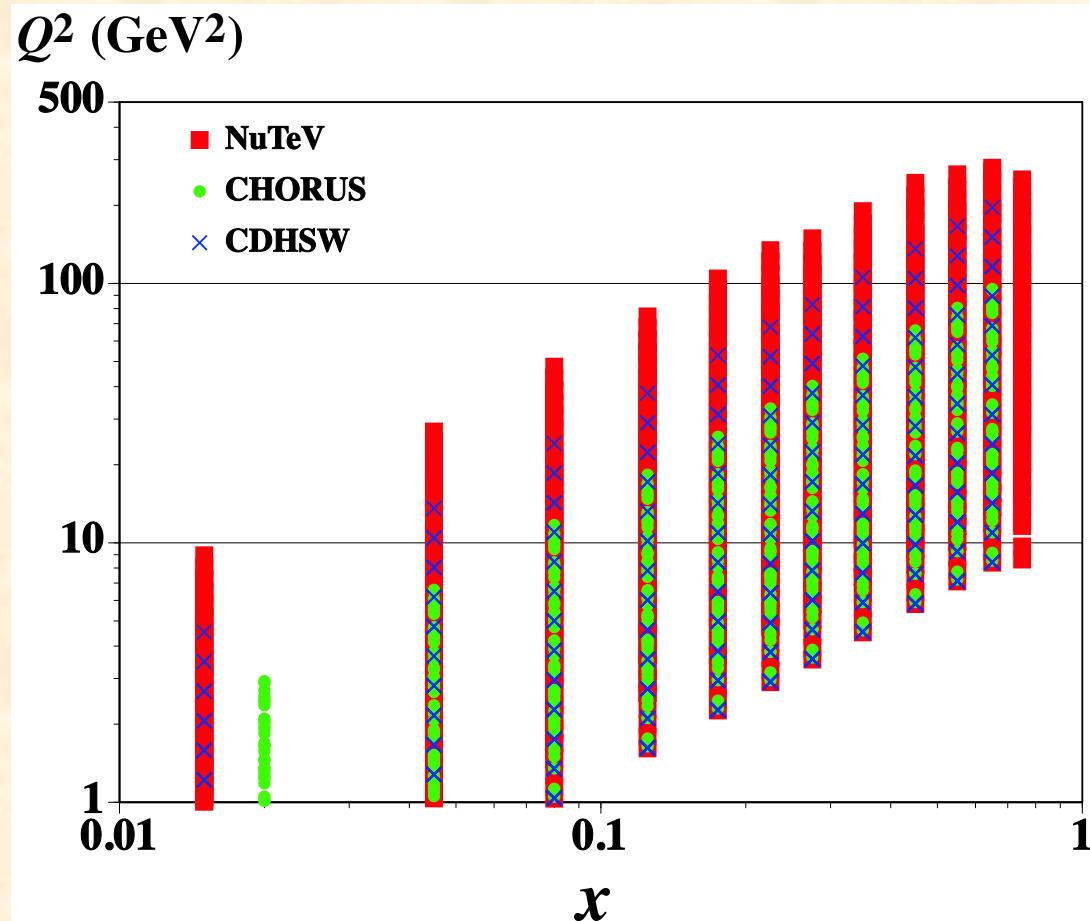


MINERvA (He, C, Fe, Pb), ...

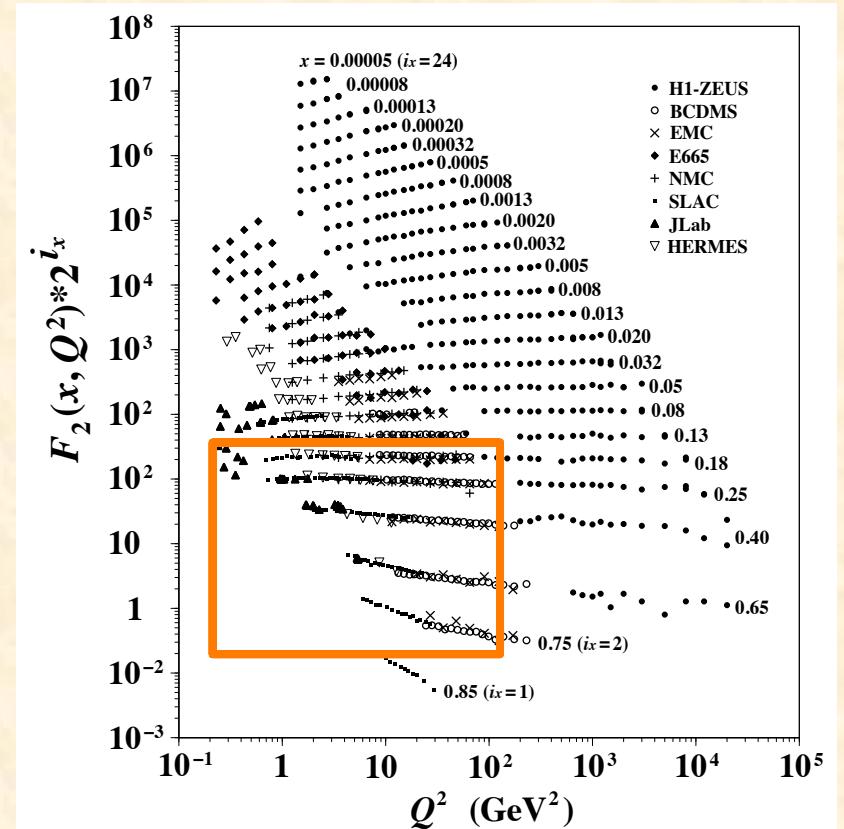


Neutrino DIS experiments: kinematical range

Neutrino DIS



Charged-lepton DIS

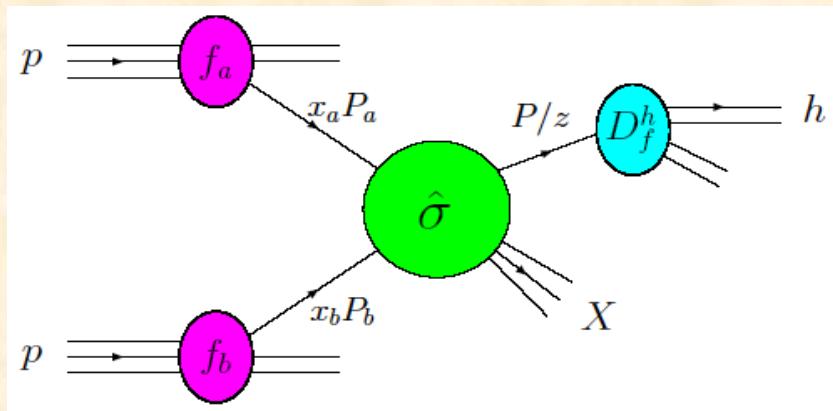


S. Kumano, Nuclear Physics (in Japanese),
KEK Physics Series, Volume 2,
Kyoritsu Shuppan (2015)

Parton distribution functions in the nucleon

High-energy hadron reactions

PDFs are needed for describing high-energy hadron reactions in order to find any new phenomena.



$$\sigma = \sum_{a,b,c} f_a(x_a, Q^2) \otimes f_b(x_b, Q^2) \otimes \hat{\sigma}(ab \rightarrow cX) \otimes D_c^h(z, Q^2)$$

$f_a(x_a, Q^2)$: parton distribution functions

$\hat{\sigma}(ab \rightarrow cX)$: partonic cross sections

$D_c^h(z, Q^2)$: fragmentation functions

Recent works on unpolarized PDFs

ABKM (Alekhin, Blümlein, Klein, Moch)

ABKM-2010, 2011, S. Alekhin *et al.*, Phys. Rev. D 81 (2010) 014032; Phys. Rev. D86 (2012) 054009;

ABM-2014, S. Alekhin *et al.*, Phys. Rev. D89 (2014) 054028 [D91 (2015) 094002, D94 (2016) 114038]

ABMP-2016: S. Alekhin *et al.*, arXiv:1609.03327.

CTEQ (Coordinated Theoretical-Experimental Project on QCD)

CTEQ6.6, P. M. Nadolsky *et al.*, Phys. Rev. D 78 (2008) 013004.

CT10, H.-L. Lau *et al.*, Phys. Rev. D 82 (2010) 074024.

CT12, J. F. Owens *et al.*, Phys. Rev. D 87 (2013) 094012. CT14, S. Dulat *et al.*, Phys. Rev. D 93 (2016) 033006.

GJR (Glück, Jimenez-Delgado, Reya)

GJR-2008, M. Gluck *et al.*, Eur. Phys. J. C 53 (2008) 355; PRD79 (2009) 074023;

JR-2014, Phys. Rev. D89 (2014) 074049.

HERA (H1 and ZEUS collaborations)

HERAPDF, F. D. Aaron *et al.*, JHEP 01 (2010) 109; Eur. Phys. J. C73 (2013) 2311;

H Abramowicz *et al.*, Phys. Rev. D 93 (2016) 092002; I. Abt *et al.*, Phys. Rev. D 94 (2016) 052007.

MSTW (Martin, Stirling, Thorne, Watt, L. A. Harland-Lang, P. Motylinski)

MSTW2008, A. D. Martin *et al.*, Eur. Phys. J. C 63 (2009) 189;

MMHT2014, A. Harland-Lang *et al.*, Eur. Phys. J. C (2015) 75.

Neural Network (Ball, Bertone, Carrazza, Del Debbio, Forte, Guffanti, Hartland, Latorre, Rojo, Ubiali, ...)

NNPDF, R. D. Ball *et al.*, Nucl. Phys. B 838 (2010) 136; B855 (2012) 153;

B867 (2013) 244; B874 (2013) 36; B877 (2013) 290;

JHEP 04 (2015) 040; JHEP 1509 (2015) 191; arXiv:1605.06515.

Parton distribution functions are determined by fitting various experimental data.

- electron/muon: $\mu + p \rightarrow \mu + X$
- neutrino: $\nu_\mu + p \rightarrow \mu + X$
- Drell-Yan: $p + p \rightarrow \mu^+ \mu^- + X$
- ...

(1) assume functional form of PDFs at fixed $Q^2 (\equiv Q_0^2)$:

$$\text{e.g. } f_i(x, Q_0^2) = A_i x^{\alpha_i} (1-x)^{\beta_i} (1+\gamma_i x),$$

where $i = u_\nu, d_\nu, \bar{u}, \bar{d}, \bar{s}, g$

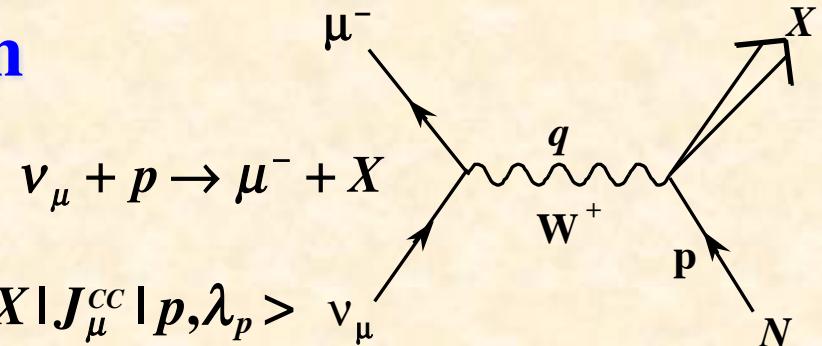
(2) calculate observables at their experimental Q^2 points.

(3) then, the parameters $A_i, \alpha_i, \beta_i, \gamma_i$ are determined so as to minimize χ^2 in comparison with data.

Determination of each distribution

Valence quark

$$q_v(x) \equiv q(x) - \bar{q}(x)$$



$$M = \frac{1}{1+Q^2/M_W^2} \frac{G_F}{\sqrt{2}} \bar{u}(k', \lambda') \gamma^\mu (1 - \underline{\gamma_5}) u(k, \lambda) \langle X | J_\mu^{CC} | p, \lambda_p \rangle$$

$$\frac{d\sigma}{dE' d\Omega} = \frac{G_F^2}{(1+Q^2/M_W^2)^2} \frac{k'}{32\pi^2 E} L^{\mu\nu} W_{\mu\nu}$$

$$L^{\mu\nu} = 8 \left[k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k \cdot k' + \underline{i \epsilon^{\mu\nu\rho\sigma} k_\rho k'_\sigma} \right] \quad \text{where } \epsilon_{0123} = +1$$

$$W_{\mu\nu} = -W_1 \left(g_{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) + W_2 \frac{1}{M_N^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) + \boxed{\frac{i}{2M_N^2} W_3 \epsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma}$$

$$MW_1 = F_1, \quad \nu W_2 = F_2, \quad \nu W_3 = F_3, \quad x = \frac{Q^2}{2p \cdot q}, \quad y = \frac{p \cdot q}{p \cdot k}$$

$$\frac{d\sigma_{\nu, \bar{\nu}}^{CC}}{dx dy} = \frac{G_F^2 (s - M^2)}{2\pi (1+Q^2/M_W^2)^2} \left[x y^2 F_1^{CC} + \left(1 - y - \frac{M x y}{2E} \right) F_2^{CC} \pm x y \left(1 - \frac{y}{2} \right) F_3^{CC} \right]$$

$$\frac{1}{2} [F_3^{\nu p} + F_3^{\bar{\nu} p}]_{CC} = \underline{u_v + d_v} + s - \bar{s} + c - \bar{c}$$

Valence: also F_2 at large x

Note: Nuclear corrections
in CCFR/NuTeV (ν +Fe).

Sea quark

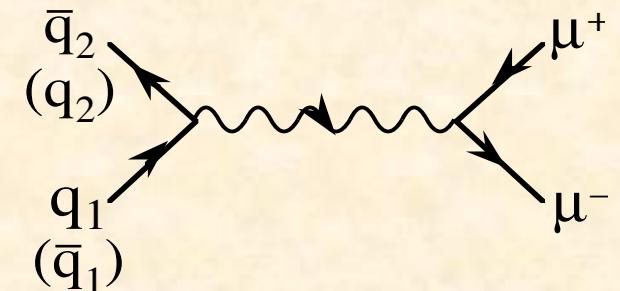
e/ μ scattering

$$\begin{aligned}
 F_2^N &= \frac{F_2^p + F_2^n}{2} = \frac{x}{2} \left[\frac{4}{9}(u + \bar{u}) + \frac{1}{9}(d + \bar{d} + s + \bar{s}) + \frac{4}{9}(d + \bar{d}) + \frac{1}{9}(u + \bar{u} + s + \bar{s}) \right] = \frac{x}{2} \left[\frac{5}{9}(u + \bar{u} + d + \bar{d}) + \frac{2}{9}(s + \bar{s}) \right] \\
 &= \frac{x}{2} \left[\frac{5}{9}(u_v + d_v) + \frac{10}{9}(\bar{u} + \bar{d}) + \frac{2}{9}(s + \bar{s}) \right] = \frac{5}{18}x(u_v + d_v) + \frac{2}{18}x \underline{\left[5(\bar{u} + \bar{d}) + (s + \bar{s}) \right]}
 \end{aligned}$$

Drell-Yan (lepton-pair production)

$$p_1 + p_2 \rightarrow \mu^+ \mu^- + X$$

$$d\sigma \propto q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2)$$



at large $x_F = x_1 - x_2$

projectile target

$$d\sigma \propto q_v(x_1)\bar{q}(x_2)$$

$\bar{q}(x_2)$ can be obtained if $q_v(x_1)$ is known.

Gluon

scaling violation of F_2

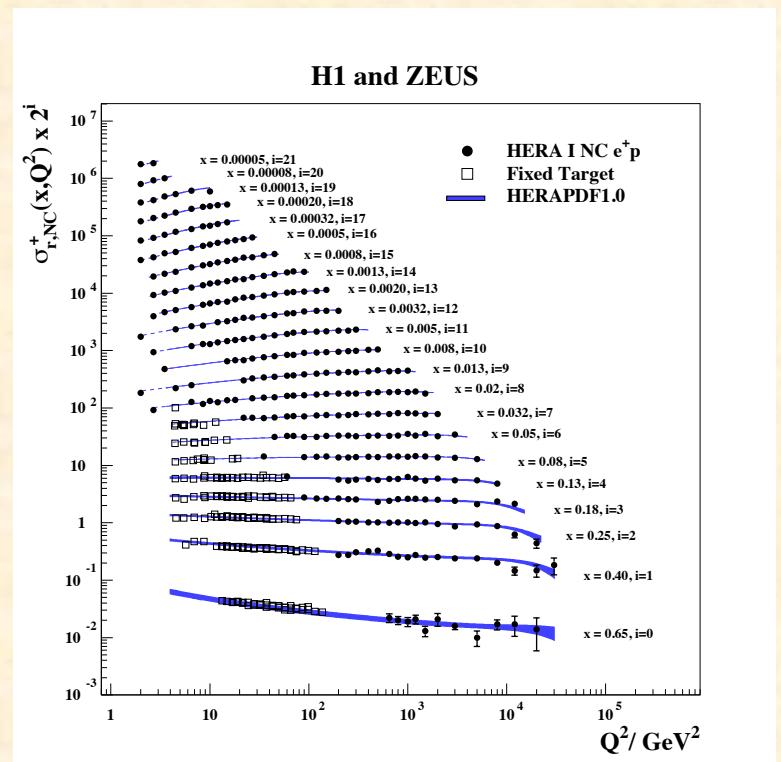
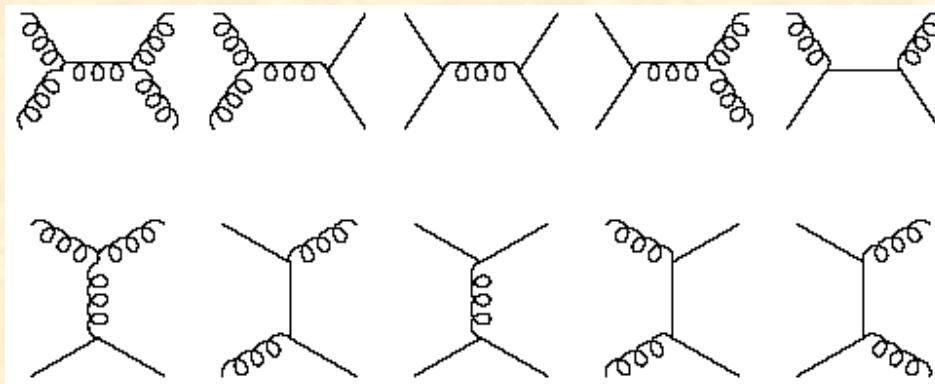
$$\frac{\partial}{\partial(\ln Q^2)} \begin{pmatrix} q_s(x,t) \\ g(x,t) \end{pmatrix} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \begin{pmatrix} P_{qq}(x/y) & P_{qg}(x/y) \\ P_{gq}(x/y) & P_{gg}(x/y) \end{pmatrix} \begin{pmatrix} q_s(y,t) \\ g(y,t) \end{pmatrix}$$

H1 and ZEUS
JHEP01(2010)109

at small x $\frac{\partial F_2}{\partial(\ln Q^2)} \approx \frac{10 \alpha_s}{27\pi} g$

K. Prytz, Phys. Lett. B311 (1993) 286.

jet production



Structure functions in parton model for neutrino-nucleon scattering

$$F_2 = 2 \times F_1$$

$$F_2^{vp} = 2 \times (d + s + \bar{u} + \bar{c})$$

$$F_2^{\bar{v}p} = 2 \times (u + c + \bar{d} + \bar{s})$$

$$F_2^{vn} = 2 \times (u + s + \bar{d} + \bar{c})$$

$$F_2^{\bar{v}n} = 2 \times (d + c + \bar{u} + \bar{s})$$

$$xF_3^{vp} = 2 \times (d + s - \bar{u} - \bar{c})$$

$$xF_3^{\bar{v}p} = 2 \times (u + c - \bar{d} - \bar{s})$$

$$xF_3^{vn} = 2 \times (u + s - \bar{d} - \bar{c})$$

$$xF_3^{\bar{v}n} = 2 \times (d + c - \bar{u} - \bar{s})$$

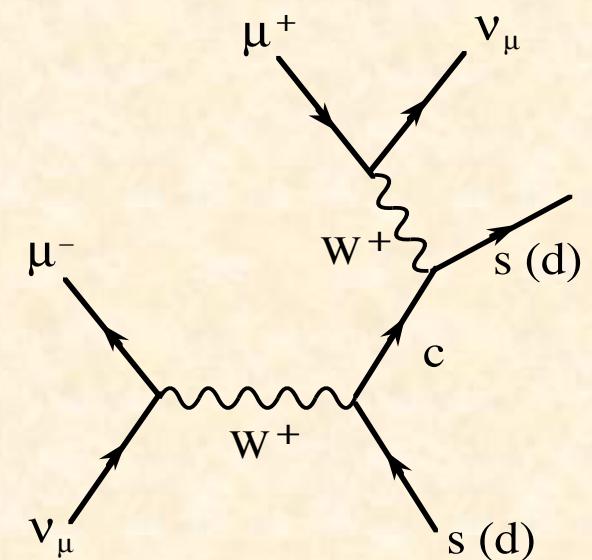
also $\nu p \rightarrow \mu^- \mu^+ X$ for finding $2 \bar{s} / (\bar{u} + \bar{d})$



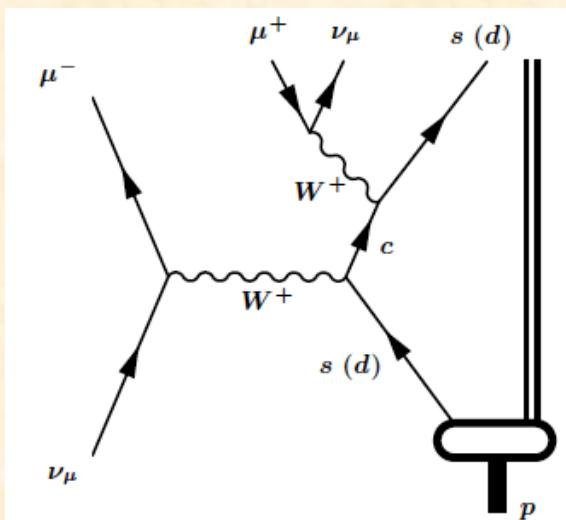
$$F_3^{vp} + F_3^{\bar{v}p} = 2 (u_v + d_v) + 2 (s - \bar{s}) + 2 (c - \bar{c})$$

valence-quark distributions

$$F_3^{v(p+n)/2} - F_3^{\bar{v}(p+n)/2} = 2 (s + \bar{s}) - 2 (c + \bar{c})$$



$s(x)$ from neutrino-induced opposite-sign dimuon events



A. Kayis-Topaksu *et al.*, NPB7 98 (2008) 1.
U. Dore, arXiv: 1103.4572 [hep-ex].

$$\kappa = \frac{\int dx x [s(x, Q^2) + \bar{s}(x, Q^2)]}{\int dx x [\bar{u}(x, Q^2) + \bar{d}(x, Q^2)]}$$

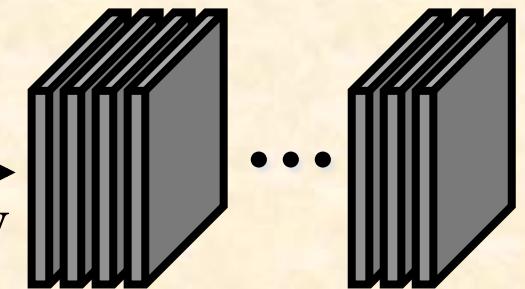
$$Q^2 = 20 \text{ GeV}^2$$

CCFR, NuTeV

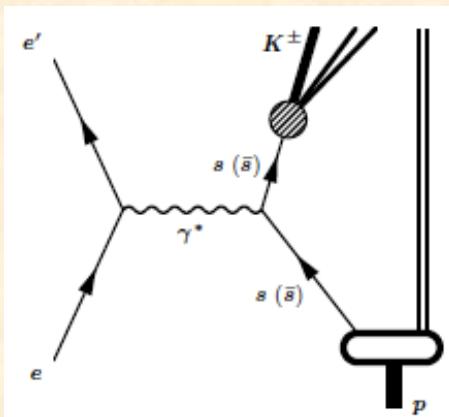
$\nu, \bar{\nu}$

$$E = 30 \sim 500 \text{ GeV}$$

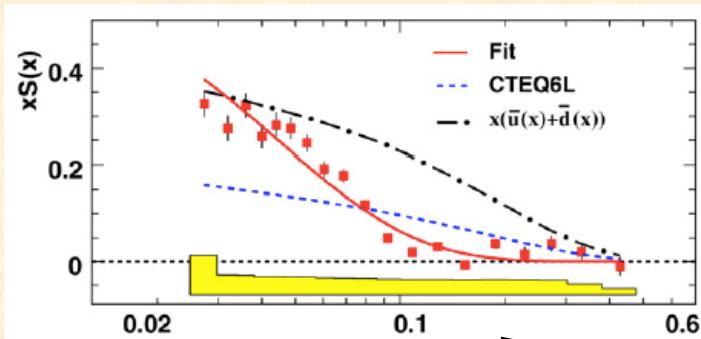
Experiment	κ
This analysis	0.33 ± 0.07
CDHS [1]	0.47 ± 0.09
CCFR [2]	0.44 ± 0.09
CHARM II [3]	0.39 ± 0.09
NOMAD [4]	0.48 ± 0.17
NuTeV [5]	0.38 ± 0.08



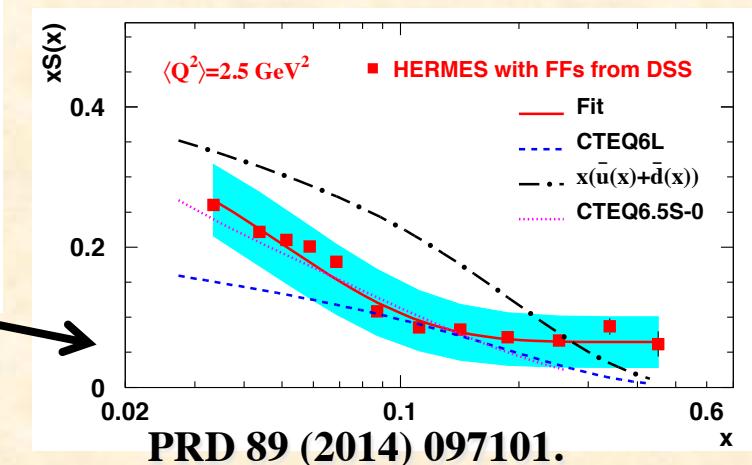
HERMES semi-inclusive measurement



A. Airapetian *et al.*,
PLB 666 (2008) 446.



Huge Fe target (690 ton)
Issue: nuclear corrections

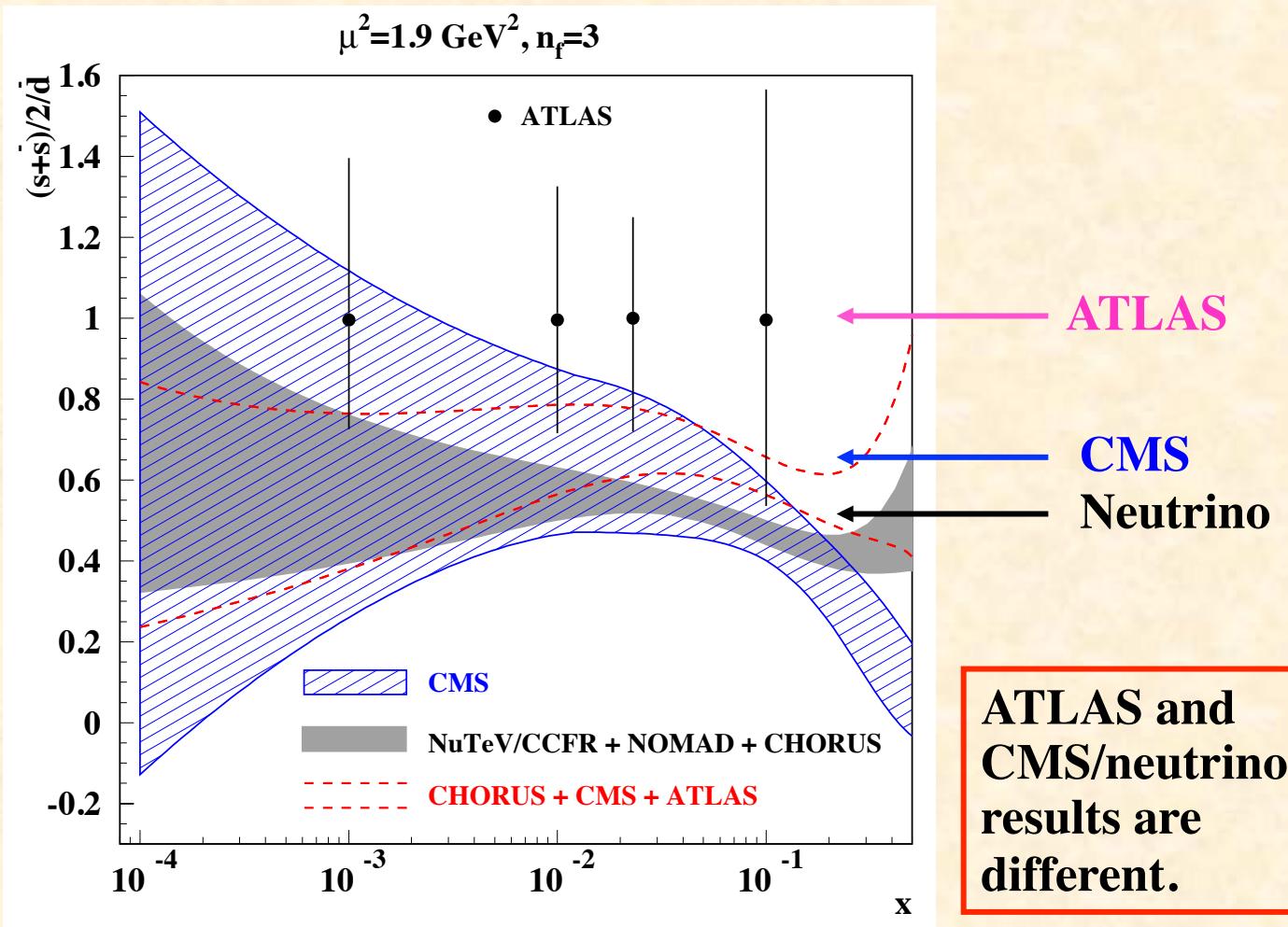


Strange-quark distribution with LHC measurements

S. Alekhin *et al.*,
PRD 91 (2015) 094002.

Neutrino: $s + W \rightarrow c$

LHC: $g + s \rightarrow W + c$



ATLAS and
CMS/neutrino
results are
different.

Situation of data for PDFs of the nucleon (CTEQ14)

ID#	Experimental data set		N_{pt}	χ^2_c	χ^2_c/N_{pt}	S_n
101	BCDMS F_2^p	[24]	337	384	1.14	1.74
102	BCDMS F_2^d	[25]	250	294	1.18	1.89
104	NMC F_2^d/F_2^p	[26]	123	133	1.08	0.68
106	NMC σ_{red}^p	[26]	201	372	1.85	6.89
108	CDHSW F_2^p	[27]	85	72	0.85	-0.99
109	CDHSW F_3^p	[27]	96	80	0.83	-1.18
110	CCFR F_2^p	[28]	69	70	1.02	0.15
111	CCFR xF_3^p	[29]	86	31	0.36	-5.73
124	NuTeV $\nu\mu\mu$ SIDIS	[30]	38	24	0.62	-1.83
125	NuTeV $\nu\mu\mu$ SIDIS	[30]	33	39	1.18	0.78
126	CCFR $\nu\mu\mu$ SIDIS	[31]	40	29	0.72	-1.32
127	CCFR $\nu\mu\mu$ SIDIS	[31]	38	20	0.53	-2.46
145	H1 σ_r^b	[32]	10	6.8	0.68	-0.67
147	Combined HERA charm production	[33]	47	59	1.26	1.22
159	HERA1 Combined NC and CC DIS	[34]	579	591	1.02	0.37
169	H1 F_L	[35]	9	17	1.92	1.7

ID#	Experimental data set		N_{pt}	χ^2_c	χ^2_c/N_{pt}	S_n
201	E605 Drell-Yan process	[37]	119	116	0.98	-0.15
203	E866 Drell-Yan process, $\sigma_{pd}/(2\sigma_{pp})$	[38]	15	13	0.87	-0.25
204	E866 Drell-Yan process, $Q^3 d^2 \sigma_{pp}/(dQ dx_P)$	[39]	184	252	1.37	3.19
225	CDF Run-1 electron A_{ch} , $p_{T\ell} > 25$ GeV	[40]	11	8.9	0.81	-0.32
227	CDF Run-2 electron A_{ch} , $p_{T\ell} > 25$ GeV	[41]	11	14	1.24	0.67
234	DØ Run-2 muon A_{ch} , $p_{T\ell} > 20$ GeV	[42]	9	8.3	0.92	-0.02
240	LHCb 7 TeV 35 pb ⁻¹ W/Z dx/dy_ℓ	[43]	14	9.9	0.71	-0.73
241	LHCb 7 TeV 35 pb ⁻¹ A_{ch} , $p_{T\ell} > 20$ GeV	[43]	5	5.3	1.06	0.30
260	DØ Run-2 Z rapidity	[44]	28	17	0.59	-1.71
261	CDF Run-2 Z rapidity	[45]	29	48	1.64	2.13
266	CMS 7 TeV 4.7 fb ⁻¹ , muon A_{ch} , $p_{T\ell} > 35$ GeV	[46]	11	12.1	1.10	0.37
267	CMS 7 TeV 840 pb ⁻¹ , electron A_{ch} , $p_{T\ell} > 35$ GeV	[47]	11	10.1	0.92	-0.06
268	ATLAS 7 TeV 35 pb ⁻¹ W/Z cross sec., A_{ch}	[48]	41	51	1.25	1.11
281	DØ Run-2 9.7 fb ⁻¹ electron A_{ch} , $p_{T\ell} > 25$ GeV	[45]	13	35	2.67	3.11
504	CDF Run-2 inclusive jet production	[49]	72	105	1.45	2.45
514	DØ Run-2 inclusive jet production	[50]	110	120	1.09	0.67
535	ATLAS 7 TeV 35 pb ⁻¹ incl. jet production	[51]	90	50	0.55	-3.59
538	CMS 7 TeV 5 fb ⁻¹ incl. jet production	[52]	133	177	1.33	2.51

CERN-BCDMS, NMC

CDHSW, CCFR, NuTeV
(ν deep inelastic)

HERA (NC, CC, charm)

Fermilab-E866 (Drell-Yan)

Tevatron (jets, W, Z)

LHC (jets, W, Z, Drell-Yan)

Recent progress

CTEQ14

Functional form: $Q_0^2 = (1.3)^2 \text{ GeV}^2$

$$xf(x, Q_0^2) = x^{a_1}(1-x)^{a_2} P(x)$$

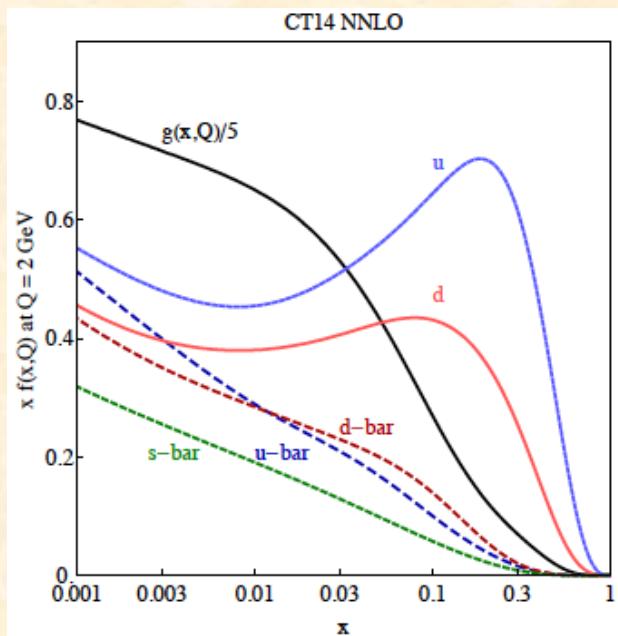
used to be $P(x) = \exp(a_0 + a_3\sqrt{x} + a_4x + a_5x^2)$

Expansion by Bernstein polynomials:

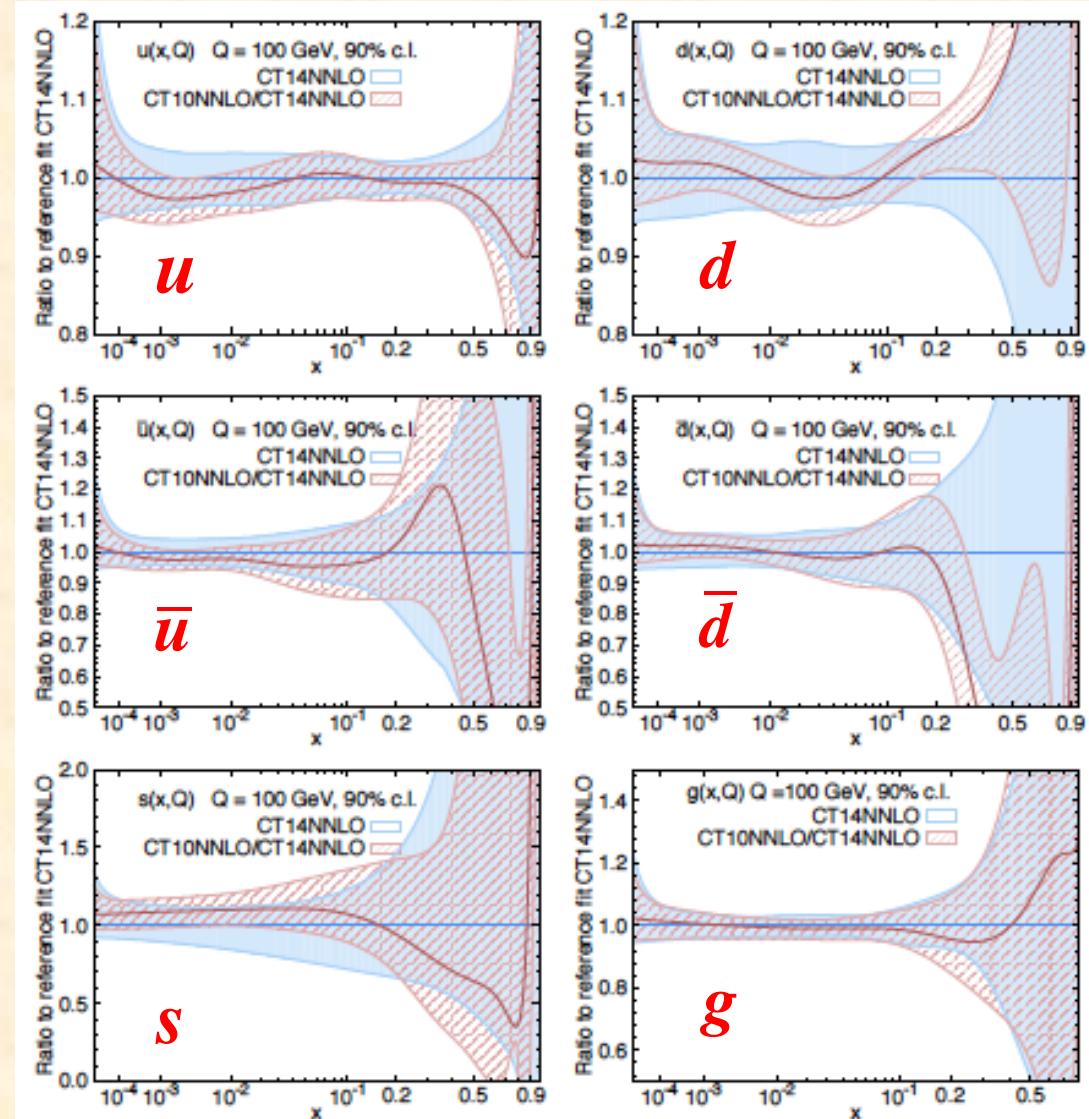
$$P(x) = \sum_{i=0}^n d_i p_i(y = \sqrt{x}), \quad p_0(y) = (1-y)^4,$$

$$p_1(y) = 4y(1-y)^3, \quad p_2(y) = 6y^2(1-y)^2,$$

$$p_3(y) = 4y^3(1-y), \quad p_4(y) = y^4$$

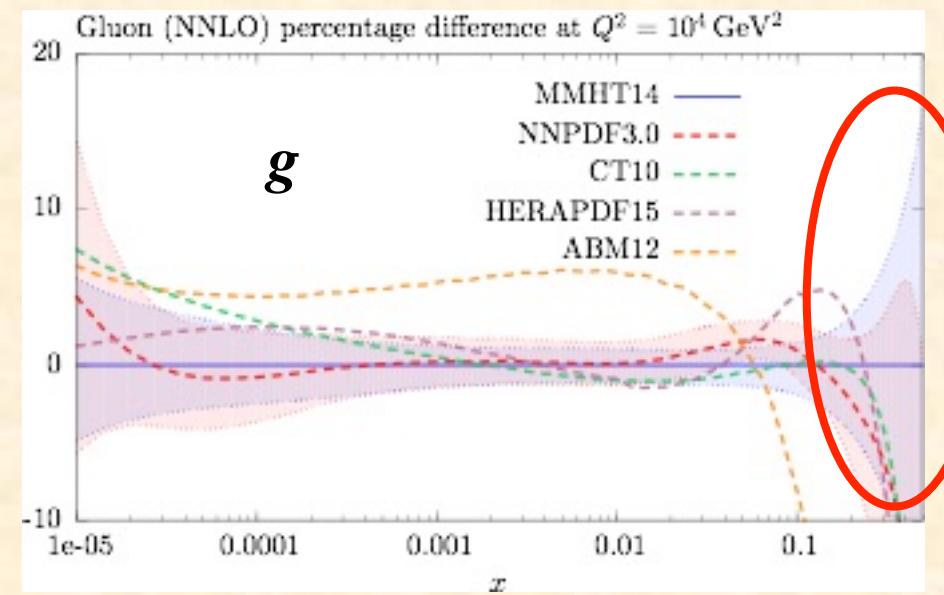
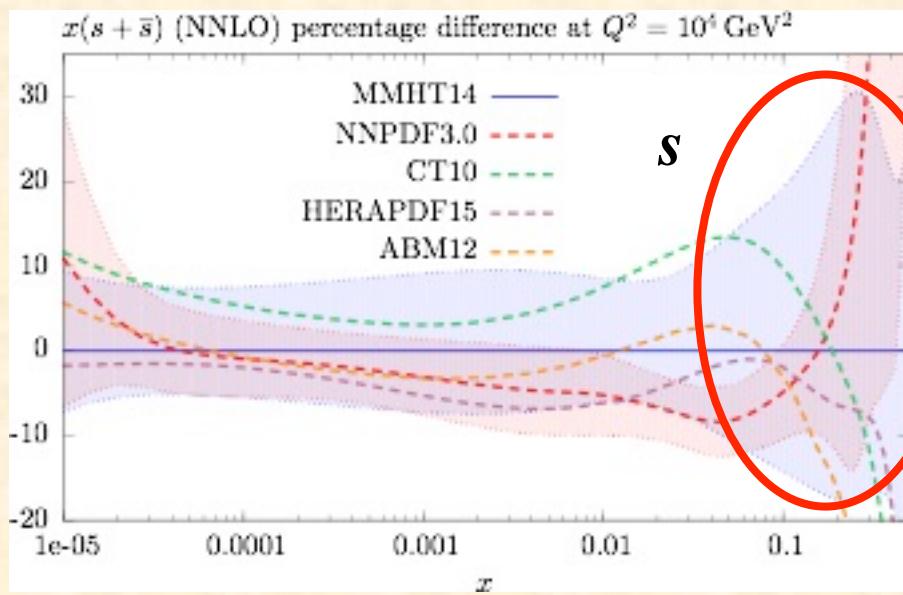
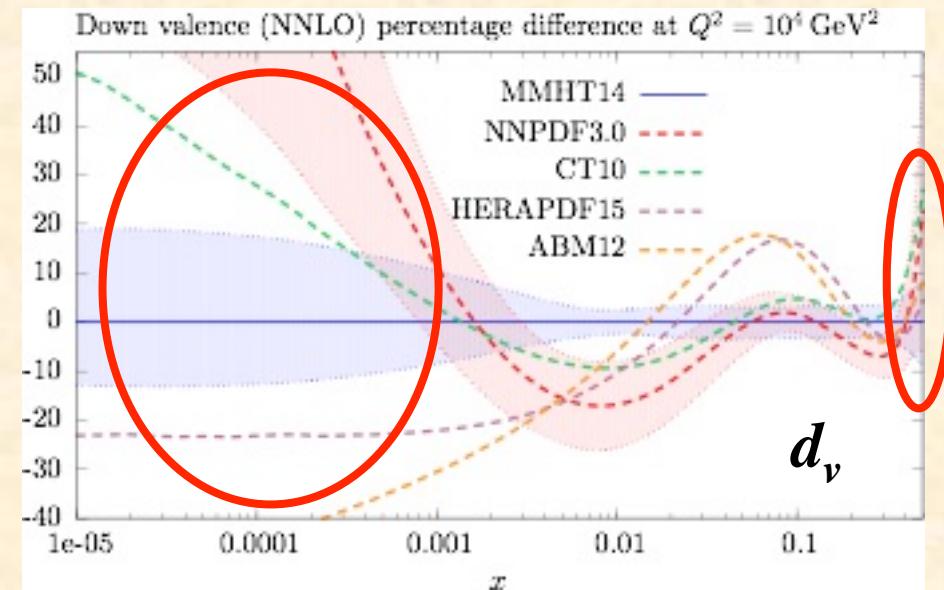
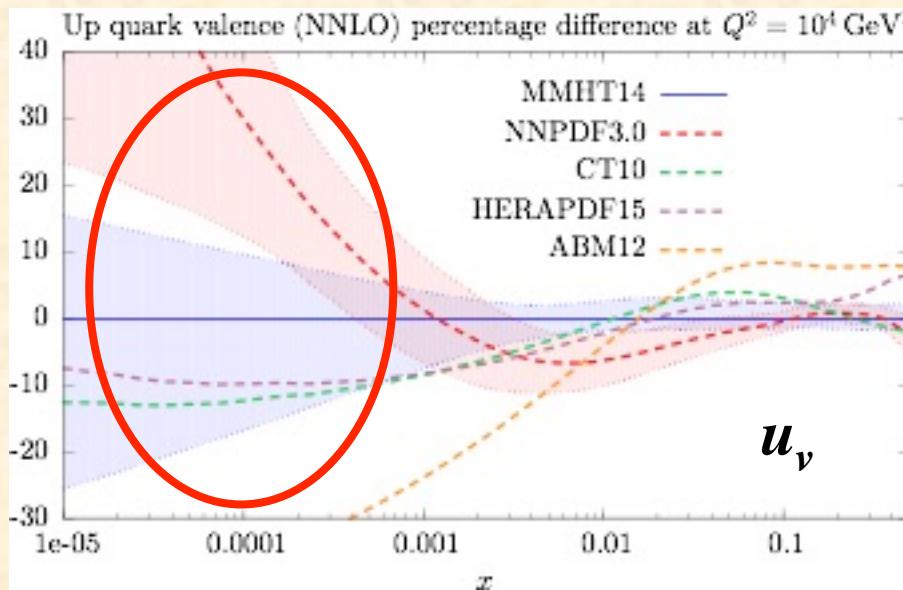


CT14, S. Dulat *et al.*, PRD93 (2016) 033006



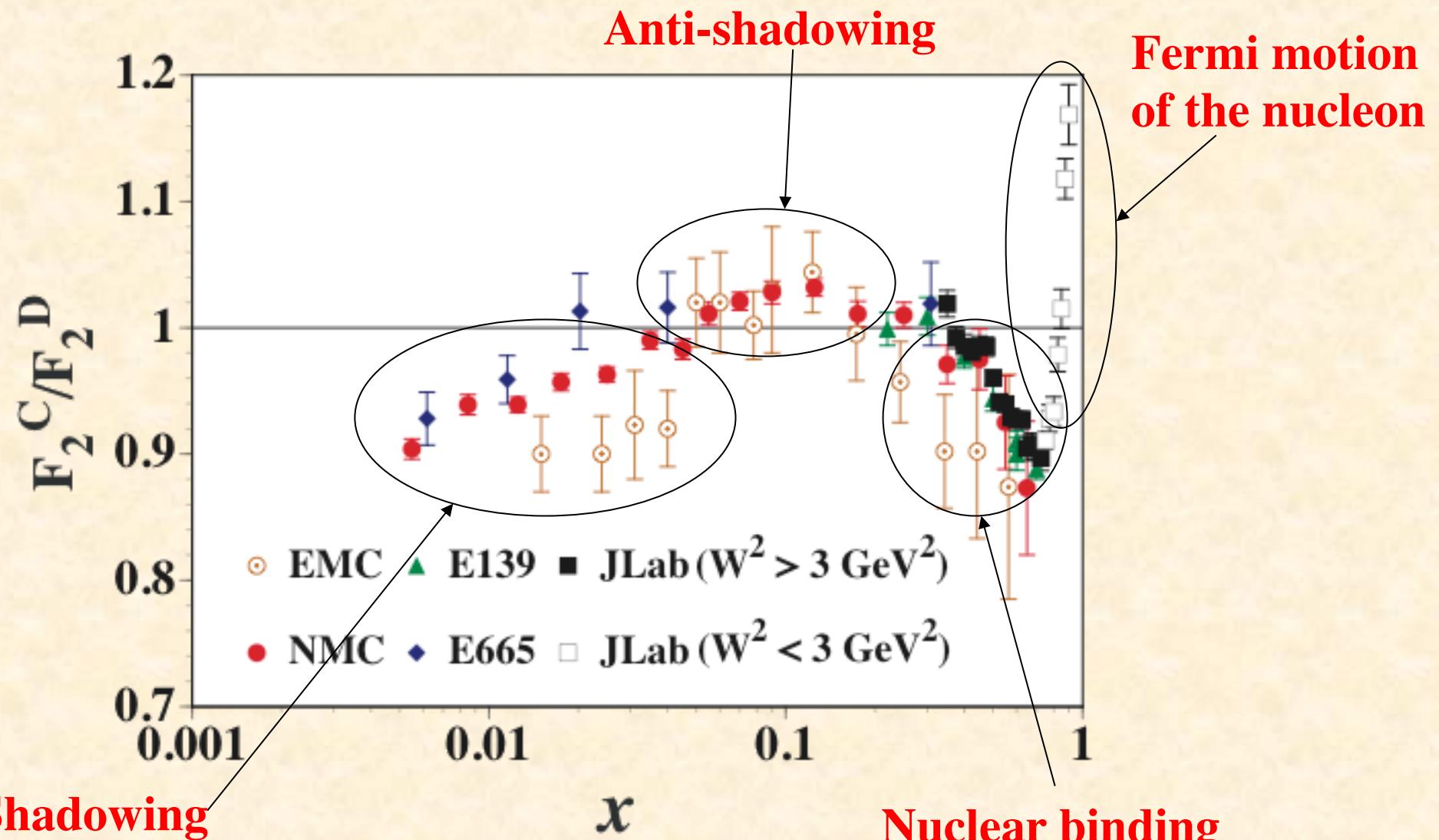
Comparisons with various PDFs

L. A. Harland-Lang, A. D. Martin,
P. Motylinkski, and R. S. Thorne,
Eur. Phys. J. C 75 (2015) 204.



Structure functions of nuclei

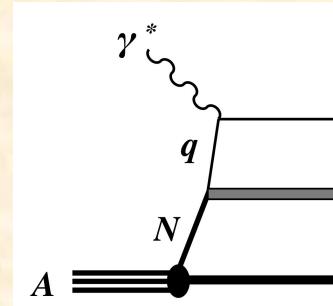
Nuclear modifications of structure function F_2



D. F. Geesaman, K. Saito, A. W. Thomas,
Ann. Rev. Nucl. Part. Sci. 45 (1995) 337

Binding and Fermi motion

Convolution: $W_{\mu\nu}^A(p_A, q) = \int d^4 p S(p) W_{\mu\nu}^N(p_N, q)$



$S(p)$ = Spectral function = nucleon momentum distribution in a nucleus

In a simple shell model: $S(p) = \sum_i |\phi_i(\vec{p})|^2 \delta(p_0 - M_N - \varepsilon_i)$

Separation energy: ε_i

$$\hat{P}_2^{\mu\nu} = -\frac{M_N^2 v}{2\tilde{p}^2} \left(g^{\mu\nu} - \frac{3\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^2} \right)$$

$$\hat{P}_2^{\mu\nu} W_{\mu\nu} = F_2$$

Projecting out F_2 : $F_2^A(x, Q^2) = \sum_i \int dz f_i(z) F_2^N(x/z, Q^2)$

$$z = \frac{p \cdot q}{M_N v} \simeq \frac{p \cdot q}{p_A \cdot q / A} \simeq \frac{p^+}{p_A^+ / A} \quad \text{lightcone momentum fraction}$$

$$p \cdot q = p^+ q^- + p^- q^+ - \vec{p}_T \cdot \vec{q}_T \simeq p^+ q^-$$

$$f_i(z) = \int d^3 p z \delta\left(z - \frac{p \cdot q}{M_N v}\right) |\phi_i(\vec{p})|^2 \quad \text{lightcone momentum distribution for a nucleon } i$$

$$a^\pm = \frac{a^0 \pm a^3}{\sqrt{2}}$$

$$q = (v, 0, 0, -\sqrt{v^2 + Q^2})$$

$$q^+ = -\frac{M x}{\sqrt{2}}, \quad q^- = \frac{2v + M x}{\sqrt{2}} = \sqrt{2}v \gg M$$

$$F_2^A(x, Q^2) = \sum_i \int dz f_i(z) F_2^N(x/z, Q^2) \quad f_i(z) = \int d^3 p z \delta\left(z - \frac{\vec{p} \cdot \vec{q}}{M_N v}\right) |\phi_i(\vec{p})|^2$$

$$z = \frac{\vec{p} \cdot \vec{q}}{M_N v} = \frac{p^0 v - \vec{p} \cdot \vec{q}}{M_N v} = 1 - \frac{|\varepsilon_i|}{M_N} - \frac{\vec{p} \cdot \vec{q}}{M_N v} \approx 1.00 - 0.02 \pm 0.20 \text{ for a medium-size nucleus}$$

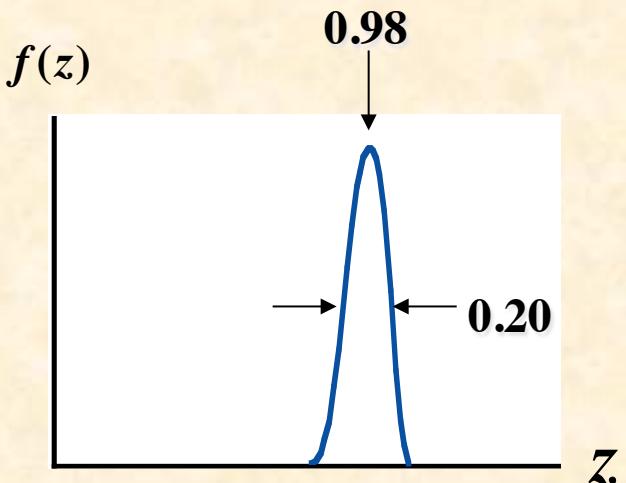
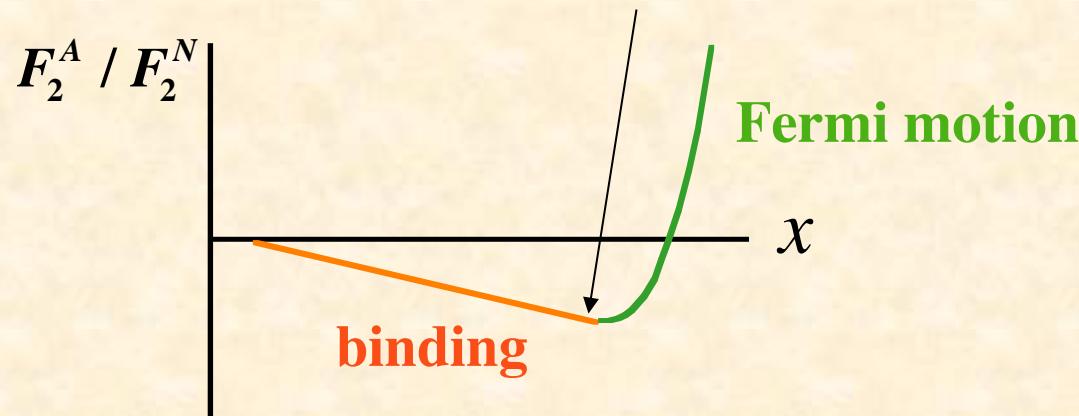
If $f_i(z)$ were $f_i(z) = \delta(z - 1)$, there is no nuclear modification: $F_2^A(x, Q^2) = F_2^N(x, Q^2)$.

Because the peak shifts slightly ($1 \rightarrow 0.98$), nuclear modification of F_2 is created.

$$F_2^A(x, Q^2) \approx F_2^N(x/0.98, Q^2)$$

$$\text{For } x = 0.60, \quad x/0.98 = 0.61$$

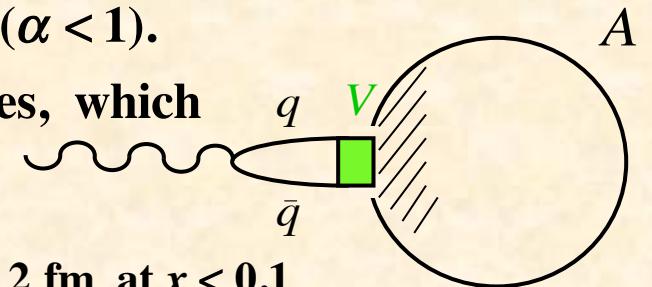
$$\frac{F_2^N(x=0.61)}{F_2^N(x=0.60)} = \frac{0.021}{0.024} = 0.88$$



Shadowing

- Shadowing means that internal constituents are shadowed due to the existence of nuclear surface ones, so that the cross section is smaller than the each nucleon contribution: $\sigma_A = A^\alpha \sigma_N$ ($\alpha < 1$).
- A virtual photon transforms into vector meson (or $q\bar{q}$) states, which then interact with a target nucleus.

Propagation length of V ($q\bar{q}$): $\lambda = \frac{1}{|E_V - E_\gamma|} = \frac{2v}{M_V^2 + Q^2} = \frac{0.2 \text{ fm}}{x} > 2 \text{ fm at } x < 0.1$

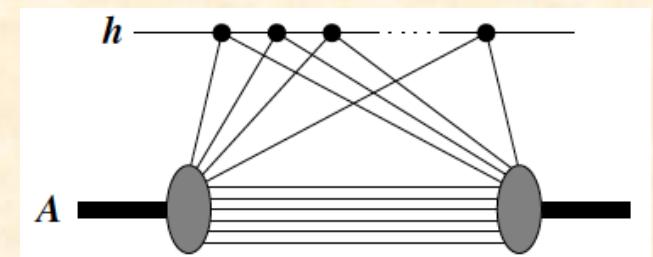


At small x , the virtual photon interacts with the target nucleus as if it were a vector meson (or $q\bar{q}$).

- Shadowing takes place due to multiple scattering.

For example, the vector meson interacts elastically with a surface nucleon and then interacts inelastically with a central nucleon.

Because this amplitude is opposite in phase to the one-step amplitude for an inelastic interaction with the central nucleon, the nucleon sees a reduced hadronic flux (namely the shadowing).



Nuclear Parton Distribution Functions

Experimental data

(1) F_2^A / F_2^D

NMC: p, He, Li, C, Ca

SLAC: He, Be, C, Al,
Ca, Fe, Ag, Au

EMC: C, Ca, Cu, Sn

E665: C, Ca, Xe, Pb

BCDMS: N, Fe

HERMES: N, Kr

+ JLab data

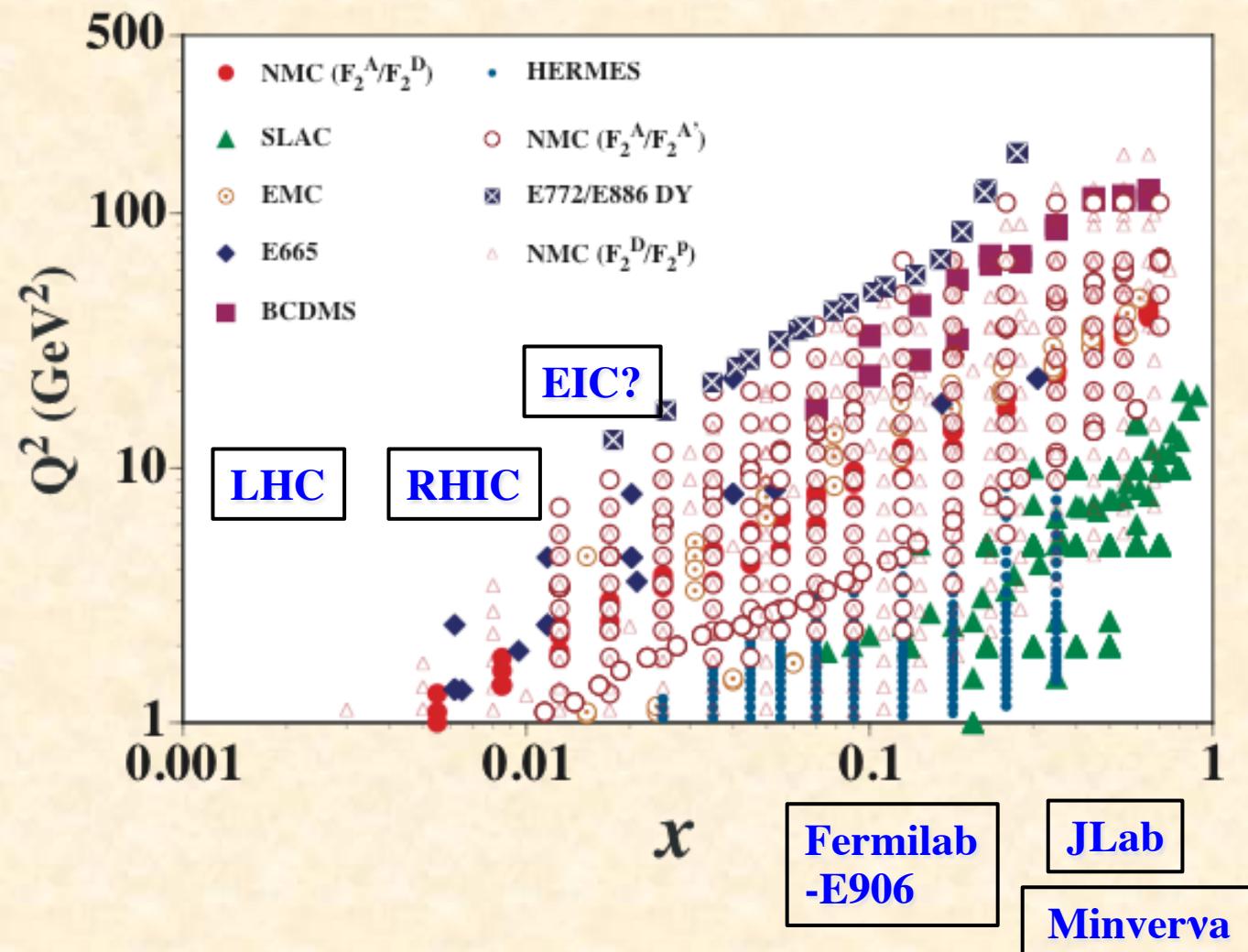
(2) $F_2^A / F_2^{A'}$

NMC: Be / C, Al / C,
Ca / C, Fe / C,
Sn / C, Pb / C,
C / Li, Ca / Li

(3) $\sigma_{DY}^A / \sigma_{DY}^{A'}$

E772: C / D, Ca / D,
Fe / D, W / D

E866: Fe / Be, W / Be

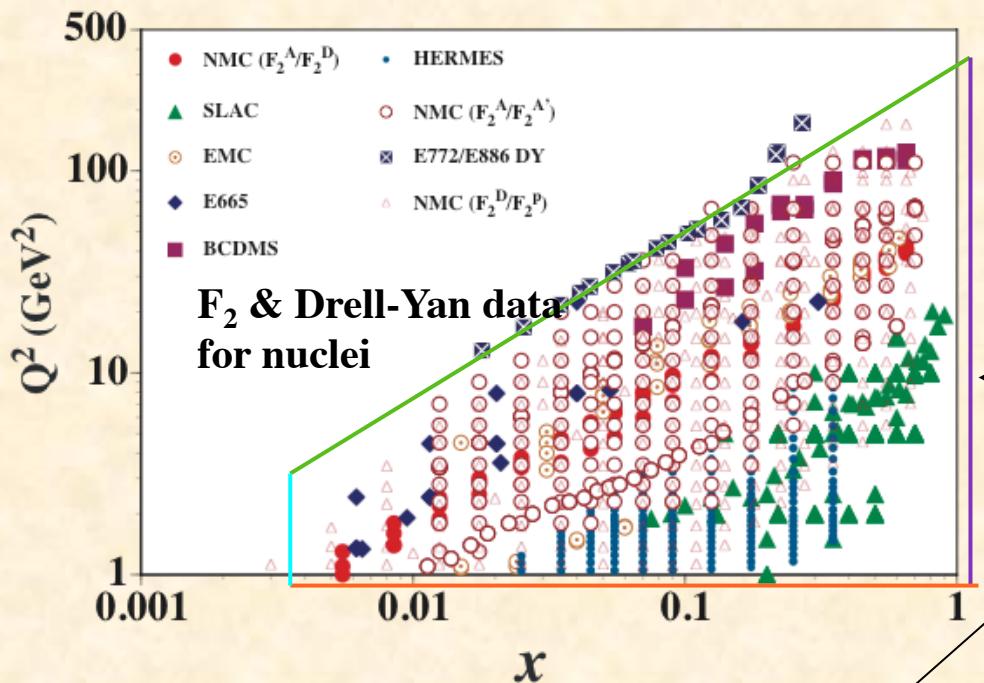


Current nuclear data are kinematically limited.

$$x = \frac{Q^2}{2p \cdot q} \approx \frac{Q^2}{ys}$$

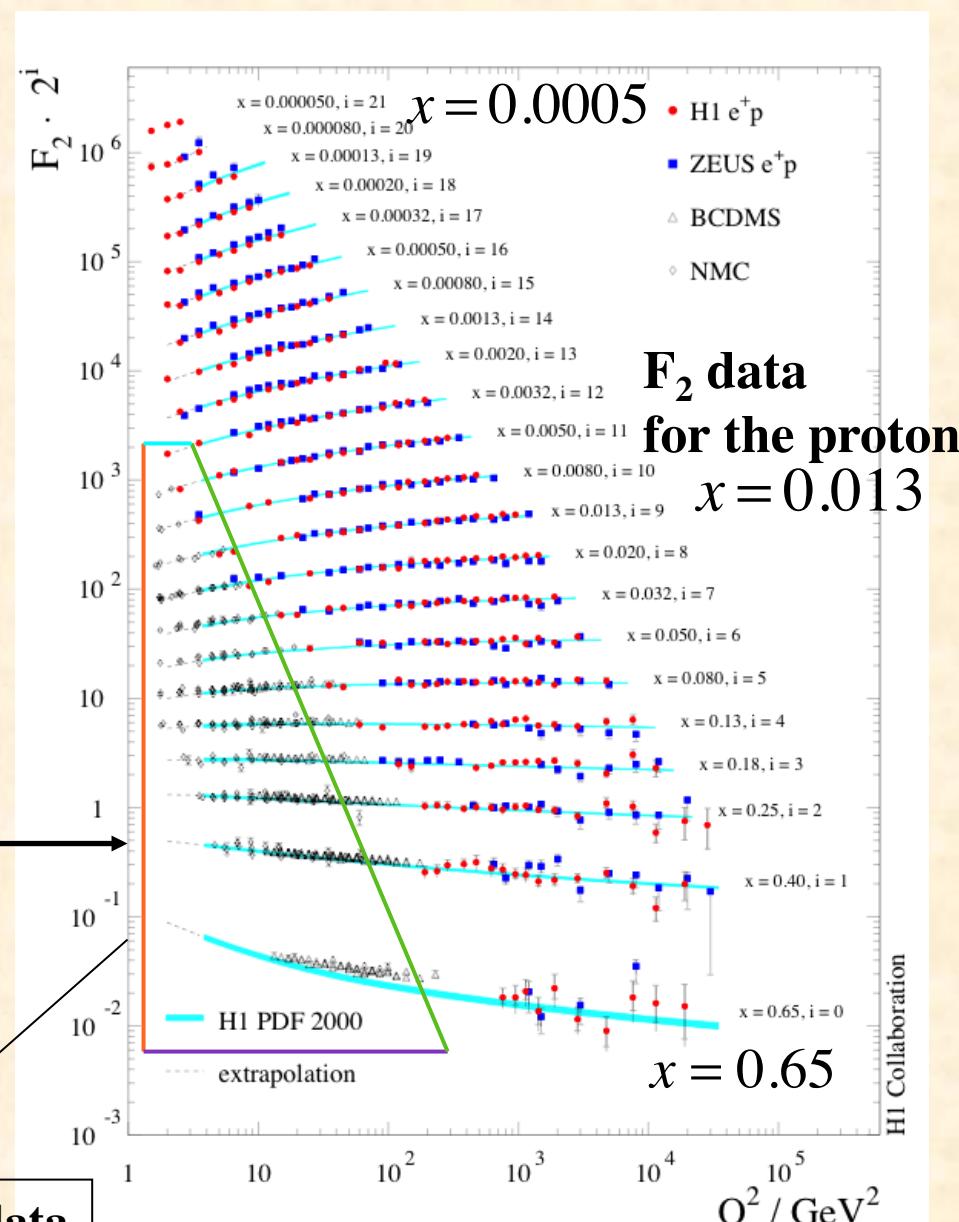
fixed target: $\min(x) = \frac{Q^2}{2M_N E_{lepton}} \leq \frac{1}{2E_{lepton}(\text{GeV})}$
if $Q^2 \geq 1 \text{ GeV}^2$

for $E_{lepton}(\text{NMC}) = 200 \text{ GeV}$, $\min(x) = \frac{1}{2 \cdot 200} = 0.003$



region of nuclear data

(from H1 and ZEUS, hep-ex/0502008)



Functional form Nuclear PDFs “per nucleon”

If there were no nuclear modification

$$Au^A(x) = Zu^p(x) + Nu^n(x), \quad Ad^A(x) = Zd^p(x) + Nd^n(x) \quad p = \text{proton}, \quad n = \text{neutron}$$

Isospin symmetry: $u^n = d^p \equiv d, \quad d^n = u^p \equiv u$

$$\rightarrow u^A(x) = \frac{Zu(x) + Nd(x)}{A}, \quad d^A(x) = \frac{Zd(x) + Nu(x)}{A}$$

Take account of nuclear effects by $w_i(x, A)$

$$u_v^A(x) = w_{u_v}(x, A) \frac{Zu_v(x) + Nd_v(x)}{A}, \quad d_v^A(x) = w_{d_v}(x, A) \frac{Zd_v(x) + Nu_v(x)}{A}$$

$$\bar{u}^A(x) = w_{\bar{q}}(x, A) \frac{Z\bar{u}(x) + N\bar{d}(x)}{A}, \quad \bar{d}^A(x) = w_{\bar{q}}(x, A) \frac{Z\bar{d}(x) + N\bar{u}(x)}{A}$$

$$\bar{s}^A(x) = w_{\bar{q}}(x, A) \bar{s}(x)$$

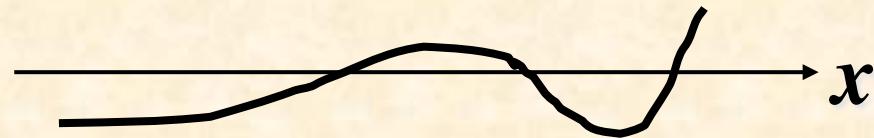
$$g^A(x) = w_g(x, A) g(x) \quad \text{at } Q^2=1 \text{ GeV}^2 (\equiv Q_0^2)$$

Nuclear modifications and constraints

$$f_i^A(x, Q_0^2) = w_i(x, A) f_i(x, Q_0^2) \quad i = u_\nu, d_\nu, \bar{u}, \bar{d}, \bar{s}, g$$

$$w_i(x, A) = 1 + \left(1 - \frac{1}{A^\alpha}\right) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^\beta}$$

Note: The region $x > 1$ cannot be described by this parametrization.



A simple function = cubic polynomial

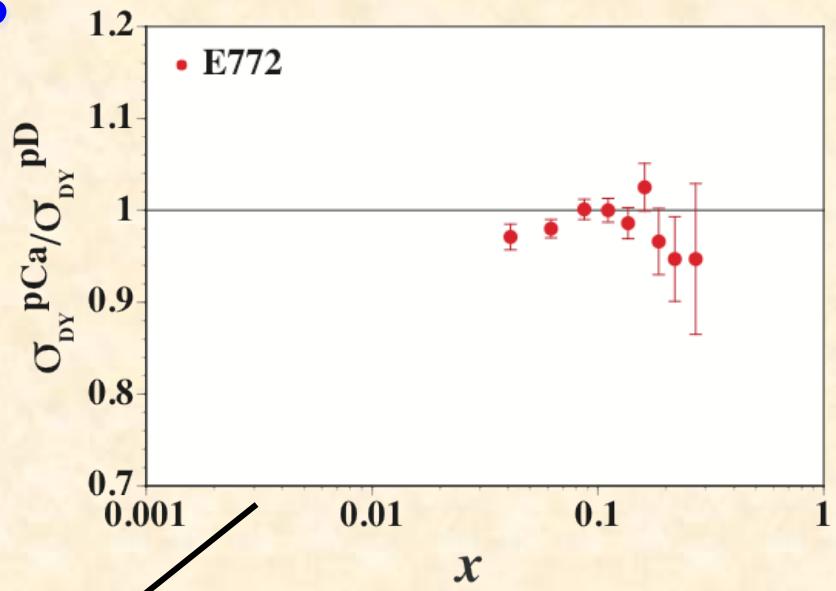
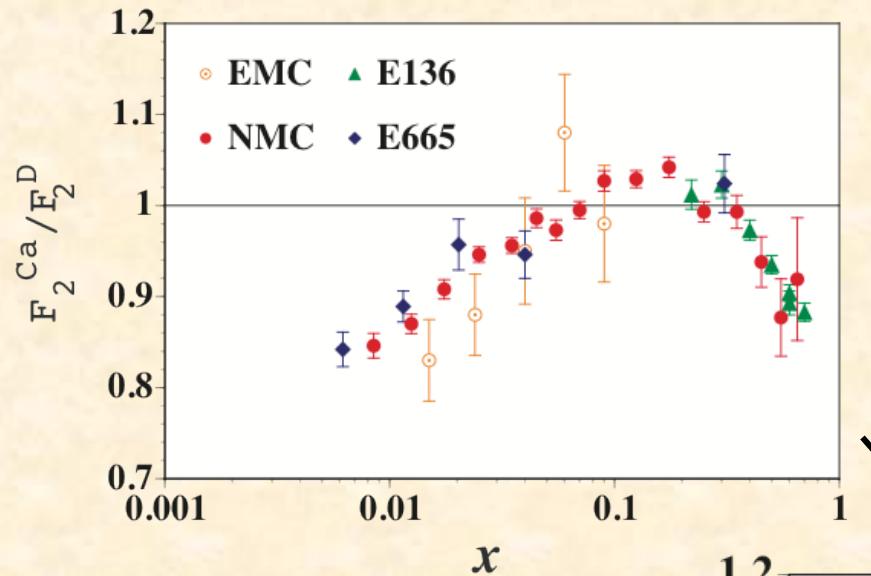
Three constraints

Nuclear charge: $Z = A \int dx \left[\frac{2}{3} (u^A - \bar{u}^A) - \frac{1}{3} (d^A - \bar{d}^A) - \frac{1}{3} (s^A - \bar{s}^A) \right] = A \int dx \left[\frac{2}{3} u_\nu^A - \frac{1}{3} d_\nu^A \right]$

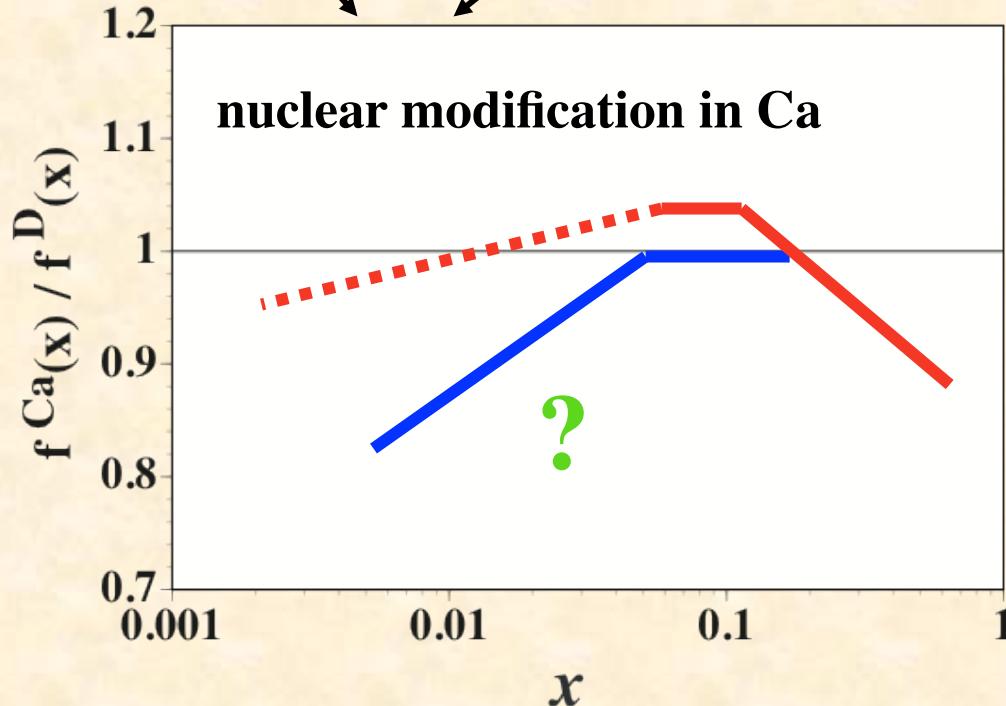
Baryon number: $A = A \int dx \left[\frac{1}{3} (u^A - \bar{u}^A) + \frac{1}{3} (d^A - \bar{d}^A) + \frac{1}{3} (s^A - \bar{s}^A) \right] = A \int dx \left[\frac{1}{3} u_\nu^A + \frac{1}{3} d_\nu^A \right]$

Momentum:
$$\begin{aligned} A &= A \int dx [u^A + \bar{u}^A + d^A + \bar{d}^A + s^A + \bar{s}^A + g] \\ &= A \int dx [u_\nu^A + d_\nu^A + 2(\bar{u}^A + \bar{d}^A + \bar{s}^A) + g] \end{aligned}$$

Nuclear modification of PDFs



- valence quark
- antiquark
- gluon



Global analyses on nuclear PDFs

I may miss some papers.

HKN

- M. Hirai, S. Kumano, and T. -H. Nagai, Phys. Rev. C **76** (2007) 065207.
- Charged-lepton DIS, DY.

EPS

- K. J. Eskola, H. Paukkunen, and C. A. Salgado, JHEP **04** (2009) 065; [arXiv:1612.05741](https://arxiv.org/abs/1612.05741).
- Charged-lepton DIS, DY, π^0 production in dAu, Neutrino

nCTEQ

- I. Schienbein, J. Y. Yu, C. Keppel, J. G. Morfin, F. I. Olness, J. F. Owens, Phys. Rev. D **77** (2008) 054013; D**80** (2009) 094004; K. Kovarik *et al.*, PRL **106** (2011) 122301; PoS DIS2013 (2013) 274; PoS DIS2014 (2014) 047; Phys. Rev. D **93** (2016) 085037.
- Neutrino DIS, Charged-lepton DIS, DY.

DSZS

- D. de Florian, R. Sassot, P. Zurita, M. Stratmann, Phys. Rev. D**85** (2012) 074028.
- Charged-lepton DIS, DY, RHIC- π

See also L. Frankfurt, V. Guzey, and M. Strikman, Phys. Rev. D **71** (2005) 054001; Phys. Lett. B**687** (2010) 167; Phys. Rept. **512** (2012) 255.
S. A. Kulagin and R. Petti, Phys. Rev. D **76** (2007) 094023; C **82** (2010) 054614; C **90** (2014) 045204; D **94** (2016) 113013.
A. Bodek and U.-K. Yang, arXiv:1011.6592.

Functional form of initial distributions at Q_0^2

Initial nuclear PDFs at

$$f_i^A(x) = \frac{1}{A} [Z f_i^{p/A}(x) + (A - Z) f_i^{n/A}(x)] \quad f_i^{N/A}(x): \text{PDF of bound nucleon in the nucleus}$$

Isospin symmetry is assumed: $u \equiv d^n = u^p, d \equiv u^n = d^p$

Functional forms

- HKN07 ($Q_0^2 = 1 \text{ GeV}^2$)

$$f_i^A(x) = w_i(x, A, Z) \frac{1}{A} [Z f_a^p(x) + (A - Z) f_a^n(x)], \quad w_i(x, A, Z) = 1 + \left(1 - \frac{1}{A^{1/3}}\right) \frac{\mathbf{a}_i + \mathbf{b}_i x + \mathbf{c}_i x^2 + \mathbf{d}_i x^3}{(1-x)^{0.1}}$$

- EPS09 ($Q_0^2 = 1.69 \text{ GeV}^2$)

$$f_i^{N/A}(x) \equiv R_i^A(x) f_i^{\text{CTEQ6.1M}}(x, Q_0^2), R_i^A(x) = \begin{cases} \mathbf{a}_0 + (\mathbf{a}_1 + \mathbf{a}_2 x)[\exp(-x) - \exp(-\mathbf{x}_a)] & (x \leq x_a : \text{shadowing}) \\ \mathbf{b}_0 + \mathbf{b}_1 x + \mathbf{b}_2 x^2 + \mathbf{b}_3 x^3 & (x_a \leq x \leq x_e : \text{antishadowing}) \\ \mathbf{c}_0 + (\mathbf{c}_1 - \mathbf{c}_2 x)(1-x)^{-\beta} & (x_e \leq x \leq 1 : \text{EMC\&Fermi}) \end{cases}$$

- CTEQ-08 ($Q_0^2 = 1.69 \text{ GeV}^2$)

$$x f_i^{N/A}(x) = \begin{cases} \mathbf{A}_0 x^{\mathbf{A}_1} (1-x)^{\mathbf{A}_2} e^{\mathbf{A}_3 x} (1+e^{\mathbf{A}_4 x})^{\mathbf{A}_5} & : i = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s} \\ \mathbf{A}_0 x^{\mathbf{A}_1} (1-x)^{\mathbf{A}_2} + (1+\mathbf{A}_3 x)(1-x)^{\mathbf{A}_4} & : i = \bar{d} / \bar{u} \end{cases}$$

- DSZS12 ($Q_0^2 = 1.0 \text{ GeV}^2$)

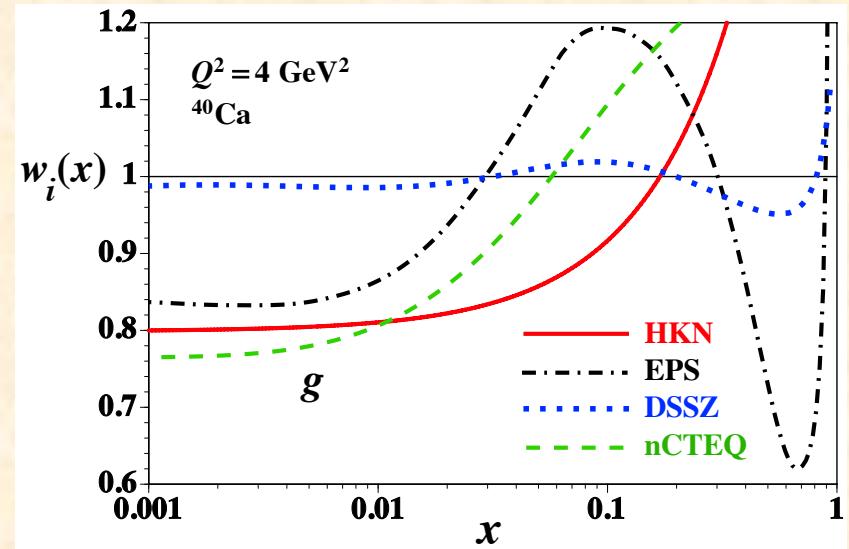
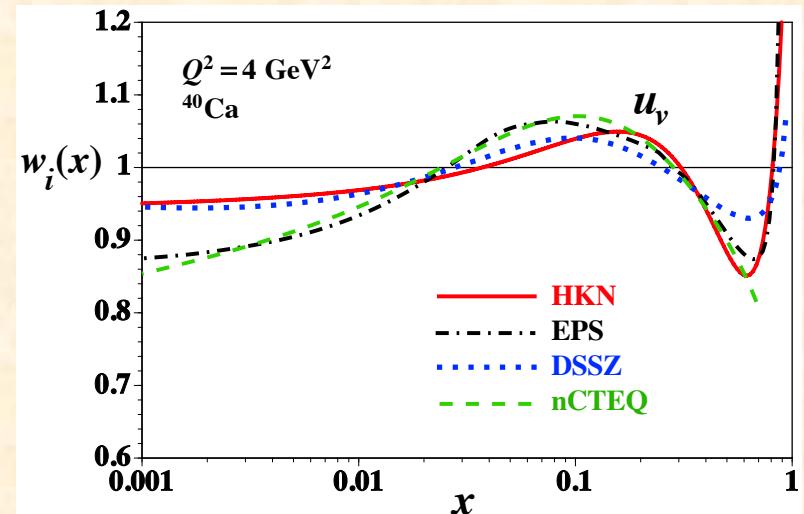
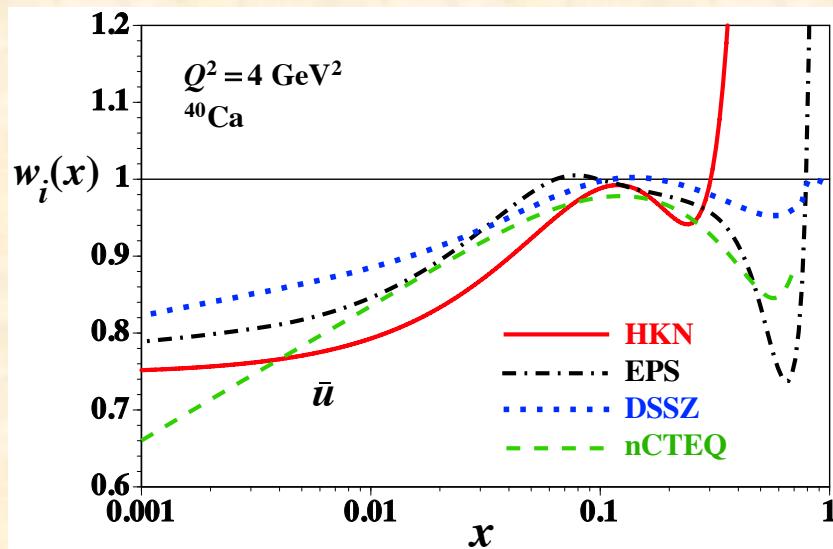
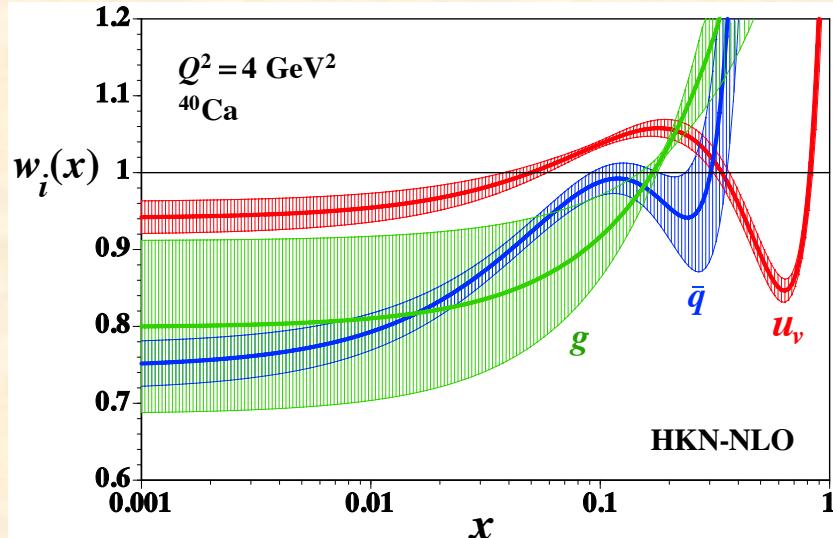
$$f_i^{N/A}(x) \equiv R_i^A(x) f_i^{\text{MSTW2009}}(x, Q_0^2), R_v^A(x) = \mathbf{\epsilon}_1 x^{\alpha_v} (1-x)^{\beta_1} [1 + \mathbf{\epsilon}_2 (1-x)^{\beta_2}] [1 + \mathbf{a}_v (1-x)^{\beta_3}]$$

$$R_s^A(x) = R_v^A(x) \frac{\mathbf{\epsilon}_s}{\mathbf{\epsilon}_1} \frac{1 + \mathbf{a}_s x^{\alpha_s}}{1 + \mathbf{a}_s}, \quad R_g^A(x) = R_g^A(x) \frac{\mathbf{\epsilon}_g}{\mathbf{\epsilon}_1} \frac{1 + \mathbf{a}_g x^{\alpha_g}}{1 + \mathbf{a}_g}$$

Review on neutrino interactions (arXiv: 1610.01464)

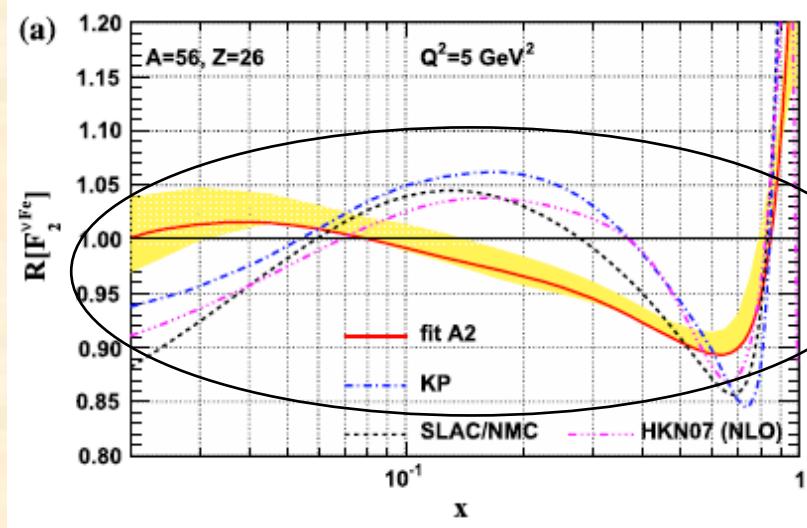
Towards a Unified Model of Neutrino-Nucleus
Reactions for Neutrino Oscillation Experiments

S.X. Nakamura¹, H. Kamano^{2,3}, Y. Hayato⁴, M. Hirai⁵,
W. Horiuchi⁶, S. Kumano^{2,3}, T. Murata¹, K. Saito^{7,3},
M. Sakuda⁸, T. Sato^{1,3}, Y. Suzuki^{9,10}



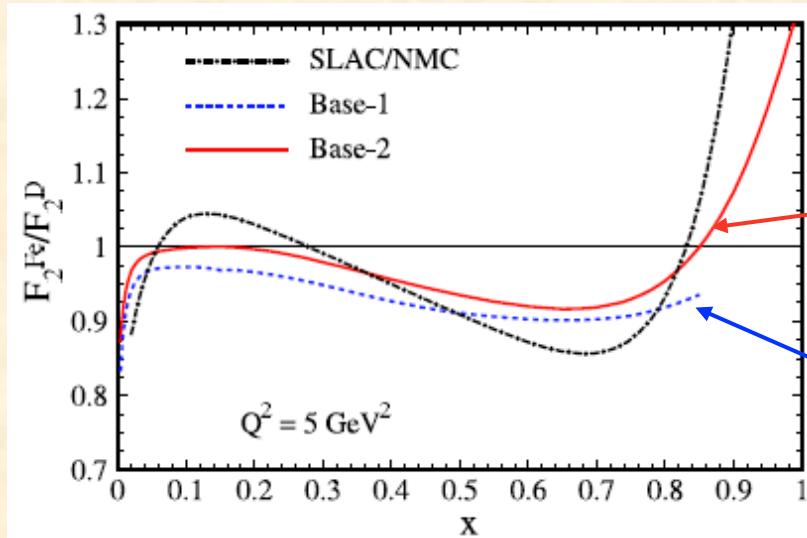
Analysis of CTEQ-2008 (Schienbein *et al.*)

I. Schienbein *et al.*,
PRD 77 (2008) 054013

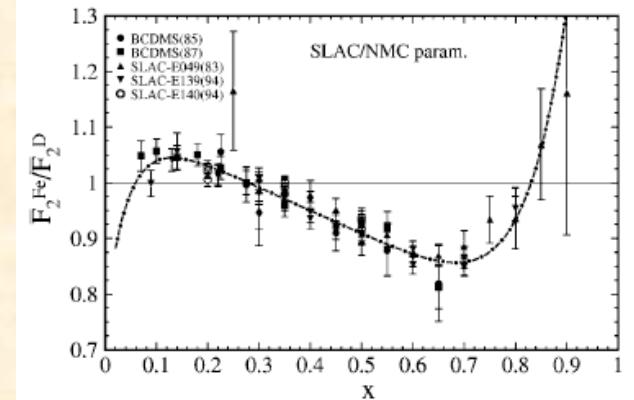


Differences
from typical NPDFs.

Neutrino scattering



Charged-lepton scattering



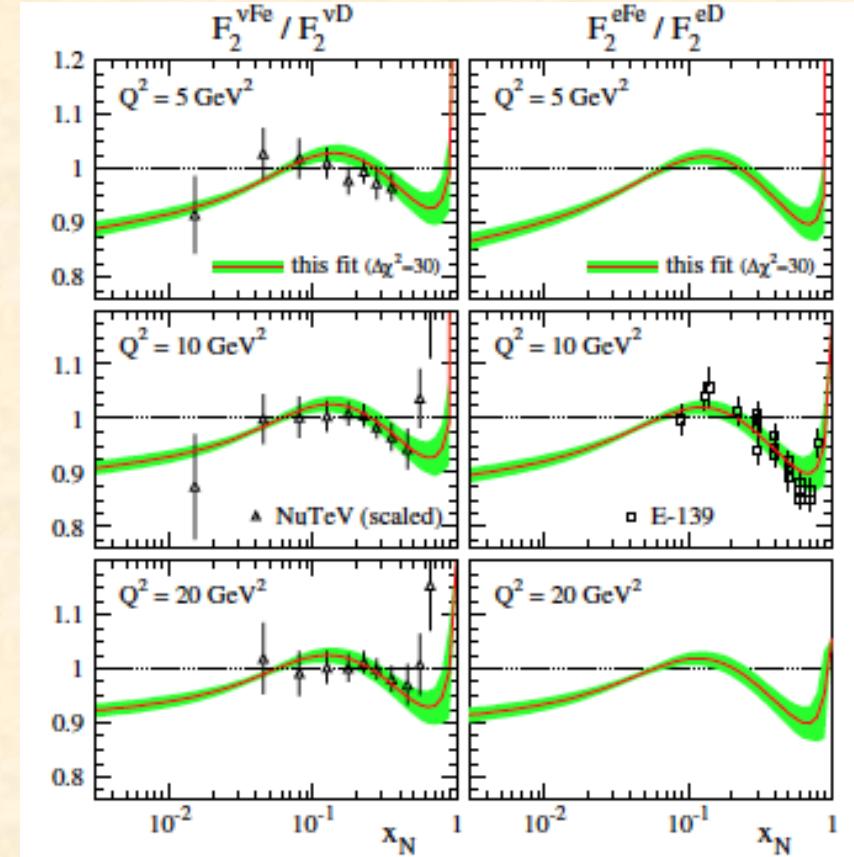
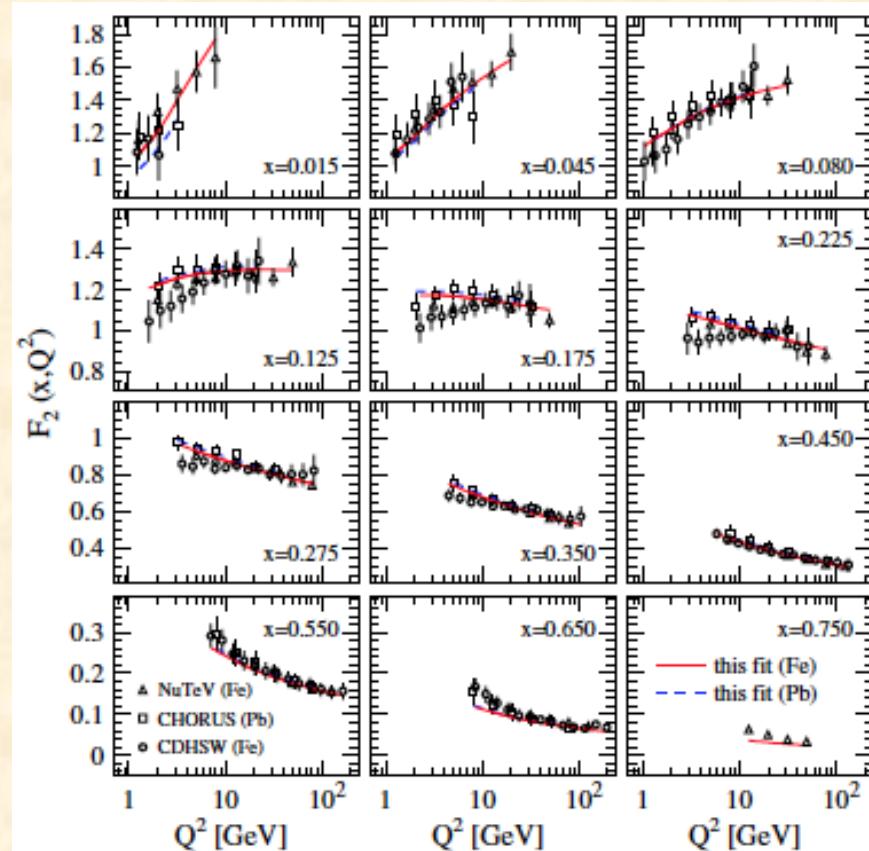
- Base-1**
 - remove CCFR data
 - incorporate deuteron corrections
- Base-2**
 - corresponds to CTEQ6.1M with $s \neq s\bar{b}$
 - include CCFR data
 - Charged-lepton correction factors are applied.
 - $s \neq s\bar{b}$

Base-2: Using current nucleonic PDFs,
they (and MRST) obtained very different
corrections from charged-lepton data.

Base-1: However, it depends on the analysis
method for determining “nucleonic” PDFs.

Neutrino DIS ⇔ Charged DIS issue

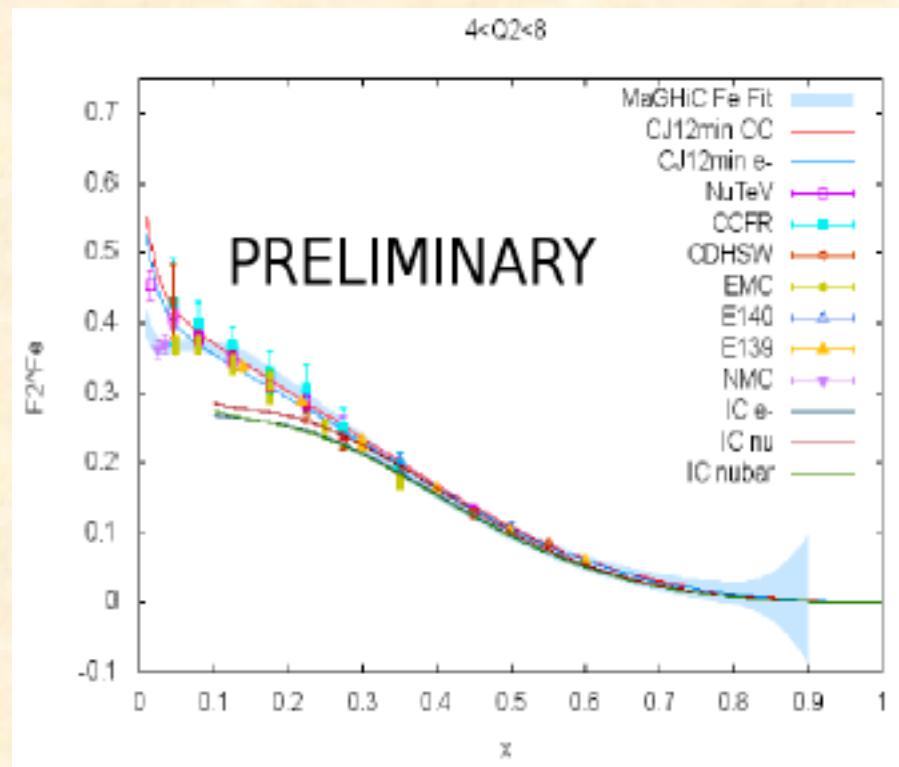
D. de Florian, R. Sassot, P. Zurita, and M. Stratmann,
Phys. Rev. D 85 (2012) 074028.



According to their analysis, the issue does not exist!?

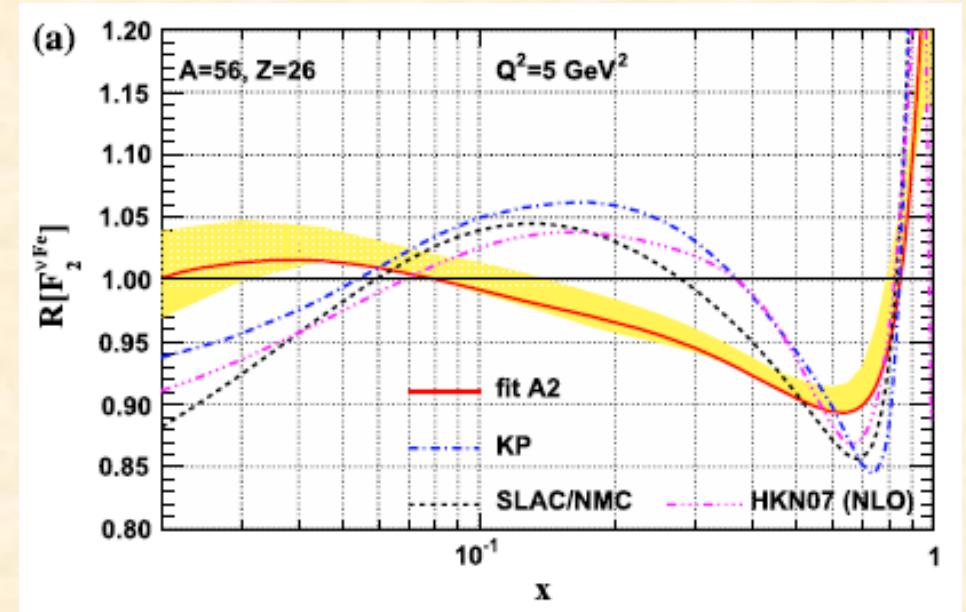
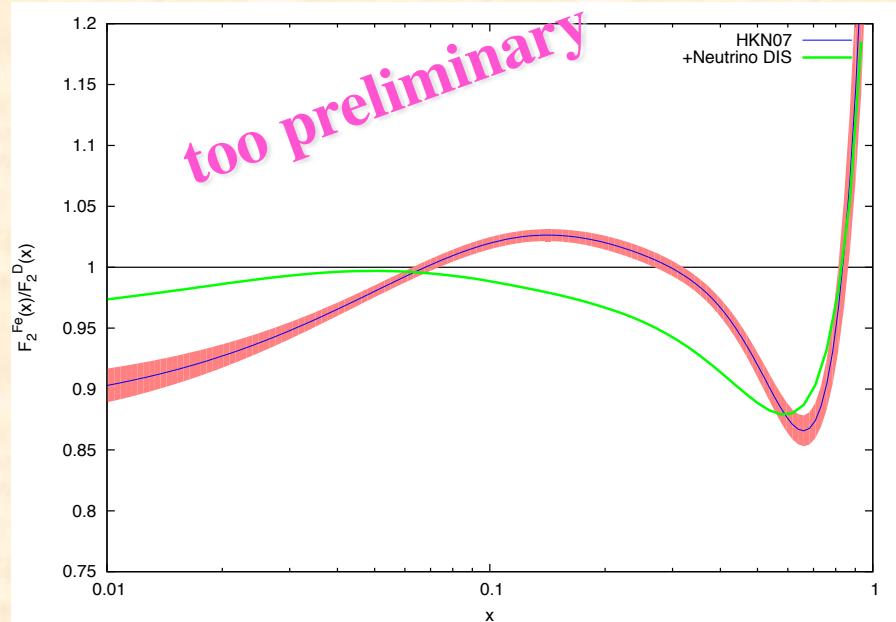
N. Kalantarians: Neutrino DIS \Leftrightarrow Charged DIS

N. Kalantarians at NuInt15
JPS Conf. Proc. 12 (2016) 010028.



According to this analysis, both structure functions
are same except for the small- x region ($x < 0.05$) !?

Our research in progress (M. Hirai, SK, K. Saito)

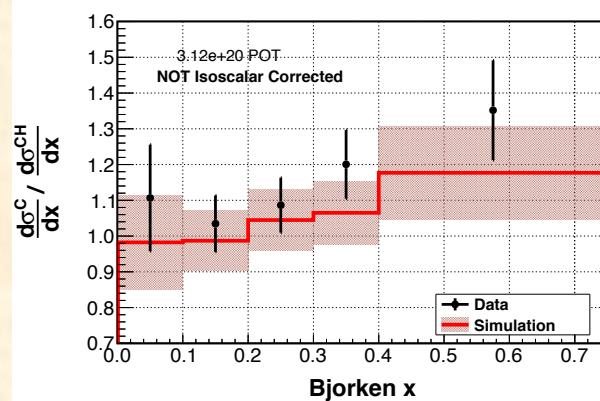


We are getting a similar modification to the nCTEQ one.

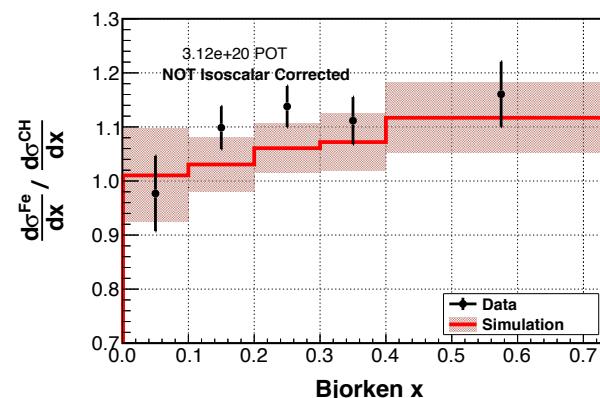
Measurements by Minerva

B. G. Tice *et al.*, PRL 112 (2014) 231801;
J. Mousseau *et al.*, PRD 93 (2016) 071101(R).

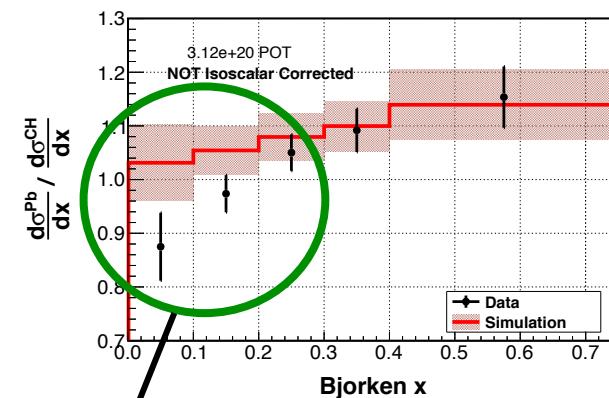
C/CH



Fe/CH



Pb/CH



Different shadowing from charged-lepton case??

Recent analysis by nCTEQ15: data set

K. Kovarik *et al.*, PRD 93 (2016) 085037

Charged-lepton DIS

F_2^A/F_2^D		# data after cuts				χ^2
Observable	Experiment	ID	Ref.	# data		
D	NMC-97	5160 [48]	292	201	247.73	
He/D	Hermes	5156 [49]	182	17	13.45	
	NMC-95,re	5124 [50]	18	12	9.78	
	SLAC-E139	5141 [51]	18	3	1.42	
Li/D	NMC-95	5115 [52]	24	11	6.10	
Be/D	SLAC-E139	5138 [51]	17	3	1.37	
C/D	FNAL-E665-95	5125 [53]	11	3	1.44	
	SLAC-E139	5139 [51]	7	2	1.36	
	EMC-88	5107 [54]	9	9	7.41	
	EMC-90	5110 [55]	9	0	0.00	
	NMC-95	5113 [52]	24	12	8.40	
	NMC-95,re	5114 [50]	18	12	13.29	
N/D	Hermes	5157 [49]	175	19	9.92	
	BCDMS-85	5103 [56]	9	9	4.65	
Al/D	SLAC-E049	5134 [57]	18	0	0.00	
	SLAC-E139	5136 [51]	17	3	1.14	
Ca/D	NMC-95,re	5121 [50]	18	12	11.54	
	FNAL-E665-95	5126 [53]	11	3	0.94	
	SLAC-E139	5140 [51]	7	2	1.63	
	EMC-90	5109 [55]	9	0	0.00	
Fe/D	SLAC-E049	5131 [58]	14	2	0.78	
	SLAC-E139	5132 [51]	23	6	7.76	
	SLAC-E140	5133 [59]	10	0	0.00	
	BCDMS-87	5101 [60]	10	10	5.77	
	BCDMS-85	5102 [56]	6	6	2.56	
Cu/D	EMC-93	5104 [61]	10	9	4.71	
	EMC-93(chariot)	5105 [61]	9	9	4.88	
	EMC-88	5106 [54]	9	9	3.39	
Kr/D	Hermes	5158 [49]	167	12	9.79	
Ag/D	SLAC-E139	5135 [51]	7	2	1.60	
Sr/D	EMC-88	5108 [54]	8	8	17.20	
Xe/D	FNAL-E665-92	5127 [62]	10	2	0.72	
Au/D	SLAC-E139	5137 [51]	18	3	1.74	
Pb/D	FNAL-E665-95	5129 [53]	11	3	1.20	
Total:			1205	414	403.70	

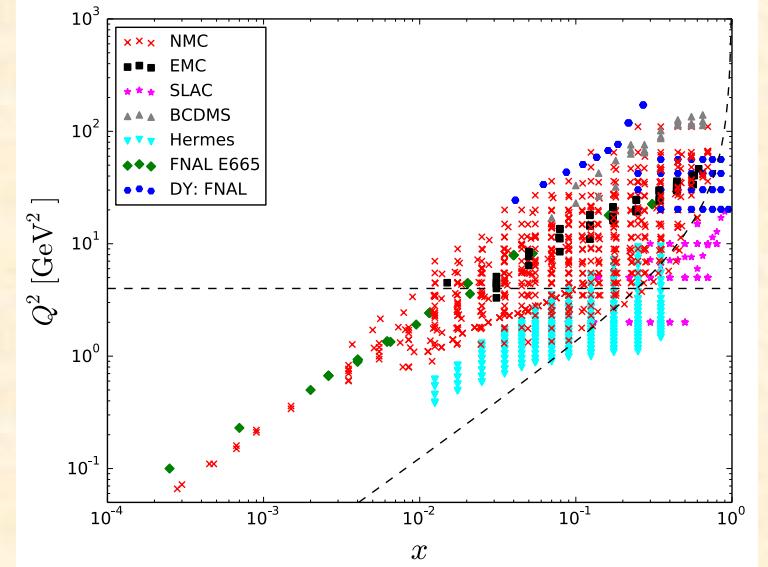
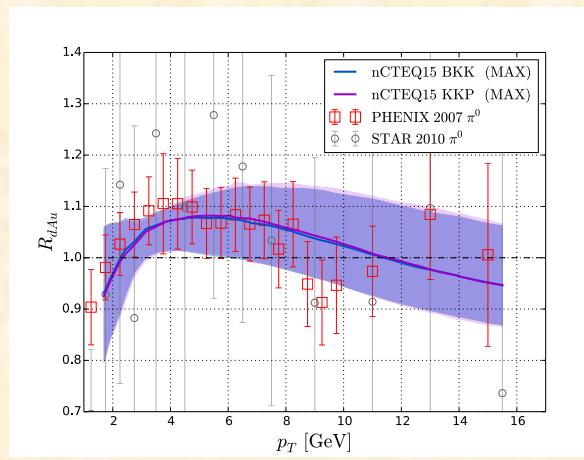
$F_2^A/F_2^{\prime A}$		# data after cuts				χ^2
Observable	Experiment	ID	Ref.	# data	after cuts	
C/Li	NMC-95,re	5123	[50]	25	7	5.56
Ca/Li	NMC-95,re	5122	[50]	25	7	1.11
Be/C	NMC-96	5112	[63]	15	14	4.08
Al/C	NMC-96	5111	[63]	15	14	5.39
Ca/C	NMC-95,re	5120	[50]	25	7	4.32
	NMC-96	5119	[63]	15	14	5.43
Fe/C	NMC-96	5143	[63]	15	14	9.78
Sn/C	NMC-96	5159	[64]	146	111	64.44
Pb/C	NMC-96	5116	[63]	15	14	7.74
Total:				296	202	107.85

Drell-Yan

$\sigma_{DY}^{PA}/\sigma_{DY}^{PA'}$		# data				
Observable	Experiment	ID	Ref.	# data	after cuts	χ^2
C/H2	FNAL-E772-90	5203	[65]	9	9	7.92
Ca/H2	FNAL-E772-90	5204	[65]	9	9	2.73
Fe/H2	FNAL-E772-90	5205	[65]	9	9	3.17
W/H2	FNAL-E772-90	5206	[65]	9	9	7.28
Fe/Be	FNAL-E886-99	5201	[66]	28	28	23.09
W/Be	FNAL-E886-99	5202	[66]	28	28	23.62
Total:				92	92	67.81

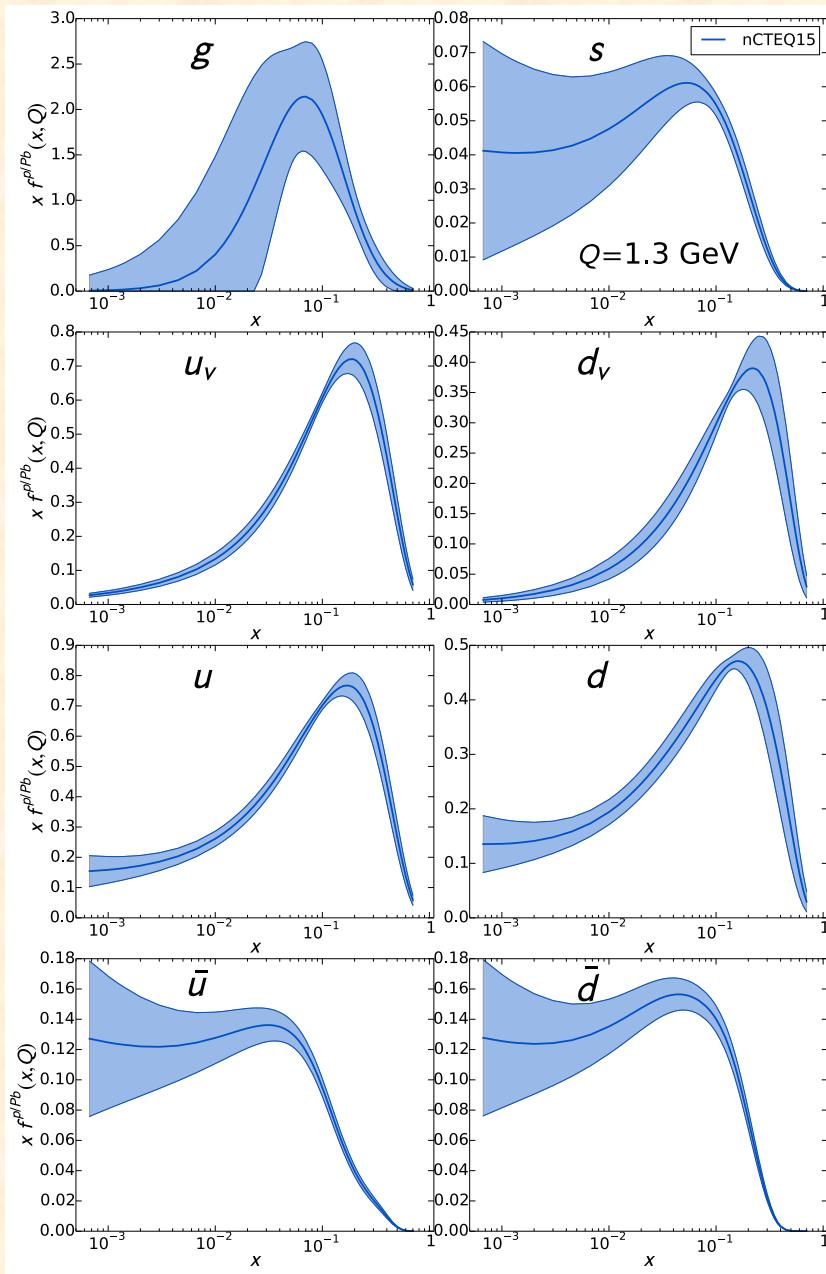
Pion-production in dA

R_{dAu}^π/R_{pp}^π		# data				
Observable	Experiment	ID	Ref.	# data	after cuts	χ^2
dAu/pp	PHENIX	PHENIX	[67]	21	20	6.63
	STAR-2010	STAR	[68]	13	12	1.41
Total:				34	32	8.04

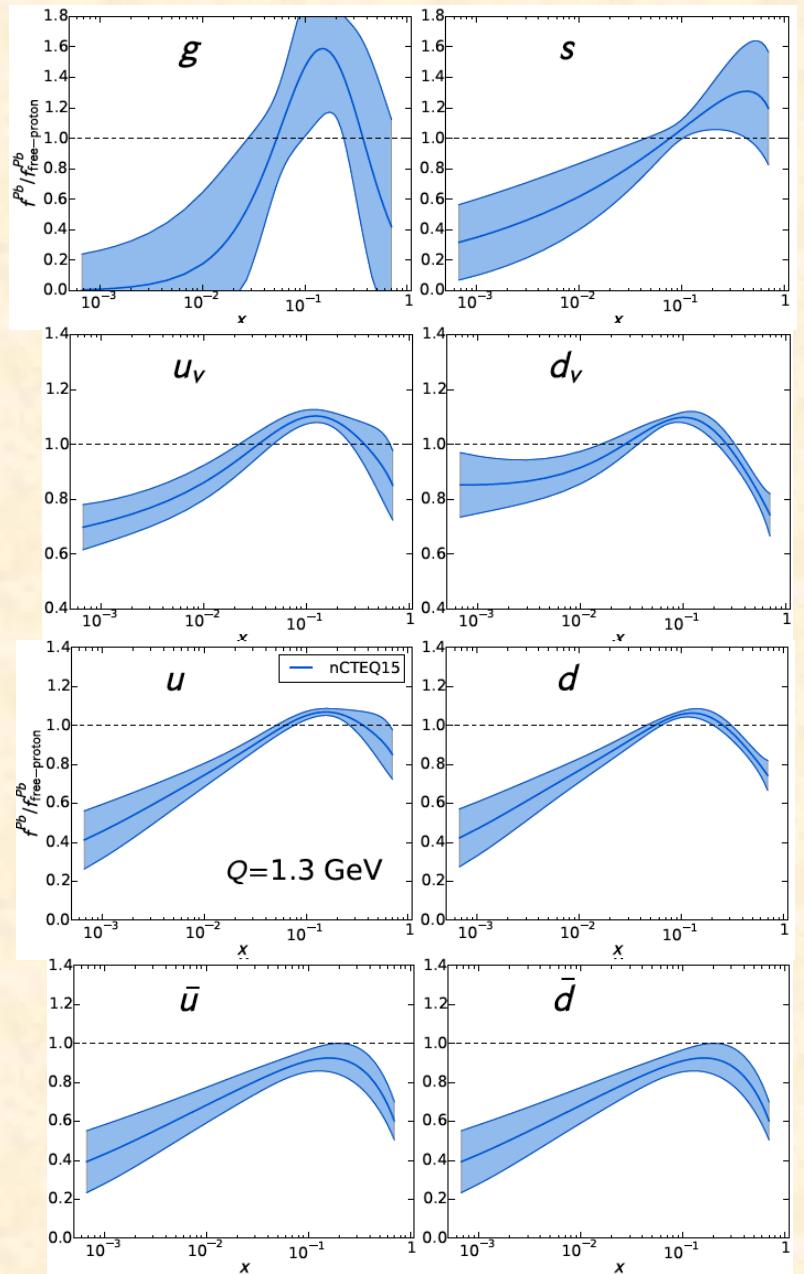


- DIS: $Q > 2 \text{ GeV}$ and $W > 3.5 \text{ GeV}$
- DY: $2 < M < 300 \text{ GeV}$
- π^0 production: $p_T > 1.7 \text{ GeV}$

nCTEQ15

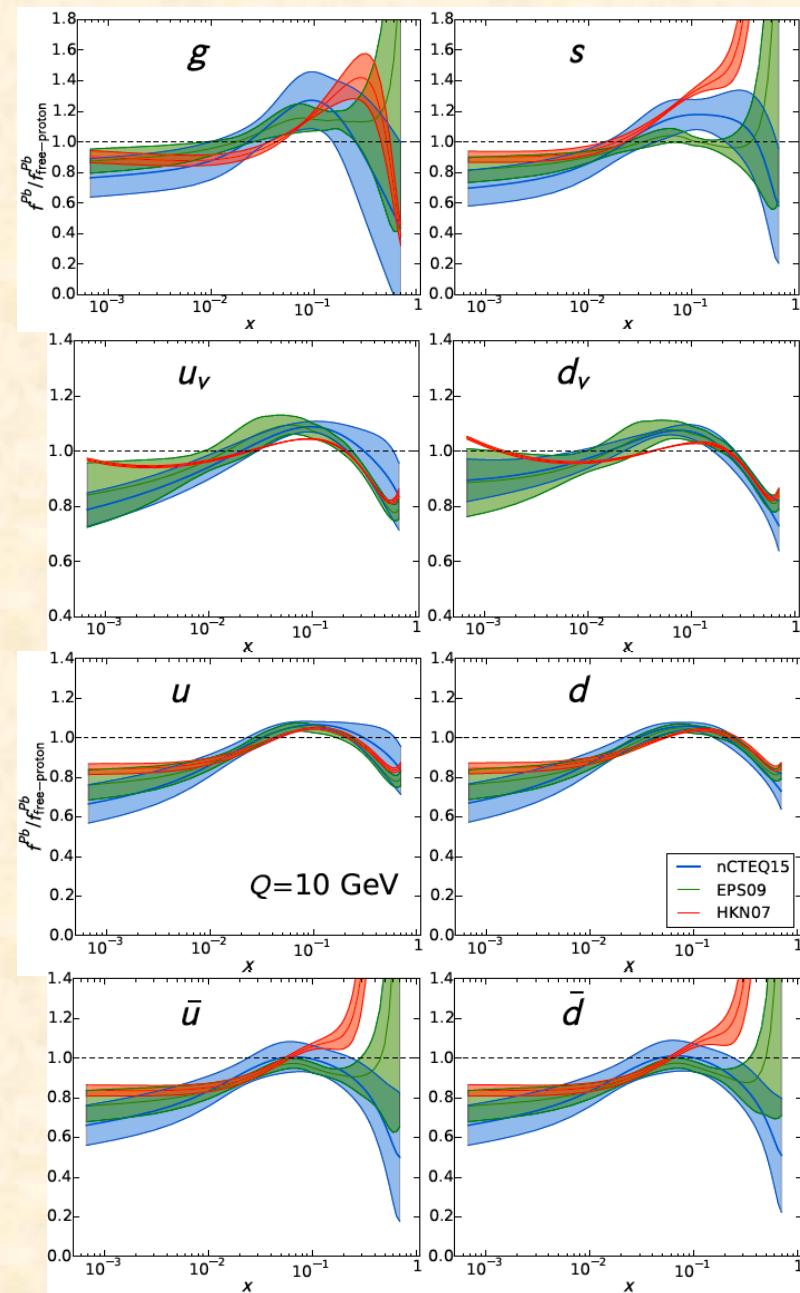
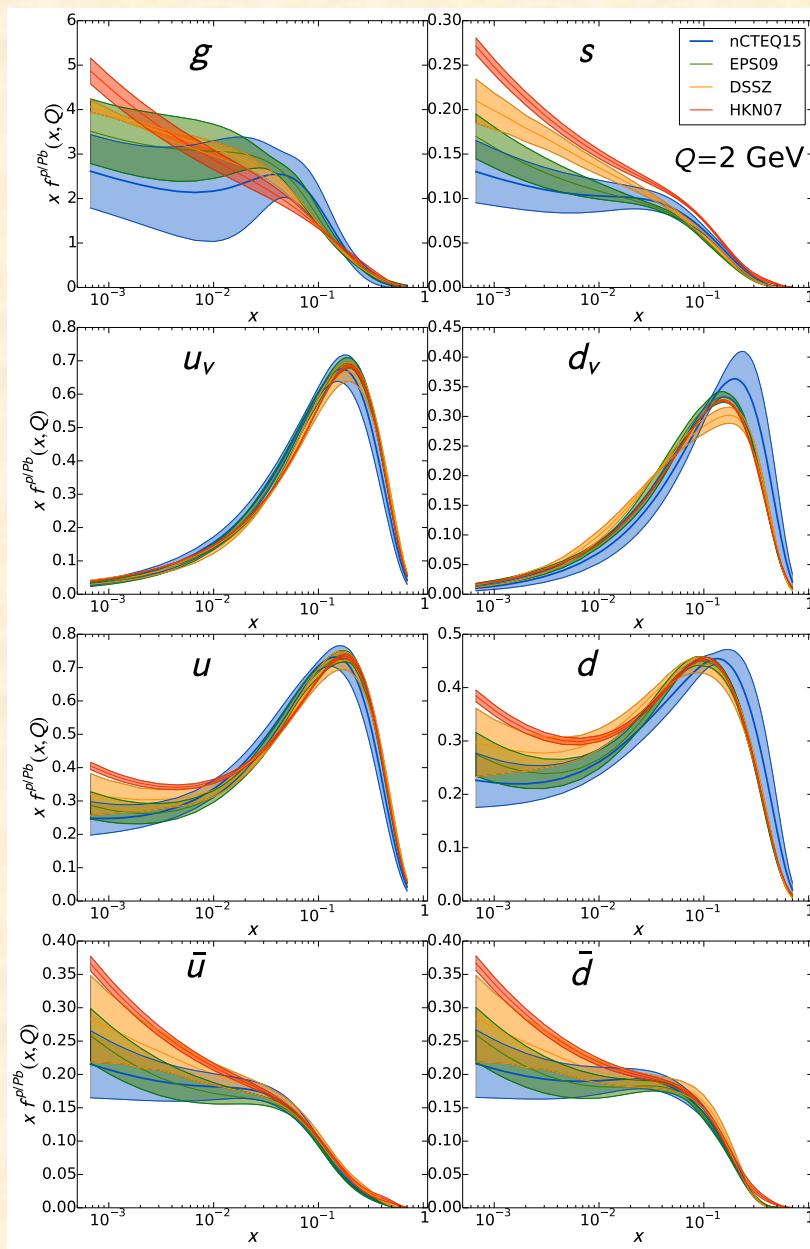


$$Q^2 = (1.3)^2 \text{ GeV}^2$$



nCTEQ15: Comparison with others

$$Q^2 = (2)^2 \text{ GeV}^2$$



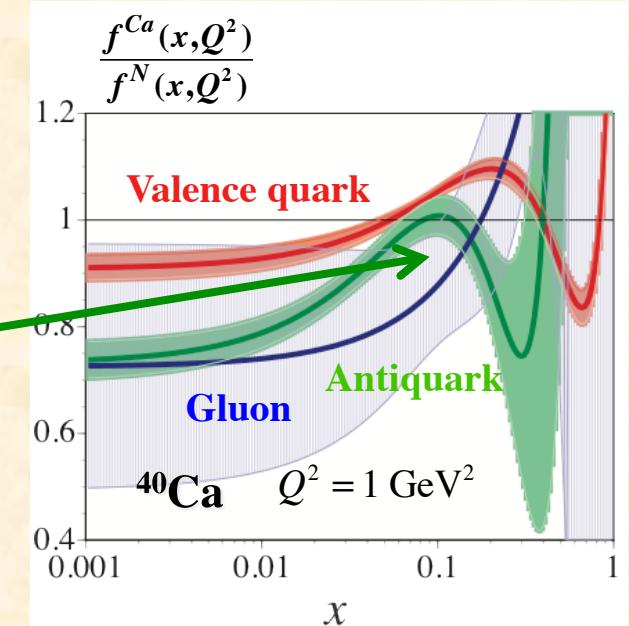
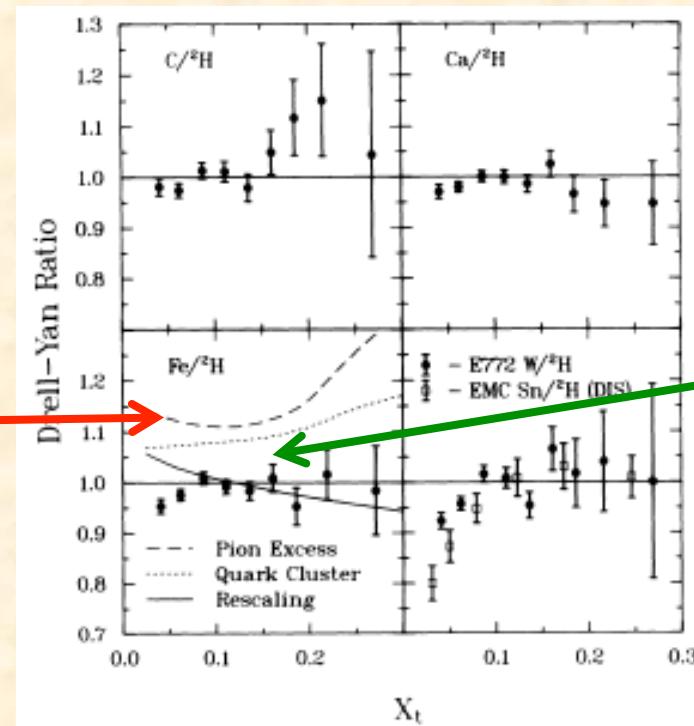
Nuclear antiquark distributions and Drell-Yan

D. M. Alde *et al.*,
PRL. 64, 2479 (1990).

$$\frac{\sigma_{pA}}{\sigma_{pD}} \approx \frac{\bar{q}_A}{\bar{q}_D}$$

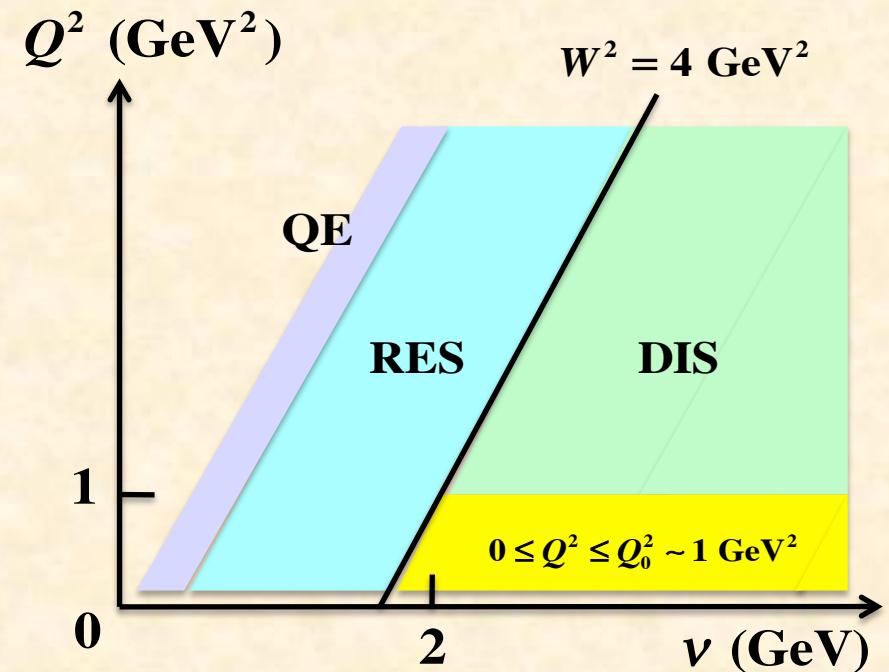
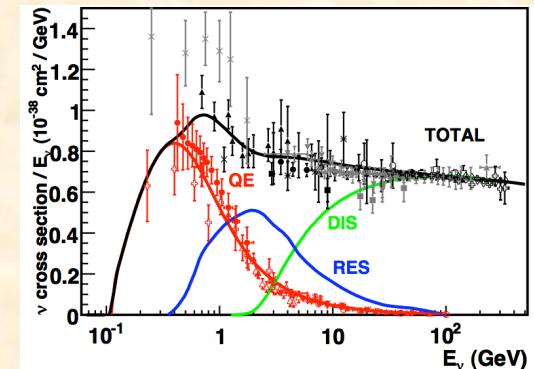
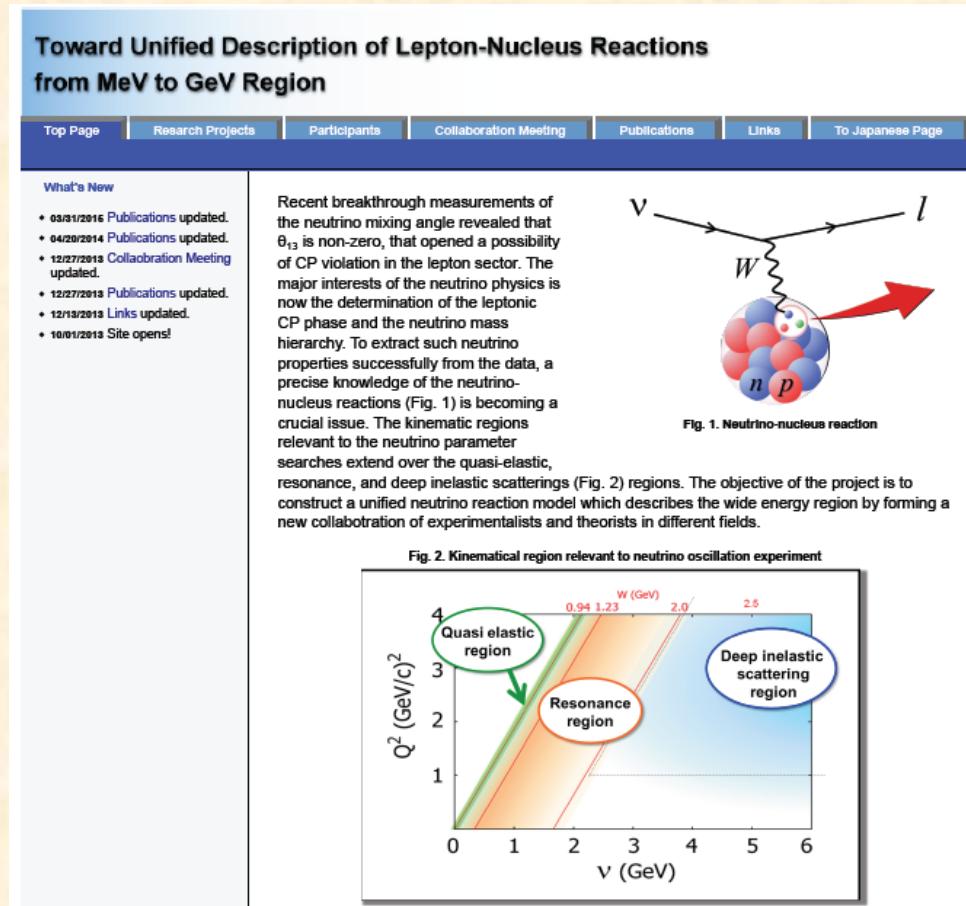
No nuclear effects
from pion contributions

E. L. Berger, F. Coester, R. B. Wiringa,
PRD 29, 398 (1984)



Fermilab-E906 in progress!

ν -interaction collaboration at J-PARC



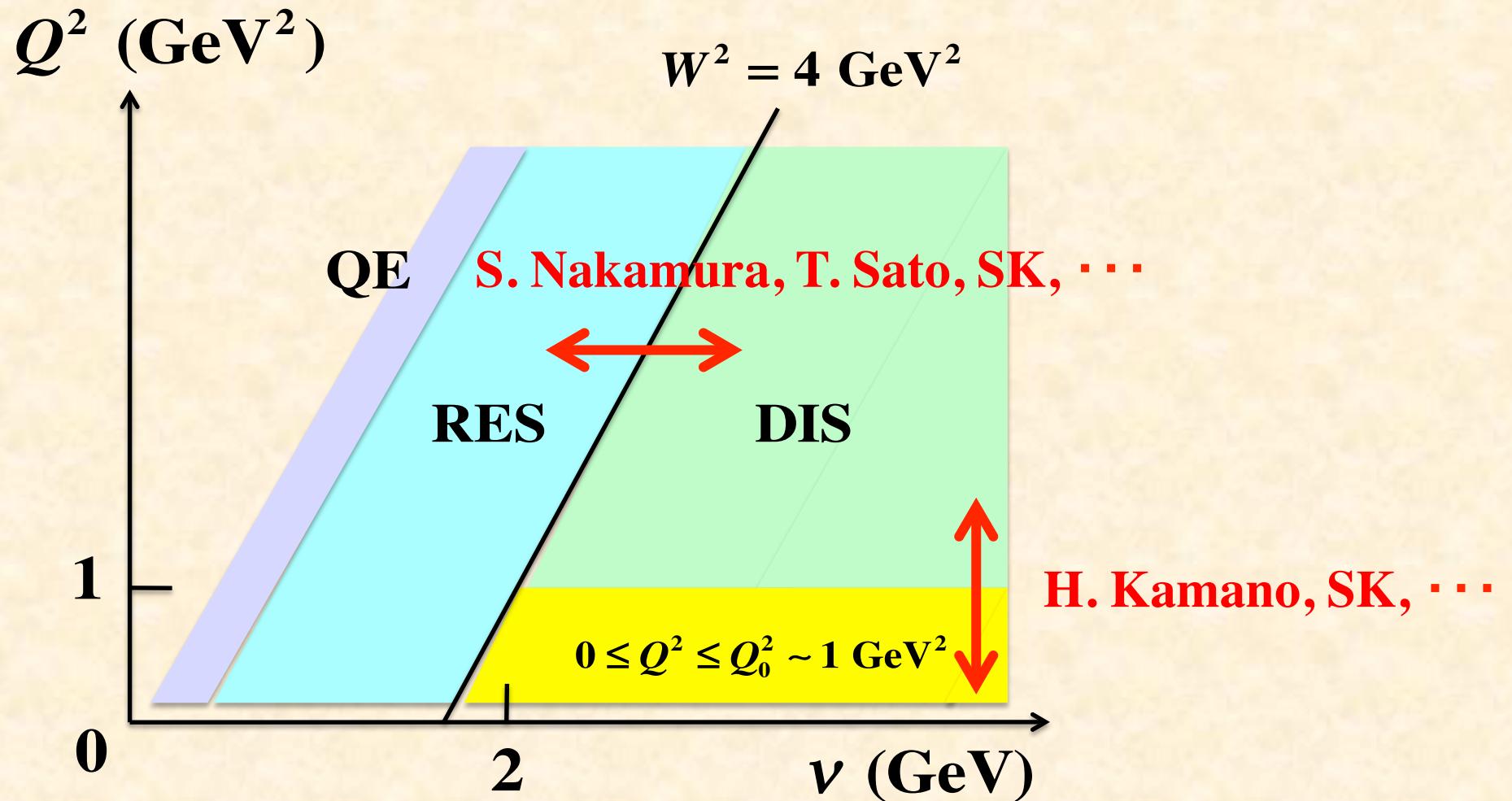
Y. Hayato, M. Hirai, W. Horiuchi, H. Kamano, S. Kumano,
T. Murata, S. Nakamura, K. Saito, M. Sakuda, T. Sato

http://nuint.kek.jp/index_e.html, ν interactions

<http://j-parc-th.kek.jp/>

J-PARC branch, KEK Theory

Research in progress at J-PARC theory



Summary

Global analyses for the nuclear PDFs by using data of charged-lepton, neutrino DIS, pA, AA collisions

Valence quark: reasonably good, in progress at JLab, Minerva for large x

Antiquark: good only at $x = 0.1$, in progress at Fermilab (E906) $x = 0.1 \sim 0.4$.

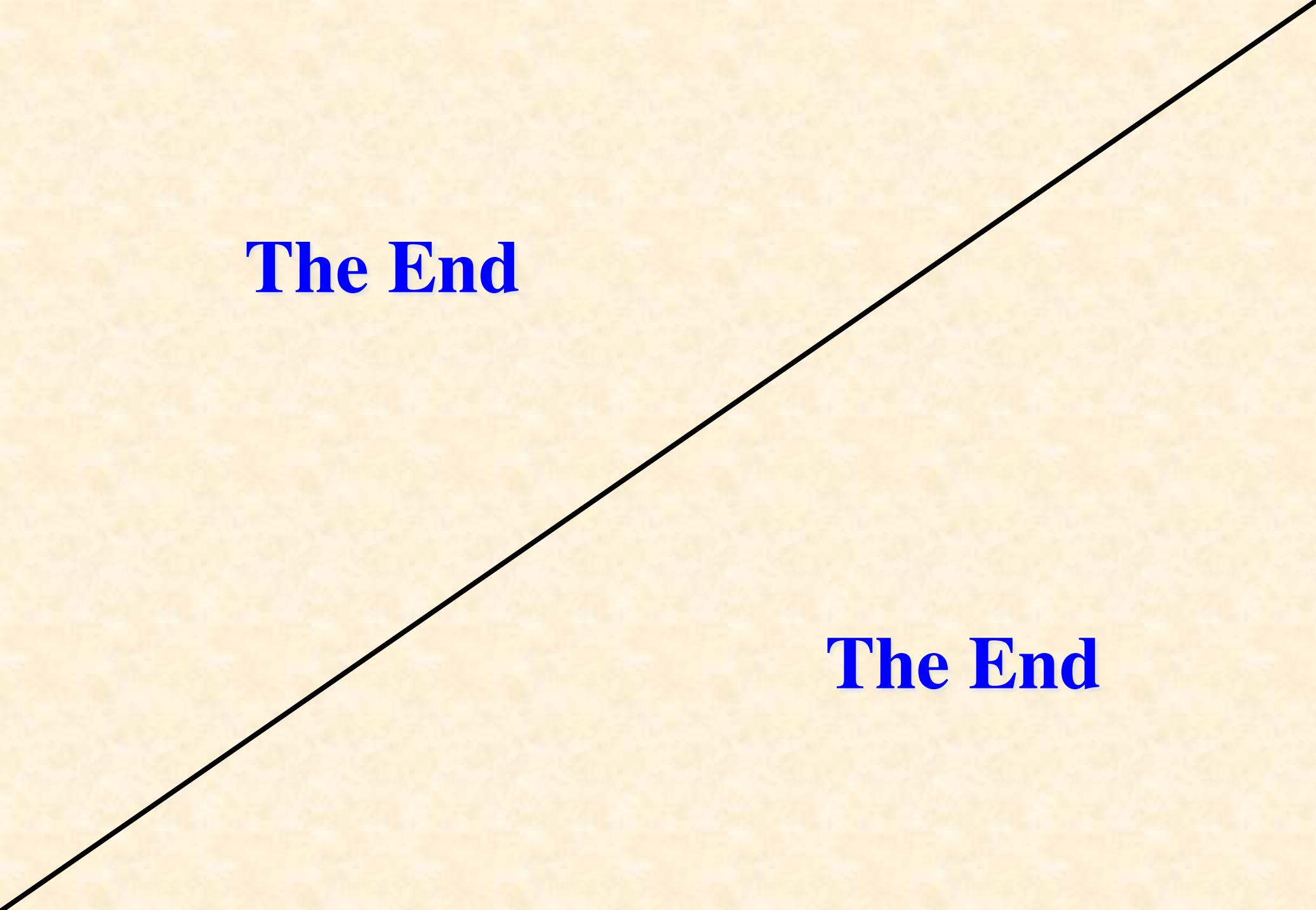
Gluon: large uncertainties in the whole- x region, LHC, RHIC

Issues

- Charged-lepton DIS \Leftrightarrow Neutrino DIS
- Matching with resonance model and $Q^2 \rightarrow 0$ region
- Gluon distributions

New experimental information

- JLab, Fermilab-DY, Minerva, LHC, ...



The End

The End